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THE
DESIGN OF SIMPLE STEEL BRIDGES

THE DESIGN OF SIMPLE STEEL BRIDGES

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PREFACE

In writing this book my idea has been to revise and rearrange my former book, entitled "Manual of Bridge Design," which has been used for some considerable time as a text-book on this subject by the Royal Engineers at the School of Military Engineering, Chatham.

I have endeavoured to depart from the generality of text-books, and to keep ever before the student the practical aim or object for which he is striving.

I have also tried always to illustrate theoretical deductions and conclusions with examples from actual practice.

The book is full of practical examples and illustrations, and contains many details of all sorts connected with bridge work.

I have confined this work to simple bridges. On all hands text-books by celebrated men abound which investigate most thoroughly every conceivable kind of structure or girder, books which go into deep mathematics, and which at their conclusion do not venture to advise any particular course of action under any set circumstances, but leave the engineer to decide for himself.

In this book I have therefore avoided any deep mathematics; a graphic solution of every problem is given where such a solution is useful, and I have in simple language endeavoured to advise or lay down a course of action in almost every case. And although the course of action recommended may sometimes be open to question as to whether it is the best, yet I think it is always good.

The book contains a very simple method of solving questions of bending moments on fixed beams, known as MacDonald's Graphic Solution. This method was originally proved by Major MacDonald, R.E., and has been in use by military engineers for some time past.

The solution of a beam continuous over several supports and loaded with several concentrated loads is also his.

The solution of a beam continuous over several spans and loaded with uniform loads is one that has been adapted from the method given by Mr. Dunn in his book, "Lectures on Reinforced Concrete," and it is to him that I am indebted for the groundwork of this method.

The treatment of rolling loads on a span, although not new, has not hitherto been set out clearly in any text-book, to my knowledge, and the treatment of shear due to rolling loads is an arrangement of my own.

The design of plate and braced girders up to 75 feet span is carefully gone into, and a great deal of detail is given with regard to bridge floors, etc.

I have found it necessary to draw the line at any masonry work, and therefore this book does not go further than the bedstones on which the girder rests.

There are twenty-two tables at the end of the book which it is hoped may be of use in daily work.

Table XVI. gives the uniform equivalent loads for bridge spans for the heaviest locomotive now running on the Great Western Railway. I have seen no other similar tables anywhere except one by Mr. Alexander Ross, M.I.C.E., in the Minutes of the Proceedings of the Institution of Civil Engineers, Supplement to Vol. CLIV., p. 22.

Table XIII., for use in estimating the dead weight of plate girders in rapid design, is compiled from various sources. This table will be found to form a useful and accurate guide.

Tables XIV. and XV. for the percentage additions to live loads for railway and road bridges to compensate for shock, are new, and are based on the practice of various engineers in England.

Table XX. of live loads on road bridges is also new, and is very complete. The authorities who have most kindly supplied the information are mentioned in the table.

Table XXII. gives the maximum sizes to which steel plates and other steel sections can be rolled, and is here given for the first time.

My reasons for adopting graphic solutions so largely are that they are so much simpler as a rule than any other method of arriving at the desired result, and they have the advantage that they show the state of the stresses in the whole beam, or throughout the whole problem at the same time, so that when the stress in any one part has been found the whole problem is completely known.

With regard to the notation employed, it is a pity that at the present time no standard system of notation exists.

Engineers have long felt the want of it, but it has so far been impracticable to arrive at any definite conclusion in this respect.

I have therefore used a notation which was at one time advocated for adoption by the Society of Engineers (Incorporated), and which coincides entirely with the notation now adopted by the Concrete Institute; it will be found to agree with that generally employed in most books of civil engineering.

A list of symbols is given at the beginning of the book.

I am greatly indebted to my friend Captain J. G. Fleming, R.E., who has written the greater part of the first five chapters, and has also helped with much useful information.

It is my regret that his name is not associated with my own in the authorship of this book.

P. O. G. USBORNE, A.I.C.E.,
LATE CAPTAIN, R.E.

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NOTATION

LETTERS.

A	Left end of a beam.
B	Right end of a beam.
C	Centre of a beam.
P, Q, X, Y	Points in any beam or figure.
O	Origin.
OO	Any axis; neutral axis.
$x, y,$	Co-ordinates of any point referred to rectangular axes.

SYMBOLS.

A	Area in square units.
a	Area of a section in square inches.
a_e	Equivalent area in square inches.
α	Constant; Gordon's formula for struts.
b	Breadth in inches.
D	Depth in feet.
D	External diameter.
d	Depth in inches; effective depth of a girder.
d	Internal diameter.
Δ	Deflection in inches.
E	Modulus of elasticity in lbs. or tons.
F	Total force or stress.
F_t	Total tensile force.
F_c	Total compressive force.
f	Stress per square inch. Modulus of rupture divided by factor of safety. Safe stress per square inch against transverse rupture.
f_c	Stress per square inch, in compression.
f_t	,, ,, ,, ,, ,, tension.
f_s	,, ,, ,, ,, ,, shear.
f_b	,, ,, ,, ,, ,, bearing.
H	Height in feet.
h	Height in inches. Least dimension of a strut in inches.
I	Moment of inertia of a plane area in inch^4 units.
k	Radius of gyration in inch units.
L	Length in feet.
l	Length in inches. Span in inches.

NOTATION

M	Any moment.
M_B	Bending moment.
Max. M_B	Maximum M_B produced at any point, or in any span.
M_R	Moment of resistance.
n	Constant in strut formula.
n,n'	Constants in deflection formula.
P	Total pressure.
p	Pressure per square inch.
R	External radius.
	Total resilience.
	Reaction.
R_A,R_B	Reactions at A and B.
r	Internal radius.
S	Shearing force, total.
S_m	Mean shear stress.
s	Intensity of shear stress.
T	Thickness in feet.
t	Thickness in inches.
W	Total load. Any concentrated load.
w	Load per unit length, or unit area.
y	Distance of extreme fibre from the neutral axis in inches.
θ	Angle of any line with the horizontal.
ϕ	Angle of any line with the vertical.
Σ	The sum of things similar to . . .

NOTE.—In the case of stresses, a small letter denotes intensity of stress per unit length or area, while a capital letter denotes a total stress.

THE
DESIGN OF SIMPLE STEEL BRIDGES

CHAPTER I

DEFINITION OF TERMS

Stress is the internal force set up in a structure by the action of external forces, tending to produce a change of form.

Strain is the change of form produced by a stress; it cannot therefore, be denoted in lbs. or other units of force. This change may be only *temporary, i.e.,* when the force is removed, the structure may resume its original form. Or the stress may be so great that the structure does not regain its original form. It is then said to be *permanently* strained. A permanent strain is frequently called a "set."

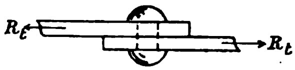

FIG. 1.

There are *three* kinds of *simple* stresses, viz.:—

(1) *Tensile stress,* tending to tear the particles of the substance away from one another.

The fibres of the structure tend to become elongated.

(2) *Compressive stress,* the converse of the above.

(3) *Shearing stress,* tending to make the particles slide past one another by the action of forces in opposite directions. Thus in Fig. 1 the rivet is subject to a shearing stress, tending to cut it in two.

Reversal of stress is alteration of the direction of the stress in the fibres of any material, as from tension to compression and *vice versâ.*

Transverse stress.

In addition to the above simple stresses there are various compound stresses. Of these the most common is *tranverse* *stress*, in which the load, acting transversely to the fibres of the structure, tends to produce compression in one set of fibres, tension in another set, and shearing stress throughout. This is the normal condition of a beam supported at both ends and loaded.

Bearing stress is another stress frequently spoken of as a separate stress; it is really made up of two stresses: (*a*) compression; (*b*) shearing.

As an example, a riveted joint may fail through an excess of bearing stress, it will then be seen that the rivets have cut into the metal forming the joint, the metal in front of the rivet having failed by compression and by shearing along the sides of the slot which tends to form in the material (Fig. 2).

Units of stress. Stress is usually measured in tons, cwts., or lbs. The *intensity of stress* or stress on a unit area is expressed in lbs., cwts., or tons per square foot or square inch.

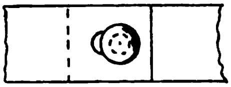

FIG. 2.

Ultimate stress, sometimes called *limiting* or *breaking* stress, is the stress at which rupture occurs.

Proof stress, which is intended to be applied only for a *short time* to test a structure, is the greatest stress that can be applied without causing permanent injury.

Safe working stress is that which can be applied for *any length of time* without fear of injury, and represents therefore the practical unit of strength of the material employed.

It must be remembered in deciding the working stress that all materials suffer in a greater or less degree from "fatigue," which is particularly noticeable in the case of members subject to reversal of stress.

The ultimate stress for any material is found by experiment, and to it the safe working stress bears a certain definite ratio. This ratio is called the *factor of safety*. It varies from 10 to 3.

The factor of safety is largely a matter of judgment and circumstance. Where a steady load has to be borne, such as the wall of a house resting on a rolled steel joist, a smaller factor will suffice, than would be the case if the load were constantly varying as on a railway bridge.

Fatigue is the deterioration produced in the fibres of any material by repeated application and removal, or increase and decrease of the load, or by reversal of stress. It has been found, for instance, that the chain of a crane will sometimes ultimately fail to carry a load which it has lifted previously with safety.

Where fatigue is likely to occur, an increased factor of safety must be given.

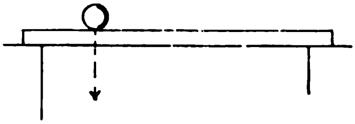

FIG. 3.

Loads.

A load may be distributed in various ways. It may be *concentrated* at a point (Fig. 3), or it may be *uniformly distributed* wholly (Fig. 4), or partially.

The effect upon the beam will vary with the distribution of the load, though the load may be the same.

Loads are commonly spoken of as *dead loads* and *live loads.* A dead load is that which is applied very gradually, and which remains steady. The weight of a structure itself is a dead load. Still water in a tank is a dead load.

A live load is that which is suddenly applied, or is accompanied with shocks or impact. A crowd of people moving on a floor, a railway train crossing a bridge, etc., constitute live loads. If liable to agitation and considerable change of level, water in a tank might be considered a live load.

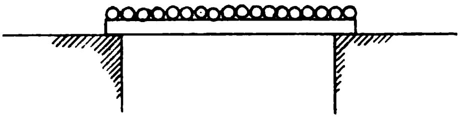

FIG. 4.

A live load produces considerably more stress in a structure than an equal dead load.

In calculating for a live load, it is therefore customary to add to it a percentage which shall cover the extra stress produced by the shock, movement or vibration.

Loads on structures divide themselves naturally into the two heads:—

(*a*) "Dead load," the weight of the structure and any permanent superimposed load.

(*b*) "Live load," which the structure is intended to carry, sometimes called the "accidental load," to which is added for purposes of calculation a percentage to allow for shock, vibration, etc., in order to convert this live load to "equivalent dead load."

Thus the actual load for calculation becomes:

(*a*) "Dead load," plus
(*b*) "Equivalent dead load."

The percentage to be added in each case is a matter for the judgment of the engineer, and each case must be decided on its own merits.

The guiding principles are that moving loads, heavy in proportion to the dead load of the structure, or loads causing much vibration, require a large percentage addition to convert them to equivalent dead load, while loads light compared to the dead load of the structure or loads causing little vibration require a proportionately small percentage addition.

Elasticity.

A material is said to be elastic when it possesses the property of regaining its original shape after having been strained by any kind of stress.

If a bar of elastic material be subjected to a gradually increasing tensile stress, it will be found that the increments of length are in direct proportion to the increments of stress up to a certain point.

This is also the case if the bar be subjected to a compressive stress. This principle is known as Hooke's Law: "Ut tensio, sic vis" as defined by the discoverer in 1676.

Yield Point.

The above law is only of limited application. A point is reached in stressing any material, beyond which the material no longer returns to its original form when the stress is removed, but shows a change in form and character termed "set."

This point is called the "yield point" or "elastic limit" of the material. Some materials have a much greater elasticity than others. Lead, for instance, has no elasticity, while glass is very elastic.

Iron and steel are elastic up to about half their ultimate stress, *i.e.*, in wrought iron about 12 tons and in mild steel about 16 tons per square inch.

Graphically, this can be easily appreciated. Fig. 5 is a graph or curve, in which the ordinates represent the stresses applied to any material, and the abscissæ represent the strains or deformations produced by those stresses.

By Hooke's Law the strains increase proportionately with the stresses, and the graph is a straight line. And if Hooke's Law were universally true, this would always be the case. But in actual fact the material under test will follow this straight line only as far as the yield point, *i.e.*, so long as the stresses and corresponding strains are low and are obedient to Hooke's Law. If at any time the stress were to be removed, the bar would regain its original length.

But at the yield point the curve departs from the straight line.

When the stressing or loading is increased beyond the yield point, the diagram shows that the strain is increasing at a greater rate than assumed by Hooke's Law.

On removal of the stress it will be found that the bar no longer regains its original length; it has assumed what is termed a "permanent set," either extended or compressed.

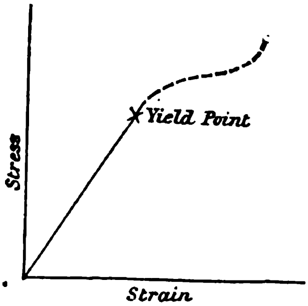

FIG. 5.

The point where a further increase of stress causes a permanent change of form is called the yield point, and if the loading be further increased a rapid change of form takes place as indicated by the curve in Fig. 5.

In deciding therefore on the safe working stress for any material, a high enough factor of safety must be allowed to prevent any possibility of the fibre stress approaching the elastic limit or yield point of the material.

Modulus of Elasticity.

This modulus, usually denoted by E, is the measure of the ratio of stress to strain produced, for any material, so long as the yield point is not passed.

It is a purely theoretical force deduced from the behaviour of test bars of a material under stress, and is that force which will, if tensile, extend a bar of initial length l to double its length, or if compressive, will reduce it to zero.

In practice this result can never be obtained, since the yield point will first be reached, and E is therefore calculated from observations on test bars thus: Let a bar of length l (inches) be subjected to a stress P (tons per square inch). Let l' be the extension produced (inches). Then since $E : l :: P : l'$, the modulus of elasticity, or $E = l \times \frac{P}{l'}$ tons per square inch.

It should be noted that the modulus of elasticity is not necessarily the same in tension as in compression for all materials, *e.g.*, concrete has very varying ratios in its capacity for resisting tension and compression, their proportion being roughly as 1 to 10.

Stiffness.

The stiffness of a material is its capacity for resisting deformation. It depends on the modulus of elasticity of the material and also upon the arrangement and shape of its constituents.

Modulus of Rupture.

When a beam is strained transversely, the material of which it is composed does not exhibit the same strength as it does when subjected to purely direct stress.

The fibres may be ruptured by excessive tension or excessive compression, whichever the material is least able to resist.

The strength per square inch or "modulus of rupture" has been found experimentally by loading to destruction small beams of different material, only 1 square inch in section, placed on supports 12 inches apart and loaded with concentrated loads suspended from their centre.

The modulus of rupture is, therefore, a property deduced experimentally for any material, and is not a scientifically accurate expression of the strength of a material to resist cross-breaking; but it is easily found, and is therefore frequently used.

Graphics.

It is frequently of value to be able to express moments or other measures of force graphically, and this system of graphic expression will be adopted wherever possible.

Such a system enables intermediate values between maximum and minimum values to be obtained by direct measurement without further calculation than that necessary to draw the graphic diagram.

It will be found a simple and expeditious system in solving complicated analytical problems, such as those of fixed and continuous beams, or beams subjected to a series of rolling loads.

It also has the advantage of enabling the stresses at any section to be determined in direction as well as in magnitude.

CHAPTER II

BENDING MOMENTS

SIMPLE BEAMS.

ANY member of a structure subjected to transverse stress is considered as a *Beam*.

To determine the correct size and proportions of the beam it is first necessary to have some measure of the effect of the loading on the beam.

This is called the "Bending moment" or "moment of flexure," and is denoted by the symbol M_B.

The symbol M_B is the general symbol for

BENDING MOMENT.

This bending moment for any given condition of loading will be at a maximum at some one point, and the bending moment at this point or the *Maximum* bending moment is denoted by the symbol *Max.* M_B.

Beams may be maintained in position by different means and can be classified as :—

(*a*) Cantilevers.
(*b*) Supported beams.
(*c*) Fixed beams.
(*d*) Continuous beams.

Bending moment or moment of flexure may be either "sagging" or "hogging."

For purposes of convenience let us call " Sagging " positive and " Hogging " negative; that is, it is assumed for convenience that a load which tends to cause a beam to deflect so that the upper surface becomes concave and the lower surface convex produces a positive bending moment.

As an example, the supported beam in Fig. 6 is subject to a positive bending moment. If, however, the load tend to cause the upper surface to become convex and the lower surface concave, then the bending moment is negative.

So the cantilever shown in Fig. 7 is subject to a negative bending moment.

In cantilevers and supported beams, the bending moments are either negative or positive throughout.

In fixed and continuous beams the bending moments vary from positive to negative.

The rules for finding the bending moment of fixed and continuous beams are therefore more complicated, and will be given later in Chapter III.

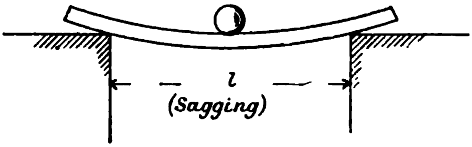

FIG. 6.

Loads may be concentrated or distributed, and the effect of any load introducing a bending moment will vary according to its method of application, that is, whether concentrated or distributed, as well as its point of application.

Concentrated loads, of course, are considered as applied at their point of contact when calculating the bending moment produced.

Distributed loads are considered as applied at their centres of gravity.

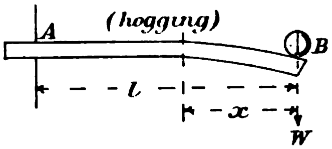

FIG. 7.

Rule for finding the bending moment at any section in *Cantilevers* and simple *Supported Beams.*

Consider either of the portions into which the beam is divided at the given section, and multiply each force acting on that portion by the distance of its point of application from the given section.

The algebraic sum of these moments is the moment required.

Therefore total load $W = wl$.

This load must be considered as acting at its centre of gravity. Consider a section at a point distant x from the outer end. The M_B at this section will be the M_B due to a concentrated load of wx acting at a distance $\frac{x}{2}$ from the section.

Then the $M_B = -wx \times \frac{x}{2} = -\frac{wx^2}{2}$.

This is obviously at a maximum when $x = l$.

So the *Max.* $M_B = -\frac{wl^2}{2}$ or $= -\frac{Wl}{2}$.

To plot this graphically it is necessary to know the nature of the curve of M_B. This curve is a parabola, and its equation is $M_B = y = -\frac{wx^2}{2}$. Then to draw the M_B curve for a uniformly loaded cantilever, it is necessary to draw a semi-parabola of height $\frac{wl^2}{2}$, having its apex at the outer end of the cantilever (Fig. 14).

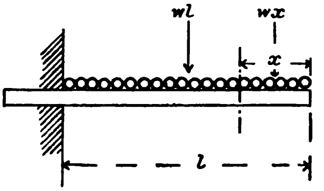

FIG. 13.

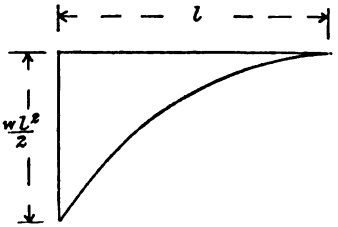

FIG. 14.

In the case of a cantilever carrying one or more concentrated loads, the M_B at any section may be found analytically, or if preferred, graphically by combining the moments for each separate load.

The system of combining graphically or analytically several loads is of great use, and a few examples are given to illustrate the method.

EXAMPLE.

Cantilever loaded as shown in Fig. 15.

Length of cantilever, 15 feet.

Load 160 lbs. per foot, uniformly distributed, over a length of 4 feet, 5 feet from the outer end.

Load of 540 lbs. at the end of the cantilever.

To solve this analytically:—

First consider the concentrated load

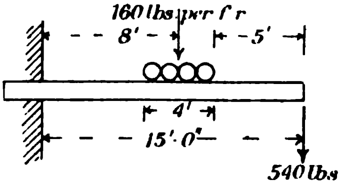

FIG. 15.

$Max.\ M_B = (540 \times 15)$ ft.-lbs.

$= 8{,}100$ ft.-lbs. (at the support).

Then consider the uniformly distributed load. This load can be considered as acting at its centre of gravity.

$$\text{Load} = 160 \times 4 = 640 \text{ lbs.}$$

Acting at a distance of 8 feet from the support.

$Max.\ M_B = (640 \times 8)$ ft.-lbs.

$= 5{,}120$ ft.-lbs. (at the support).

Then the total M_B at the support

$= 8{,}100 + 5{,}120$

$= 13{,}220$ ft.-lbs.

To represent this graphically, it is necessary to make a M_B diagram for each load.

The M_B diagram for the concentrated load is shown on Fig. 16, and requires no comment.

To understand the diagram for the distributed load some explanation is necessary. The diagram is shown in Fig. 17.

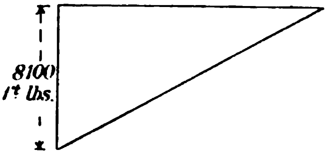

FIG. 16.

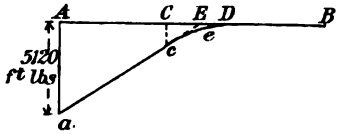

FIG. 17.

First consider the load as concentrated at its centre of gravity, E, distant 8 feet from the support, and for this load considered as concentrated draw a M_B diagram *aEA*, in which $aA = 5{,}120$ ft.-lbs.

Then on *CD* construct a M_B diagram *CDec*, for a cantilever 4 feet long, uniformly loaded with a distributed load of 160 lbs. per foot, the ordinate *Cc* being 1,280 ft.-lbs.,

$$\left(\frac{Wl'}{2} = 1{,}280 \text{ ft.-lbs.}\right).$$

The point *c* will fall upon the line *aE* and the final diagram is *aceDA*.

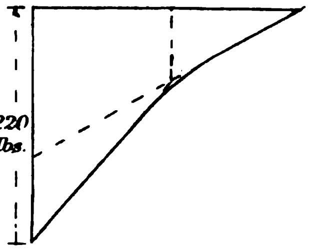

FIG. 18.

Fig. 19 shows this construction on a larger vertical scale.

The two diagrams, Figs. 16 and 17, can now be compounded, and Fig. 18 shows the exact state of M_B at any point in the given beam.

This apparently elaborate graphical process is usually unnecessary, but is of value at times in enabling the exact stresses at any section to be determined. The example given is a simple one, but it is the method which is of importance, not the definite result, which could in this case be more easily obtained analytically.

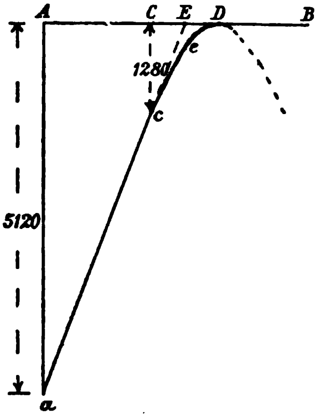

FIG. 19.—Enlargement of Fig. 17.

4. Supported beam.

Single concentrated eccentric load, Fig. 20.

Load = *W* *Span* = *l*.

Load distant *k* from support *A*.

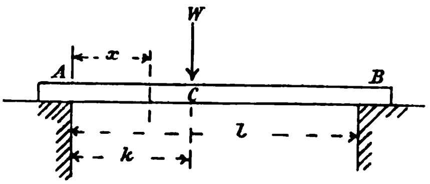

FIG. 20.

By the rule for determining bending moments the M_B at a section distant *x* from the support *A* is equal to the reaction at *A*, multiplied by *x*.

Let this reaction be called R_A.

The reaction is found by taking moments about B, the alegebraic sum of the moments being zero for equilibrium.

Hence $$R_A \times l = W(l - k) = o.$$

or $$R_A = \frac{W(l-k)}{l}.$$

Then the M_B at the section distant x from the support A

$$= R_A \times x$$

$$= W\frac{(l-k)}{l}x.$$

This is a maximum between A and the point of application of the load when x is a maximum or when $x = k$. The *Max.* M_B between A and the point of application of the load is under the load and is equal to

$$W\frac{(l-k)}{l}k.$$

Similarly, the position of the *Max.* M_B between the support B and the load is also under the load and is equal to

$$W\frac{(l-k)}{l}k.$$

Therefore the *Max.* M_B *on the beam* for this condition of loading is under the load and is equal to

$$W\frac{(l-k)}{l}k.$$

To plot this graphically;

The general equation for M_B is of the form—

$$M_B = \frac{W(l-k)}{l}x.$$

Now W, l, and k are constants for any given loading, span, and position of load, so that the curve is a straight line. All that is necessary in order to draw this curve is to plot the M_B to some convenient scale, in the correct position, at right-angles to a line drawn to represent the span.

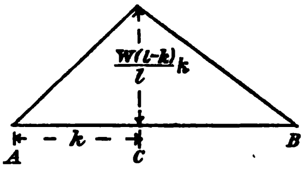

FIG. 21.

This is done in Fig. 21, which is the M_B diagram. The M_B curve is positive, as in a simple supported beam the bending moment is positive from end to end.

If the load is central, $k = \frac{l}{2}$, and

$$Max.\ M_B = W\frac{\left(l - \frac{l}{2}\right)}{l} \times \frac{l}{2}$$

$$= \frac{Wl}{4}.$$

For this load, this central position gives the greatest M_B on the beam.

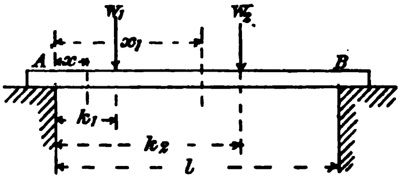

FIG. 22.

5. Supported beam. Two concentrated loads (Fig. 22).

Loads W_1 and W_2 at distances k_1 and k_2 from the support A. Span $= l$.

Consider a section distant x from the support, where x is less than k_1, Fig. 22.

M_B due to $\quad W_1 = \frac{W_1(l - k_1)}{l}x.$

M_B due to $\quad W_2 = \frac{W_2(l - k_2)}{l}x.$

M_B due to W_1 and $W_2 = \left\{\frac{W_1(l - k_1)}{l} + \frac{W_2(l - k_2)}{l}\right\}x.$

This is clearly the equation to a straight line as x is the only variable.

If the M_B is required at a section lying between W_1 and W_2 distant x_1 from the support A,

M_B due to $W_1 = R_{A_1} \times x_1 - W_1(x_1 - k_1)$

M_B due to $W_2 = R_{A_2} \times x_1$

Total $M_B \quad = R_{A_1} \times x_1 - W_1(x_1 - k_1) + R_{A_2} \times x_1$

$$= \frac{W_1(l - k_1)}{l}x_1 - W_1(x_1 - k_1) + \frac{W_2(l - k_2)}{l}x_1,$$

which again is the equation to a straight line as x is the only variable.

Hence the curve is composed of straight lines.

To avoid mathematics the position of $Max.\ M_B$ can best be found graphically.

To construct the diagram for the combined loads draw a M_B diagram for W_1, Fig. 23, and a M_B diagram for W_2, Fig. 24, and combine these two diagrams in Fig. 25.

It is sufficient to add the ordinates of each diagram at the points of application of the loads only.

In the combined figure $ACQB$, Fig. 25, the ordinate OC is the sum of the ordinate OC from Fig. 23 and the ordinate OC from Fig. 24.

And the ordinate PQ is the sum of the ordinates PQ from Fig. 23 and PQ from Fig. 24.

It will be seen from this that the M_B must occur under one or other of the loads.

If graphics have to be avoided, the M_B under each load can be computed and the *Max.* M_B found in this way.

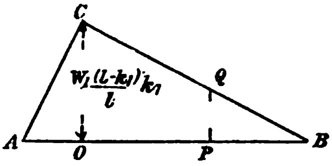

FIG. 23.

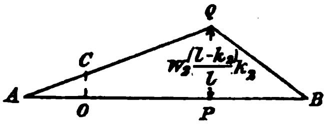

FIG. 24.

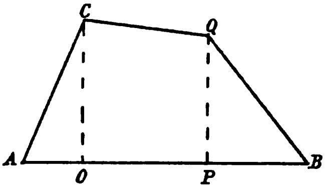

FIG. 25.

EXAMPLE.

This method can be extended to any number of loads. As an example take a beam with 25 feet span, with loads of 5 tons, 3 tons, and 7 tons at distances 7 feet, 15 feet, and 18 feet from the left support as in Fig. 26.

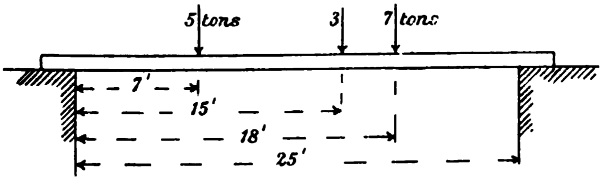

FIG. 26.

$$M_B \text{ due to the 5-ton load} = \frac{5 \times 7 \times 18}{25} \text{ ft.-tons}$$
$$= 25{\cdot}2 \text{ ft.-tons.}$$

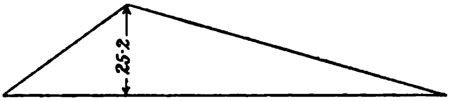

FIG. 27.

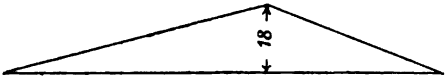

FIG. 28.

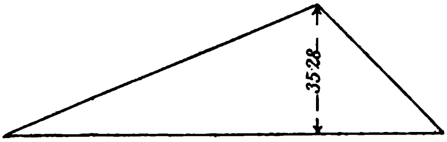

FIG. 29.

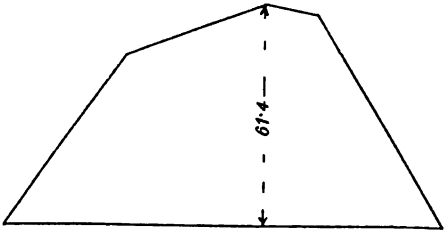

FIG. 30.

This is plotted on Fig. 27.

M_B due to the 3-ton load $= \frac{3 \times 15 \times 10}{25}$ ft.-tons $= 18$ ft.-tons.

This is plotted on Fig. 28.

M_B due to the 7-ton load $= \frac{7 \times 18 \times 7}{25}$ ft.-tons $= 35{\cdot}28$ ft.-tons.

This is plotted in Fig. 29.

These three figures are combined in Fig. 30.

The *Max.* M_B is under the 3-ton load, and is equal to 61·4 ft.-tons.

It must be remembered that the M_B curve can never become concave or have a re-entrant angle, for a supported beam.

6. Uniformly distributed load on a supported beam.
Load $= w$ per unit length, span $= l$.
Then total load $W = wl$.

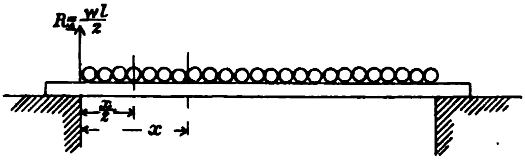

FIG. 31.

Taking a section distant x from support A, Fig. 31,

$$M_B = R_A x - wx\frac{x}{2}.$$

But $$R_A = \frac{wl}{2}.$$

Therefore
$$M_B = \frac{wl}{2}x - \frac{wx^2}{2}$$
$$= \frac{wx}{2}(l - x).$$

This is a maximum when $x = \frac{l}{2}$

and
$$Max.\ M_B = \frac{wl^2}{8},$$

or
$$= \frac{Wl}{8}.$$

To plot this graphically it is necessary to know the nature of curve.

The general equation is:
$$M_B = \frac{wlx}{2} - \frac{wx^2}{2}.$$

This is the equation for a parabola, so that the M_B curve is a parabola of height $\frac{wl^2}{8}$ or $\frac{Wl}{8}$ on a base l, plotted above the line since the M_B is positive (Fig. 32).

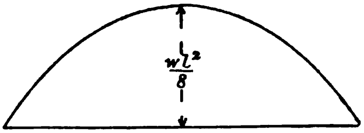

FIG. 32.

7. The combination of various loads, as has already been indicated in the case of concentrated loads, may be applied to all cases of supported beams whatever the loading.

EXAMPLE.

A beam has a span of 30 feet, and is loaded with a concentrated load of 2 tons at 6 feet from the left support and a distributed load of 1 cwt. per foot over the whole span of 30 feet. Also a uniform load of 5 cwts. per foot over a length of 8 feet at a distance of 4 feet from the right support as shown in Fig. 33.

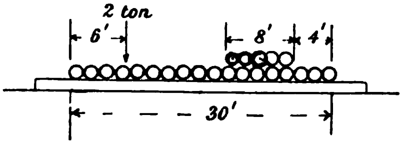

FIG. 33.

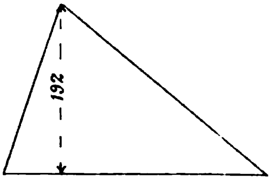

FIG. 34.

First take the concentrated load.

$$Max.\ M_B = \frac{2 \times 6 \times 24}{30} \text{ ft.-tons}$$
$$= \frac{2 \times 6 \times 24 \times 20}{30} \text{ ft.-cwts.}$$
$$= 192 \text{ ft.-cwts.}$$

The diagram for this is shown in Fig. 34.

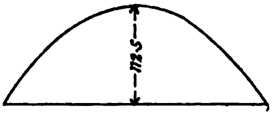

FIG. 35.

Then take the uniformly distributed load of 1 cwt. over the span.

$$Max.\ M_B = \frac{(1 \times 30) \times 30}{8} \text{ ft.-cwts.}$$
$$= 112{\cdot}5 \text{ ft.-cwts.}$$

This is shown in Fig. 35.

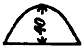

FIG. 36.

Then the distributed load of 5 cwts. per foot.

First draw a M_B diagram for this load on a span of 8 feet.

$$Max.\ M_B = \frac{5 \times 8 \times 8}{8} \text{ ft.-cwts.}$$
$$= 40 \text{ ft.-cwts.}$$

This is shown in Fig. 36.

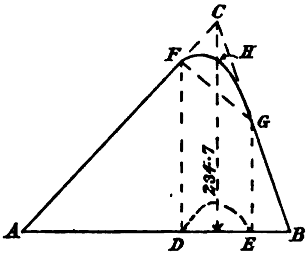

FIG. 37.

Then consider this load of 40 cwts. as a concentrated load acting at its centre of gravity.

$Max.\ M_B$ (on a span of 30 ft.)

$$= \frac{40 \times 8 \times 22}{30} \text{ ft.-cwts.}$$
$$= 234{\cdot}7 \text{ ft.-cwts.}$$

This is shown in Fig. 37.

Now Fig. 36 must be superposed on this.

On this figure plot the points D and E, 12 feet and 4 feet from B respectively.

At D and E erect perpendiculars DF and EG cutting AC and BC in F and G respectively.

Join FG.

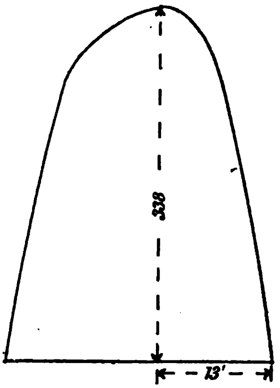

FIG. 38.

On *FG* plot vertically the ordinates of the parabola in Fig. 36, giving the Fig. *AFHGB*.

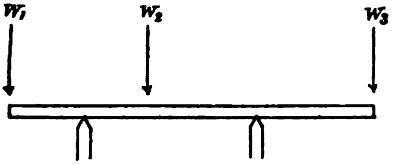

FIG. 39.

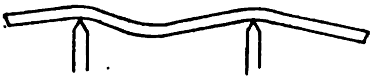

FIG. 40.

This is the M_B diagram for the 5 cwts. per foot load.

Figs. 34, 35, and 37 can now be combined in Fig. 38, which is the M_B diagram for all the loads, and the *Max.* M_B is found by measurement to be 338 ft.-cwts. at a point 13 feet from *B*.

8. Another case which is easily solved graphically is a beam supported on two supports which are not at the extremities of the beam, as in Fig. 39.

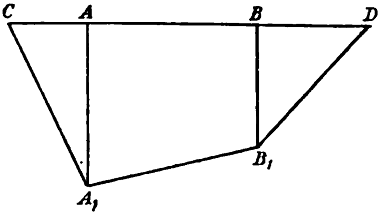

FIG. 41.

This is a beam made up of two cantilevers and a supported beam, and will tend to bend as shown in Fig. 40.

Draw separately M_B diagrams for the two cantilever portions of the beam, neglecting entirely the load on the centre portion, viz., CAA_1 and DBB_1 (Fig. 41).

Join A_1 B_1. Then CA_1B_1D is the M_B diagram for the beam without any load on the centre portion, and the ordinates between *A* and *B* give the values of negative M_B between *A* and *B* caused by the loads on the cantilever portions.

Now draw a M_B diagram for the central portion of the beam with its load treated as supported only, neglecting the cantilever portions (Fig. 42).

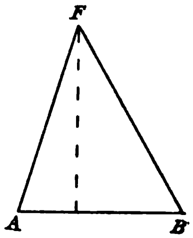

FIG. 42.

The ordinates of this diagram represent the M_B on the central portion of the beam due to its own load only.

These two diagrams, 41 and 42, must now be compounded, having due regard to the sign of the ordinates, those in Fig. 41 being negative and those in Fig. 42 being positive.

The two diagrams are compounded in Fig. 43.

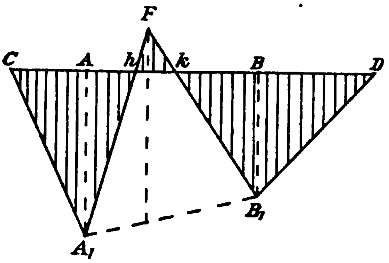

FIG. 43.

It will be seen that the M_B changes from negative to positive at two points, *h* and *k*, in Fig. 43.

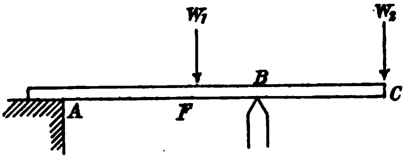

FIG. 44.

These are points of no bending moment, and as the bending changes from hogging to sagging at these points they are called "*points of contraflexure.*"

These points of contraflexure are of great importance, as will be seen in dealing with fixed beams.

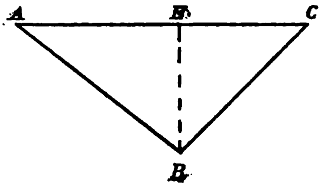

FIG. 45.

9. Another case of a similar nature is that of a beam fastened at one end by a hinge or similar fitting, and strained over a support as shown in Fig. 44.

Draw diagrams for the two portions of the beam treated separately, Figs. 45 and 46.

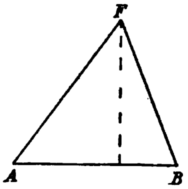

FIG. 46.

Fig. 45 shows the bending moment throughout the beam due to the cantilever portion, and Fig. 46 shows that due to the supported portions only.

Combine these two diagrams, 45 and 46, and obtain the diagram shown in Fig. 47.

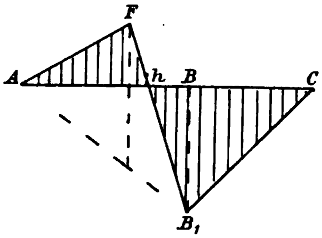

FIG. 47.

All bending moments are measured vertically from AC, h being the point of contraflexure.

CHAPTER III

MOMENTS OF FLEXURE—FIXED AND CONTINUOUS BEAMS

FIXED BEAMS.

On fixed beams the load produces a very different combination of bending moments to that on supported beams.

The ends of the beam being unable to move, portions of the beam at each end act as cantilevers, and the centre portion of the beam acts as a supported beam. The cantilever portions of the beam are subjected to hogging stresses, or negative bending moment, and the central portion is subjected to sagging or positive bending moment. Fig. 48 shows diagrammatically the tendency of a fixed beam to bend when loaded. There can be no bending moment where the hogging stress changes to sagging.

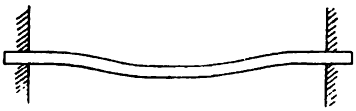

Fig. 48.—Fixed beam.

The positions of these changes of stress, or points where the $M_B = 0$ are called the "*points of contraflexure.*"

The bending moment of the central portion is the bending moment due to the load on a span equal to the distance between the points of contraflexure; and the bending moment in each cantilever at each end can be found from the dead weight or reaction due to the load on the centre portion of the beam between the points of contraflexure, with the addition of any bending moment due to the load on the cantilever portion itself.

If we can determine the position of these points the solution of the beam follows easily.

Problems of this nature are most easily solved by graphics, and this is the method that will be adopted here, the solutions being extremely simple and rapid.

Now it can be proved that the bending moment at either point of fixation is equal to what would be the *Max.* M_B on the beam, if treated as supported only, multiplied by the ratio of the distance between the other point of fixation and the load, to the span.

One concentrated load.

Suppose the beam to be loaded with one concentrated load.

Then drawing the M_B diagram AFB for the beam, Fig. 49, as if supported only, the M_B at A for this load when the beam is fixed is: $CF\frac{CB}{AB}$

Similarly the M_B at B is: $CF\frac{AC}{AB}$

(The proof of this rule will not be entered upon here).

These values AK and BL for the M_B at A and B can be plotted as shown in Fig. 49.

If K and L are joined, cutting AF and FB in D and E respectively, the complete diagram is obtained for the beam when fixed. The line KL is called the "datum line" or "closing line."

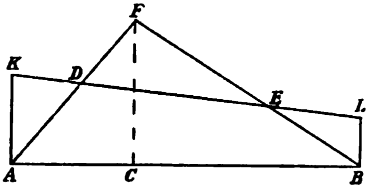

FIG. 49.

All M_B ordinates are measured vertically from KL.

It will be noticed on Fig. 49 that both the bending moments on the cantilever portions are negative, when measured from KL.

The curve of M_B being $KADFEBL$, and the ordinates being measured from the line KL, it is clear that all ordinates from K to D are measured downwards and are negative; similarly from E to L.

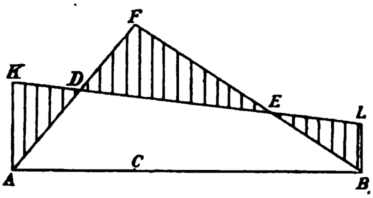

FIG. 50.

From D to E all ordinates are measured upwards, and are therefore positive.

The M_B at these points D and E, where KL cuts the M_B curve is $= 0$, and D and E are the points of contraflexure.

Fig. 50 shows the actual area enclosed by the datum line and the M_B curve shaded.

This shaded area is the portion of the diagram on which the ordinates are measured to determine the M_B at any required point.

The practical graphical construction is as follows:—

Draw AFB, the M_B diagram for the beam as if supported only, Fig. 51; through A and B draw verticals.

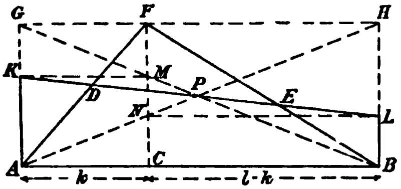

FIG. 51.

Through F draw GFH parallel to AB, cutting the vertical at A in G and the vertical at B in H.

Join GB, HA, cutting FC in M and N.

Through M draw MK parallel to AB cutting AG in K, and through N draw NL parallel to AB cutting BH in L.

Join KL cutting AF in D and BF in E.

$KDEL$ is the "datum" or "closing line," D and E are the points of contraflexure, and the diagram is complete.

PROOF.

AK must be shown to be equal to $\dfrac{CF \times CB}{AB}$.

Now $AK = MC$ by construction; also $AG = FC$.

Then from the similar triangles AGB, CMB

$$CM : CB :: AG : AB.$$

$$CM = \frac{CB \times AG}{AB} = \frac{CB \times CF}{AB},$$

or
$$AK = \frac{CF \times CB}{AB},$$

similarly
$$BL = \frac{CF \times AC}{AB}.$$

Also the points of contraflexure must lie on the line KL, and the co-ordinates of the point of contraflexure D can be proved to be $\dfrac{kl}{2k+l}$ and $\dfrac{Wk(l-k)}{2k+l}$ referred to A as origin and AB and AG as axes.

In Fig. 51a
$$\frac{AK}{FP} = \frac{KD}{DP} = \frac{AO}{OC}$$

also $$AO + OC = AC.$$

$$FP = FM + MP.$$

$$\frac{MP}{KG} = \frac{PR}{KR} = \frac{SC}{SA},$$

also $KG = \dfrac{CF \times AC}{AB}.$

$$\therefore MP = \frac{CF \times AC \times SC}{AB \times SA},$$

also $FM = \dfrac{CF \times AC}{AB}.$

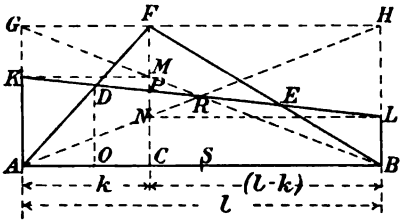

FIG. 51*a*.

Then $FP = \dfrac{CF \times AC}{AB} + \dfrac{CF \times AC}{AB} \times \dfrac{CS}{SA} = \dfrac{CF \times AC}{AB} \left[\dfrac{SA + CS}{SA}\right].$

Then since $$\frac{OC}{AO} = \frac{FP}{AK},$$

$$\therefore \frac{OC}{AO} = \frac{\dfrac{CF \times AC}{AB}\left[\dfrac{SA + CS}{SA}\right]}{\dfrac{CF \times CB}{AB}},$$

$$\therefore \frac{OC}{AO} = \frac{AC}{CB}\left[\frac{SA + CS}{SA}\right]$$

$$= \frac{k}{l-k}\left[\frac{\frac{l}{2} + \frac{l}{2} - k}{\frac{l}{2}}\right]$$

$$= \frac{2k}{l}.$$

$$\therefore OC = \frac{2k}{l} AO,$$

But $$AO + OC = k.$$

$$\therefore AO + \frac{2k}{l} AO = k.$$

$$AO = \frac{kl}{2k + l}.$$

also $$\frac{OD}{OA} = \frac{CF}{AC}.$$

$$\therefore OD = \frac{kl}{2k+l} \times \frac{Wk(l-k)}{lk},$$
$$= \frac{Wk(l-k)}{2k+l}.$$

Then the point D found by construction is the correct point of contraflexure. Similarly E can be proved to be a correct point of contraflexure, and therefore $KDEL$ is the correct datum.

A particular case of the single concentrated load is that of a *central* concentrated load; the *Max.* M_B is then $\frac{Wl}{8}$ at the centre or at either end.

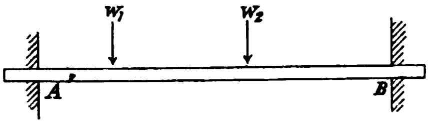

FIG. 52.

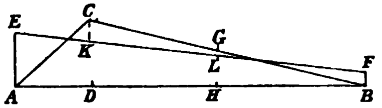

FIG. 53.

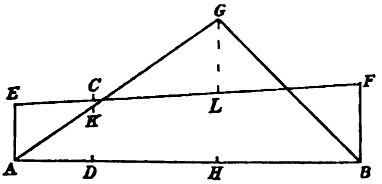

FIG. 54.

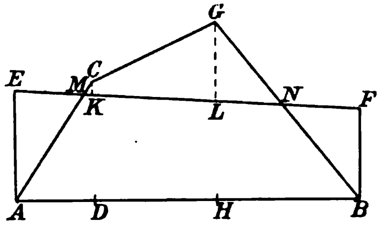

FIG. 55.

Two concentrated loads.

With two concentrated loads (Fig. 52), the simplest plan is to draw the M_B diagram for each load on the fixed span, as above, and then combine these two diagrams.

Draw the diagram shown in Fig. 53 for the load W_1, and the diagram shown in Fig. 54 for the load W_2.

Then combine these two diagrams.

In Fig. 55, draw AE equal to the sum of AE from Fig. 53 and AE from Fig. 54; and also BF in Fig. 55 equal to the sum of BF from Fig. 53 and BF from Fig. 54.

Then in Fig. 55 join EF.

EF is the new datum.

The ordinates of the principal points on the new combined curve can now be drawn from the datum line having due regard to sign.

At D the ordinate of the new curve is the algebraic sum of the ordinates CK from Fig. 53, and CK from the Fig. 54 measured from the datum EF.

It will be observed that in Fig. 54 CK is negative, and is also smaller than CK in Fig. 53. Then CK on the combined figure is equal to the difference between CK from Fig. 53 and CK from Fig. 54, and is positive.

Therefore on Fig. 55 CK is drawn upwards from the datum line, and C is a point on the required curve.

Similarly at H on the combined figure, LG is equal to the sum of the ordinates LG from Fig. 53 and LG from Fig. 54, measured from the datum EF; in this case both are positive, and LG in Fig. 55 is drawn upwards from the datum line.

On the combined figure join AC, CG, GB, and the figure is complete. The M_B diagram is $AEMCGNFB$, and M and N are the points of contraflexure, the M_B being negative from E to M and N to F, and positive from M to N.

Any number of loads can be dealt with in the same way, by drawing a diagram for each load separately and then combining them into one.

Uniform distributed load.

For a distributed load the construction is as follows:

Draw a M_B diagram for the beam as if supported only, Fig. 56.

Draw AE and BD verticals at A and B to meet the tangent to the vertex of the parabola at E and D.

Join CB and AD, cutting at F.

Through F draw KFL parallel to AB.

KL is the datum cutting the curve at G and H, which are the points of contraflexure.

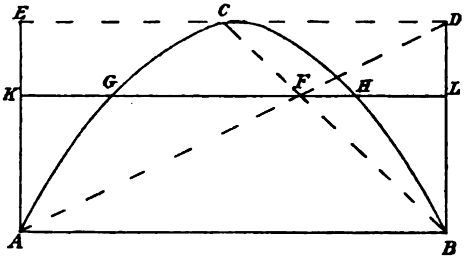

Fig. 56.

It can be proved that BL and AK are each two-thirds of the height of the parabola.

In practice it is generally simpler to take advantage of this fact and plot the datum at once without construction.

The height of the parabola being $\frac{wl^2}{8}$ or $\frac{Wl}{8}$

AK and BL are each equal to $\frac{wl^2}{8} \times \frac{2}{3} = \frac{wl^2}{12}$ or $\frac{Wl}{12}$

and the M_B at the centre is equal to $\frac{wl^2}{24}$ or $\frac{Wl}{24}$.

The M_B diagram is shown shaded on Fig. 57.

This diagram can be combined with any other form of loading, as is shown in the example.

EXAMPLE.

A beam, span 25 feet, fixed at both ends, is loaded with a uniform load of 2 cwts. per foot, and three concentrated loads of 1 ton, 1½ tons, and ¾ tons, 6, 13, and 20 feet from the end. First treat each of these loads separately.

Draw the M_B diagram for the uniformly distributed load (Fig. 57).

The height of the parabola

$$= \frac{Wl}{8} = \frac{2 \times 25 \times 25}{8} \text{ ft.-cwts.}$$

$$= 156\tfrac{1}{4} \text{ ft.-cwts.}$$

M_B at the ends is

$$= \frac{2 \times 25 \times 25}{12} \text{ ft.-cwts.}$$

$$= 104\tfrac{1}{6} \text{ ft.-cwts.}$$

This gives the height of the datum line, and Fig. 57 is complete, when the datum line has been plotted.

Draw the M_B diagram for the 1-ton load (Fig. 58).

Ends supported,

$$Max. M_B = \frac{20 \times 6 \times 19}{25} \text{ ft.-cwts.}$$

$$= 91\tfrac{1}{5} \text{ ft.-cwts.}$$

Complete the diagram for "ends fixed."

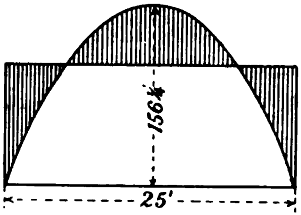

FIG. 57.

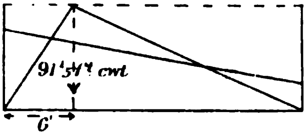

FIG. 58.

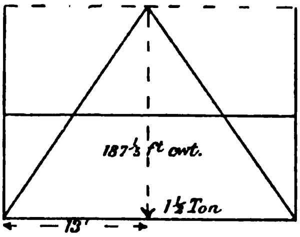

FIG. 59.

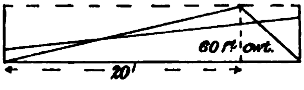

FIG. 60.

Draw the M_B diagram for the $1\frac{1}{2}$-ton load (Fig. 59).

Ends supported,

$$Max.\ M_B = \frac{30 \times 13 \times 12}{25} \text{ ft.-cwts.}$$

$$= 187\tfrac{1}{5} \text{ ft.-cwts.}$$

Complete the diagram for ends fixed.

Draw the M_B diagram for the $\frac{3}{4}$-ton load (Fig. 60).

Ends supported,

$$Max.\ M_B = \frac{15 \times 20 \times 5}{25} \text{ ft.-cwts.}$$

$$= 60 \text{ ft.-cwts.}$$

Complete the diagram for ends fixed.

Combining these four diagrams by adding ordinates at (say) every $2\frac{1}{2}$ feet across the span, we obtain the diagram shown on Fig. 61, from which we see that the $Max.\ M_B$ is at K and is equal to 270 ft.-cwts. (by measurement).

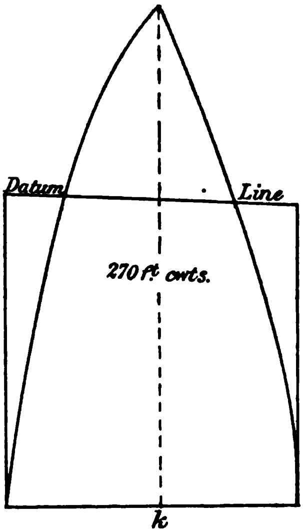

FIG. 61.

GENERAL REMARKS.

In actual practice it is very difficult to obtain complete fixation; materials perish, the fastenings may not be sound, or may give.

For these reasons it is frequently desirable to consider a beam as only partially fixed. The exact state of affairs is usually very indeterminate, being some intermediate stage between fixed and supported.

To obtain an exact knowledge of the stresses caused by partial fixation is very complicated and is not here attempted. It is sufficient for ordinary cases to find the $Max.\ M_B$ where the beam is considered as "half fixed." This can be found graphically from the "fixed" diagram.

Half fixed beam.

Bisect the ordinates at the ends of a M_B diagram and join these points of bisection; this line is the "half fixed" datum.

Fig. 62 represents the M_B diagram for a fixed beam, AD and DE being the M_B ordinates at the ends.

Bisect AD in F and DE in G; join FG. Then FG is the half fixed datum, and the M_B diagram is shown shaded.

It should be noted that the M_B at the middle of the beam is greatest when the beam is only half fixed, but that at the ends is greatest when the beam is wholly fixed.

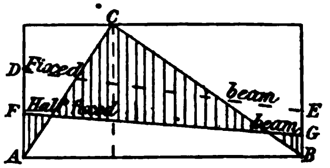

FIG. 62.

For purpose of calculation when there is doubt as to the fixation, it is necessary to draw the M_B diagram, and to insert the datum lines, both for " fixed ends " and for " half fixed ends."

The *Max.* M_B for any point must then be selected from both of these diagrams, and must then be taken as the M_B for calculation.

The M_B values for a few loadings are given in Table XI.

Fixed at one end only.

In the case of a beam fixed at one end, and supported only at the other, the same process is applied.

First draw the M_B diagram for the beam as if fixed at both ends; then add to the M_B at the fixed end, ½ of the ordinate at the supported end, and join the point so found to the support at the supported end.

If Fig. 63 is the M_B curve for a beam AB, ends fixed, and B is really supported and not fixed, add to AE the value of half BF obtaining a point G (Fig. 64).

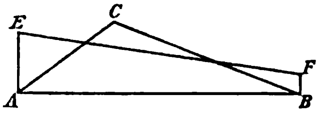

FIG. 63.

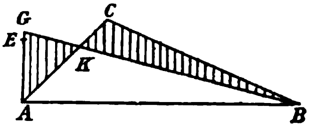

FIG. 64.

Join G to B; GB is then the new datum line, and the completed figure is as Fig. 64.

The M_B at B, ends fixed, is distributed between B and A, one half increasing the sagging or positive moment, between the point of contraflexure K, and B, and the other half increasing the hogging or negative M_B between K and G.

Subsidence ; fixed both ends.

A common case occurs when one end of a fixed beam subsides. The stresses in this beam are by no means the same as before the subsidence, particularly if the fixation remain complete.

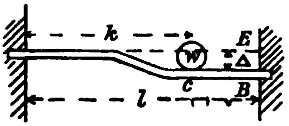

FIG. 65.

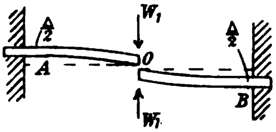

FIG. 66.

Take the case of the beam shown on Fig. 65 to illustrate the method of solution.

First disregard the load W, and examine the stresses produced by subsidence. The beam can be considered as two cantilevers each equal to half the span as shown on Fig. 66. Each cantilever is loaded with an unknown load producing a known deflection equal to half the subsidence, which is known. Then if W_1 is the unknown load, and the section and material of the beam are known, W_1 can be found from the deflection equation, *vide* Chap. VI. p. 69.

$$\Delta = \frac{nW_1l^3}{EI}$$

Where—

Δ = half the subsidence,

n for a cantilever loaded at the end $= \frac{1}{3}$

l = one half the span.

E and I are known, *vide* Chap. IV. pp. 47—55.

Then having solved for W_1, the M_B diagrams for these two cantilevers can be drawn as on Fig. 67, where they are shown together for convenience, the maximum ordinate being $\pm \frac{W_1l}{2}$.

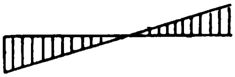

FIG. 67.

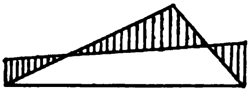

FIG. 68.

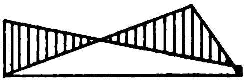

FIG. 69.

Due attention must of course be paid to the sign of the bending moment.

Then having drawn the M_B diagram for the superimposed load W, Fig. 68, the two Figs. 67 and 68 are compounded in the ordinary way and Fig. 69 is the result.

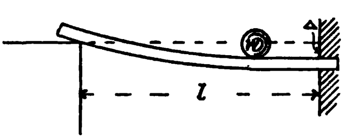

FIG. 70.

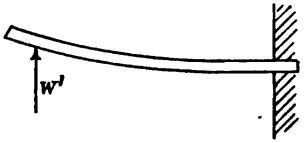

FIG. 71.

Subsidence ; fixed one end.

A similar case is that of a beam fixed at one end and supported at the other with a subsidence at the fixed end, as in Fig. 70.

In this case the subsidence causes the beam to act as a cantilever of length equal to the span as shown in Fig. 71, acted upon by an upward pressure W' at the end.

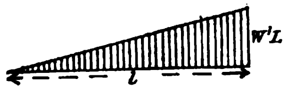

FIG. 72.

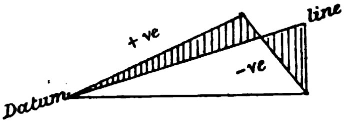

FIG. 73.

W' can be found from the deflection formula where the span l is the whole span of the beam, and the deflection is equal to the total subsidence.

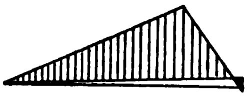

FIG. 74.

For this an M_B diagram is drawn, Fig. 72, and also an M_B diagram for the beam, as loaded with the load W, fixed at one end and supported at the other (Fig. 73).

These two Figs. 72 and 73 are compounded, giving Fig. 74, showing the actual M_B in the beam taking the subsidence into account.

CONTINUOUS BEAMS.

Beams continuous over one or more intermediate supports, are common in structural work, and their solution is necessary to enable advantage to be taken of the extra strength secured over independent beams, extending merely from one support to the next.

This solution is not easy analytically, as the bending moments are somewhat difficult to determine, the chief reason being that the reactions at the supports are not necessarily vertical.

There is also a danger in the use of continuous beams, which causes many engineers to fight shy of them; this danger is the enormous reversal of stress that takes place if any of the supports subside.

For example, we have a beam length $2l$ continuous over one support in the centre, loaded with a uniform load w per unit length; the *Max.* M_B if the central support subsides is $= +\frac{wl^2}{2}$ at the centre.

If the end support subsides the bending moment changes sign and $= -\frac{wl^2}{2}$ at the centre support, making in fact a complete reversal of stress.

Such a complete reversal of stress is unlikely, but the possibility must always be considered.

The method of calculation is based upon the deflection, as it is very difficult to determine the direction and exact magnitude of the reactions when there are several supports.

REACTIONS AT SUPPORTS.

Before entering on the graphical solutions, it is necessary to show that the reactions under a loaded beam, supported at both ends and at the middle, are $\frac{3}{16}W$, $\frac{5}{8}W$, and $\frac{3}{16}W$.

Suppose the centre support to be removed; then the deflection in the beam due to a distributed load of w per foot run over the whole beam $=\frac{5}{384}\frac{wl^4}{EI}$. (Chap. VI.)

Now, if we replace the distributed load, wl, on the beam by a single central load, W_1, the deflection is given by the expression,

$$\Delta=\frac{1}{48}\frac{W_1l^3}{EI}. \quad \text{(Chap. VI.)}$$

Now suppose W_1 to be such as to cause the same deflection as the distributed load; then the two expressions for deflection may be equated, and $\frac{5}{384}\frac{wl^4}{EI}=\frac{1}{48}\frac{W_1l^3}{EI}$.

or

$$W_1=\frac{5}{8}wl.$$

$$=\frac{5}{8}W.$$

But since the beam is actually supported at the middle, and no deflection takes place, the reaction at this central support preventing deflection must be equal and opposite to the load W_1.

Hence R at the central support $= W_1$.

$$=\frac{5}{8}W.$$

or

$$=\frac{5}{8}wl.$$

The reactions at the ends are equal to each other and each

$$=\frac{3}{16}wl.$$

$$=\frac{3}{16}W.$$

GRAPHIC SOLUTIONS OF PROBLEMS IN CONTINUOUS BEAMS.

Concentrated Loads.

McDonald's graphic solutions of problems in continuous beams are among the best and simplest approximations that have yet been devised, and are sufficiently close for practical purposes.

They also have the great advantage of showing graphically the position of the points of contraflexure and the sign of the bending moment at any section.

There are three steps:—

(1) Consider each span separately as fixed, and draw the diagram for such a case.

(2) Consider the outside ends as supported only, and modify the diagrams accordingly.

(3) Consider the ends resting on the piers as supported only, and modify the diagrams accordingly, working from each end in turn and taking the mean.

If the ends are fixed in reality step 2 may be omitted.

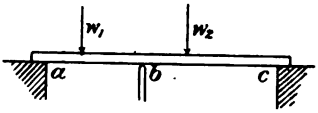

FIG. 75.

TWO SPANS.

Taking the continuous beam shown in Fig. 75, first draw the diagram for each span, ends fixed as Fig. 76. Now release the two outer ends of the whole beam and consider them as supported only, obtaining Fig. 77; where $ak_1 = 0$ and bl_2 in Fig. 77 $= bl_1 + \frac{1}{2}\, ak_1$ from Fig. 76, and bk_2 in Fig. 77 $= bk_2 + \frac{1}{2}\, cl_2$ from Fig. 76.

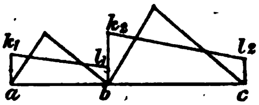

FIG. 76.

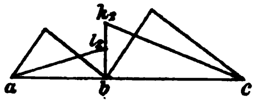

FIG. 77.

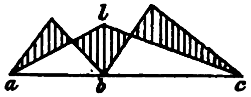

FIG. 78.

To complete the whole figure, as in Fig. 78, bl must be the mean of bl_2 and bk_2 in Fig. 77. This is obvious since the beam is continuous over the support b, and the bending moments on each side of the support must be the same and equal to one another in magnitude.

Therefore in Fig. 78, plot $bl = \frac{bl_2 + bk_2}{2}$.

MORE THAN TWO SPANS.

For more than two spans the principle is the same. Consider the beams and loading as shown on Fig. 79.

(1) Draw the M_B diagram for each span, treated as fixed at each point of support as in Fig. 80, obtaining the following tabulated results for the ordinates at the point of fixation :—

A.	B.	C.	D.
$ak_1 = 75$	$bl_1 = 75$ $bk_2 = 84{\cdot}4$	$cl_2 = 28{\cdot}1$ $ck_3 = 108$	$dl_3 = 72$

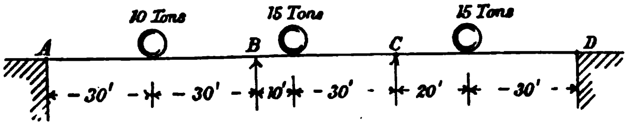

FIG. 79.

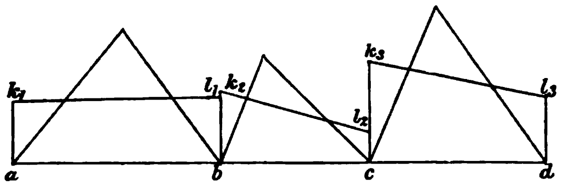

FIG. 80.

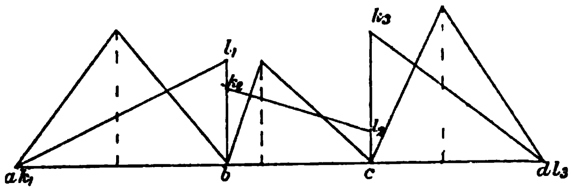

FIG. 81.

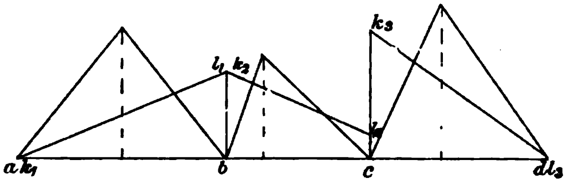

FIG. 82.

(2) Then, unfixing *A* and *D*, bl_1 and ck_3 are each increased as shown on the next table. (*Vide* Fig. 81.)

A.	B.	C.	D.
$ak_1 = 0$	$bl_1 = 75 + \frac{75}{2}$ $= 112{\cdot}5$ $bk_2 = 84{\cdot}4$	$cl_2 = 28{\cdot}1$ $ck_3 = 108 + \frac{72}{2}$ $= 144$	$dl_3 = 0$

(3) It is now necessary to unfix *B* and *C* in turn, starting with *B*. If *B* is unfixed, and the beam is considered as continuous over *B*, the moments represented by bl_1 and bk_2

as given in (2) must be equal to one another, and each equal to the mean between them. This alters the datum line k_2l_2, and any increase in bk_2 must to some extent affect cl_2. If bk_2 is increased cl_2 must be decreased by half the amount of this increase, and if bk_2 is decreased, cl_2 must be increased by half the amount of the decrease.

Therefore, unfixing *B*, we obtain results as tabulated below. (*Vide* Fig. 82):—

A.	B.	C.	D.
$ak_1 = 0$	$bl = \frac{112{\cdot}5 + 84{\cdot}4}{2}$ $= 98{\cdot}5$	$cl_2 = 28{\cdot}1 - \frac{98{\cdot}5 - 84{\cdot}4}{2}$ $= 21{\cdot}05$ $ck_3 = 144$	$dl_3 = 0$

(4) Then, unfixing *C*, the ordinate *cl* (Fig. 83) is the mean of cl_2 and ck_3 (Fig. 82). Also since the datum on the span *BC* is affected by the alteration to cl_2, *bl* is affected by half this increase or decrease. Thus taking the values in (3) we get fresh values for *cl* and *bl* as shown on (4) (Fig. 83).

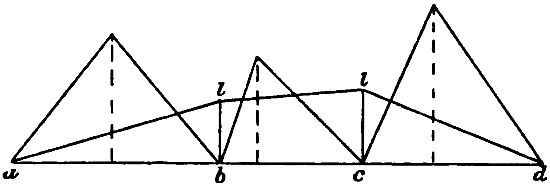

FIG. 83.

Beginning with the point *C*:—

A.	B.	C.	D.
$ak_1 = 0$	$bl = 98{\cdot}5 - \frac{82{\cdot}52 - 21{\cdot}05}{2}$ $= 67{\cdot}76$	$cl = \frac{21{\cdot}05 + 144}{2}$ $= 82{\cdot}52$	$dl_3 = 0$

(5) This is the first result, Fig. 83, but it is necessary to repeat the process, starting with C, instead of B, and to work in the opposite direction, that is from right to left and back to C.

Starting from the values in (2) and unfixing C we obtain the values in (5):—

A.	B.	C.	D.
$ak_1 = 0$	$bl_1 = 112{\cdot}5$ $bk_2 = 84{\cdot}4 - \frac{86{\cdot}05 - 28{\cdot}1}{2}$ $= 55{\cdot}42$	$cl = \frac{28{\cdot}1 + 144}{2}$ $= 86{\cdot}05$	$dl_3 = 0$

(6) Then unfixing B and compensating at C:—

A.	B.	C.	D.
$ak_1 = 0$	$bl = \frac{112{\cdot}5 + 55{\cdot}42}{2}$ $= 83{\cdot}96$	$cl = 86{\cdot}05 - \frac{83{\cdot}96 - 55{\cdot}42}{2}$ $= 71{\cdot}78$	$dl_3 = 0$

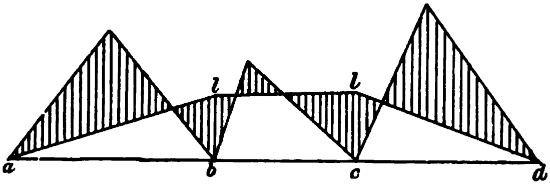

FIG. 84.

Then taking a mean between results (4) and (6).

$$bl = \frac{67{\cdot}76 + 83{\cdot}96}{2}.$$

$$= 75{\cdot}86.$$

and

$$cl = \frac{82{\cdot}52 + 71{\cdot}78}{2}.$$

$$= 77{\cdot}15.$$

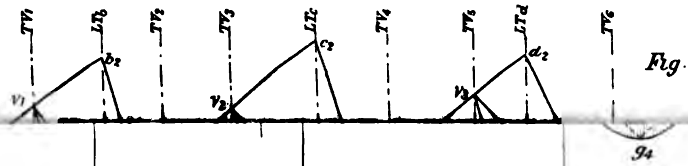
TV1
LTb
TV2
TV3
LTc
TV4
TV5
LTd
TV6
v1
b2
v2
c2
v3
d2
g4

Fig. 85.

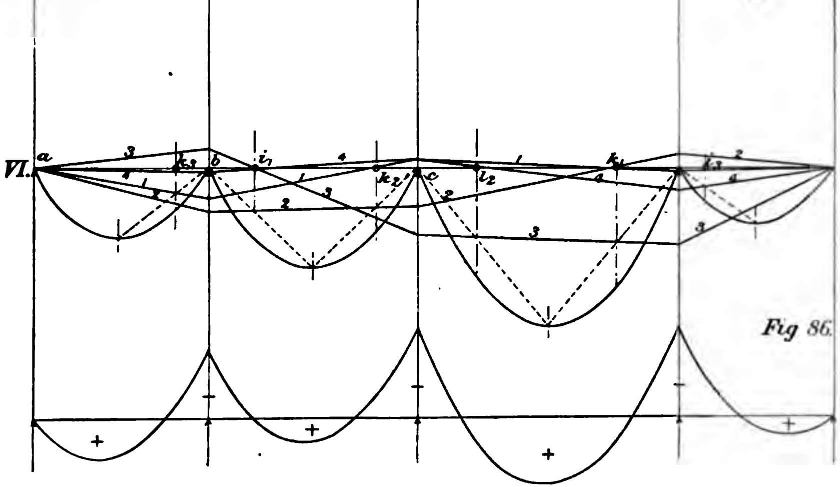
VI.
a
k3
b
i1
k2
c
l2
k1
1
2
3
4
+
−

Fig 86.

These results are plotted on Fig. 84, and give the M_B diagram which is shown shaded, the M_B being measured vertically from the datum line *alld*.

CONTINUOUS BEAMS WITH A UNIFORM LOAD.

For continuous beams uniformly loaded the best solution is that given below. It is derived from Dunn's solution for continuous beams under concentrated loads, taken from Dubois' "Graphical Statics," Mehrten's "Forlesunger über Statite der Baukonstruktionen und Festigkeitslehre," and Church's "Mechanics of Materials."

In this example four spans are considered:—

Draw them out to scale. See Fig. 85 (I. to VI.).

The points of support are marked *a*, *b*, *c*, *d*, *e*.

Divide each span into three equal parts, and at these points of division, erect verticals, called "third verticals" lettered TV_1, TV_2, TV_3, TV_4, TV_5, Fig. 85 (I.).

The outer verticals of the end spans are not required as these spans are supported at their outer ends.

The verticals on each side of any point of support form a pair, *e.g.*, TV_1 and TV_2.

Divide the distance between each pair into two parts in the inverse ratio of the spans on either side of the point, and at these points draw verticals LT_b, LT_c, LT_d, etc. These verticals are called "Limited third verticals"; when this has been done, the first step is to find the "inflection points" of the different spans when they are loaded; to do this, draw from *a* any line ab_2, cutting LT_b at b_2.

Let this cut TV_1 at v_1.

Join v_1b and produce it to cut TV_2 at f_1.

Join b_2f_1, cutting *bc* at i_1.

Then i_1 is an inflection point.

To obtain the next inflection point it is necessary to start again from the point just found, namely, i_1.

Draw any line i_1c_2 to cut LT_c at c_2, and let the line i_1c_2 cut TV_3 at v_2.

Join v_2c and produce it to cut TV_4, at f_2, cutting *cd* at i_2; this point will be another inflection point.

For a third inflection point start again from i_2.

Draw any line i_2d_2, cutting LT_d at d_2 and TV_5 at v_3; join v_3d and produce to meet TV_6 at f_3, cutting de at i_3.

This will be another inflection point.

All these inflection points so found are on the right hand side of the points of support, and to find those on the left it is necessary to start again from the opposite end, namely e.

Join ef_3 and produce it to meet LT_d at d_3; join v_3d_3, cutting cd at k_1.

This will be an inflection point.

Join k_1f_2 and produce to meet LT_c at c_3; join c_3v_2 cutting bc at k_2.

This will be another inflection point.

Join k_2f_1 and produce to meet LT_b at b_3; join b_3v_1, cutting ab at k_3.

This is the last inflection point.

When this has all been done it is simple to draw in the M_B diagram for any or all of the loads.

Suppose that it is required to draw the diagram for the load on the span bc.

Draw the M_B parabola on the line bc as if for a supported uniform load only. Fig. 85 (III.).

Let g_2 be the apex of the parabola; join it to the two ends of the span line, b and c.

Through i_1 and k_2, the inflection points of the span, draw verticals cutting g_2b at i'_1, and g_2c at k'_2.

These points are points on the datum line for the beam bc when considered as fixed.

Draw b_1c_1 through i'_1 and k'_2, cutting the verticals at b and c, in b_1 and c_1.

Join ab_1.

Join c_1k_1, k_1 being the further inflection point of the span cd; and produce it to meet the vertical at d in d_1.

Join d_1e.

The $ab_1c_1d_1e$ is the datum for this condition of loading.

This figure is shown drawn in Fig. 85 (III.).

Again, suppose the same is required for the end span de. Fig. 85 (V.).

Draw the parabola for the load as if supported only.

Let g_4 be the apex.

Join it to d.

Draw a vertical from the inflection point i_3 as before, and obtain the point on the datum i'_3.

Join i'_3 to e and obtain the datum line for the span.

The datum line is then ei'_3d_1.

Join d_1i_2. Produce it to c_1.

Join c_1i_1. Produce it to b_1.

Join b_1a.

The same applies to all the spans.

When the diagram for the whole is required, draw the diagrams for all the spans and superpose them, as in Fig. 85 (VI.).

The ordinates of these diagrams can then be added together and the final diagram is obtained as in Fig. 86.

A little practice will enable this method to be used very rapidly, and with excellent results.

The scale adopted should be as large as possible, as this enables all lines to be clearly distinguished, and also makes the small M_B values on the spans which are considered as being not loaded of sufficient size to be able to be measured.

Where the load is the same on all the spans, the *Max.* M_B may be found in Table XII.

CHAPTER IV

MOMENTS OF RESISTANCE

DEFINITION.

When a beam or member of a structure is loaded transversely with a load or force, a certain "bending moment" or "moment of flexure" is produced upon this member.

In order to prevent this bending moment from bending or breaking the member, a contrary force must be produced in the member in order to resist it.

This contrary force, commonly called the strength of the member, produces a contrary moment, known as the Moment of Resistance of the member; this is generally denoted by the symbol

$$M_R$$

GENERAL INVESTIGATION.

In order to arrive at some measure of this moment of resistance, certain facts or laws have to be known. The first is called Hooke's Law; as originally expressed by him, it is "Ut tensio, sic vis," or freely translated, "the strain is proportional to the stress."

The second is, that transverse planes or sections which are right or straight, before bending, remain straight after bending.

Both these laws are true as long as the materials composing the members are not stressed beyond the "yield point." If then, we consider a member subject to transverse stress or commonly a beam, it is clear without proof that some of the theoretical layers or fibres composing the beam, are in compression, and some in tension.

Whether the upper fibres are in tension or compression depends on the method of support or fixation or on the sign of the moment of flexure.

Where this tension changes to compression there must be some fibre or layer in the beam which is not stressed either in tension or compression.

This layer or fibre is called the "neutral layer," and the intersection where this layer intersects any vertical cross-section of the beam, is called the "neutral axis" of that section.

Consider a beam as shown in Fig. 87, and any section of this beam XY, and apply a load upon it until it bends as in Fig. 88. Then the vertical section $X'Y'$ will still remain a straight line, in accordance with the second law given above.

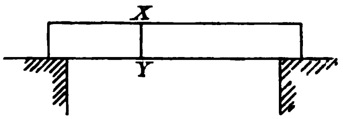

FIG. 87.

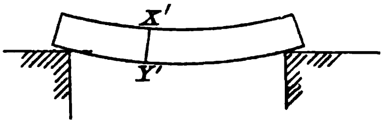

FIG. 88.

If we now enlarge these two sections, and superimpose them as shown on Fig. 89, it is clear that the upper fibres of the beam have been shortened by the amount XX', and the lower fibres have been elongated by the amount YY'.

The lengths XX' and YY' represent the amount of compression or tension, or the strain, due to the stress on the beam.

The lines XY and $X'Y'$ cut at O, which is a fibre of no stress, and is therefore the neutral axis. Then as the strain is proportional to the stress (Hooke's Law) and we measure our strains from XY for convenience, we can make a stress diagram showing the stresses set up in the fibres from Fig. 89.

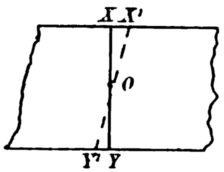

FIG. 89.

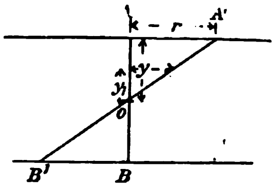

FIG. 90.

Fig. 90 will be such a figure, where AA' is proportional to XX', and BB' is proportional to YY'. Also as XY and $X'Y'$ are straight, in accordance with our original assumption, AB and

$A'B'$ are also straight lines. This being so, the stress at any layer is clearly proportional to the distance from the neutral axis.

The moment of resistance of the section is obviously equal to the sum of the moments of all the stresses, shown in each fibre in the figure, about the neutral axis OO, both above and below it. If we take any fibre, distant y_1 from O the moment of this fibre about O is $y_1 \times$ stress at y_1. Now the stress is proportional to the distance from the neutral axis, so that if f is the stress per unit area in AA', and $OA = y$, the stress in the fibre distant y_1 from O is equal to $\frac{f \times y_1}{y}$.

Hence the moment of that fibre stress round O the neutral axis is :—

$$= \frac{f \times y_1}{y} \times y_1$$

$$= \frac{fy_1^2}{y}.$$

This is the moment of resistance due to that fibre. But every beam has some breadth, and each fibre is stressed equally throughout its breadth, so that considering this fibre as being a layer across the beam, the moment of resistance of the layer is :—$\frac{f}{y} y_1^2 \times$ breadth of beam (on the particular horizontal section under consideration) (Fig. 91). Let this breadth be called b_1. Then the moment of resistance of this particular layer under consideration is :—

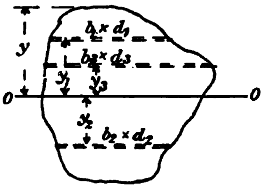

FIG. 91.

$$\frac{f}{y} \times y_1^2 \times b_1.$$

The moment of resistance of the whole beam is the sum of the moments of all such layers which is :—

$$\frac{f}{y} [y_1^2 \times b_1 \times d_1 + y_2^2 \times b_2 \times d_2 + y_3^2 \times b_3 \times d_3 + \text{etc.}]$$

where d_1, d_2, etc., are the thicknesses of the layers under consideration and are infinitely small.

This expression includes all layers both above and below the neutral axis.

The expression $[y_1^2 \times b_1 \times d_1 + y_2^2 \times b_2 \times d_2 + y_3^2 \times b_3 \times d_3]$ is called the "moment of inertia" of the section and is usually denoted by the symbol "I."

Therefore substituting I we obtain the moment of resistance of the section :—

$$M_R = f\frac{I}{y}$$

I, or the moment of inertia of any section is a symbol, used to denote briefly a certain property of that section, which depends on its shape alone.

I is entirely independent of the material of the beam or member under consideration.

It is rarely that I will have to be worked out for any beam, as it can be found for all standard sections in tables published by the British Standards Committee, and also by nearly all engineering firms.

In the case of large beams or girders, which are not of standard dimensions, a different formula, not involving I, will probably be used, for finding the moment of resistance.

MOMENT OF INERTIA—GENERAL DEFINITIONS.

The moment of inertia of any section is a property of that section with reference to an axis, depending only on the shape of the section. It should not be confused with the moment of inertia which concerns mass, and with which it has nothing whatever to do.

The term moment of inertia has been applied above to the expression $y_1^2\, b_1\, d_1 + y_2^2\, b_2\, d_2 +$ etc.

This represents the moment of inertia of a figure or section such as shown in Fig. 91 referred to its axis OO. It is a general case, in which y_1 and y_2, etc., increase up to a limit called y.

b_1 and b_2, etc., may be equal, or unequal, also d_1, d_2, etc., may be taken of the same or different values but must be very small.

This may be translated into a definition thus :—

DEFINITION.

"The moment of inertia of any section round any axis of reference is the sum of the products of areas into the squares of their *C.G.* distances from the axis of reference, provided that the areas are very small."

This includes all areas in the section whether above or below the axis.

To find I for any section from this general definition, for the purpose of determining the M_R of a section, the moment of inertia of the section round its own neutral axis is the value of I required.

Now, it can be proved for beams of homogeneous material, that is a material offering the same resistance per square inch to tension as compression, that the neutral axis passes through the centre of gravity of any cross-section since the sum of the tensile stresses must be equal to the sum of the compressive stresses.

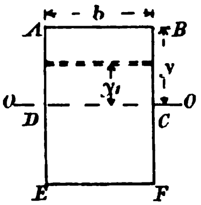

FIG. 92.

Hence the neutral axis can be found and drawn for any section.

We are not often concerned with irregular sections in simple engineering problems, and therefore the neutral axis is always an easy matter to locate.

In Fig. 92, let OO be the neutral axis passing through the C of G.

It is required to find I for the whole section $ABFE$. Consider I for the upper portion $ABCD$ referred to the axis OO; then the rule stated above applies; viz., $I =$ sum of products of areas into squares of *c.g.* distances from axis OO.

$$= \int_0^v y^2 b\,dy,$$

and when the thickness of areas is very small

$$= \frac{1}{3} b y^3.$$

This is the moment of inertia of a rectangle with reference to its base as axis.

Similarly for the figure $DEFC$

$$I = \frac{1}{3} b y^3.$$

Hence I for the whole section equals the sum of the moments of inertia of its two component rectangles $ABCD$ and $DEFC$ referred to OO.

$$2 = \frac{1}{3}by^3$$
$$= \frac{2}{3}by^3.$$

Now let the total depth of this rectangular section shown in the figure be called d.

Then $$d = 2y$$

or $$y = \frac{d}{2}.$$

Substituting for y above

$$I = \frac{2}{3}\,b\left(\frac{d}{2}\right)^3$$
$$= \frac{1}{12}\,bd^3.$$

This is the moment of inertia of a rectangular section, $b \times d$, referred to its own neutral axis.

This result is an important one, being that used in order to find M_R.

It was shown above that the M_R of a beam

$$= f\frac{I}{y}.$$

Hence the M_R for a rectangular beam $b \times d$

$$= \frac{f}{\frac{d}{2}} \times \frac{1}{12}\,bd^3$$
$$= \frac{1}{6}\,fbd^2.$$

This formulæ is of everyday use.

In a similar manner I has been worked out for other common simple sections, and the values are tabulated here.

Rectangle, breadth b, depth d	$I = \frac{1}{12}bd^3$.
Isosceles triangle, base b, height d . .	$I = \frac{bd^3}{36}$.
Circle, diameter d	$I = \frac{\pi d^4}{64}$.
Annulus, external diameter D ,, internal diameter d . .	$I = \frac{\pi}{64}(D^4 - d^4)$.
Semi-circle, diameter d	$I = {\cdot}0017\,d^4$.

NOTE.—I being the product of areas into distances squared is in inch 4 units.

So far I for simple sections only has been found.

In the finding of I for a rectangular section above two important rules are exemplified.

1. The moment of inertia of the whole section is equal to the sum of the moments of inertia of the component rectangles; all being referred to the same axis OO. This is obvious from the original assumption that I is the sum of certain products.

2. The moment of inertia of an area or section round any axis OO, is equal to the moment of inertia round its own neutral axis, + its area multiplied by the square of the distance of its neutral axis from the axis OO. For example, the moment of inertia of the rectangle $ABCD$ (Fig. 93) round OO is equal to its I round KK plus the area $ABCD$ multiplied by $\frac{y}{2}$ squared.

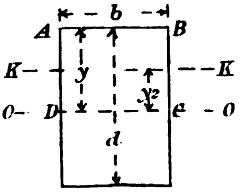

FIG. 93.

The formula for I for a rectangular section round its own neutral axis is,

$$I = \frac{1}{12} bd^3.$$

Hence the moment of inertia of $ABCD$ round KK

$$= \frac{1}{12} by^3.$$

Its area $= by$.

Its neutral axis KK is distant $\frac{y}{2}$ from OO.

Hence by rule 2 its I round OO,

$$= \frac{1}{12} by^3 + by \times \left(\frac{y}{2}\right)^2$$

$$= \frac{by^3}{3}.$$

The two rules stated above enable us with the aid of the simple formulæ given in the Table above to find I for almost any rectilinear or symmetrical section whatever.

The two important rules are stated again.

1. *The moment of inertia of any section about its own neutral axis is the sum of the moments of inertia of its own component areas round that axis.*

2. *The moment of inertia of a section or area round any axis of reference is its moment of inertia round its own neutral axis plus the product of its area into the square of the distance of its own neutral axis from the axis of reference.*

The neutral axis of the component areas must be parallel to the axis of reference.

These rules are now applied to find I for some more complex sections.

Let Fig. 94 be the section of a beam.

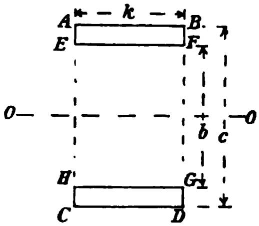

FIG. 94.

It is desired to find I for the two rectangles $ABFE$ and $HGCD$ round their neutral axis OO.

Now the two sections named are composed of the rectangle $ABDC$ less the rectangle $EFGH$, and the moment of inertia required is I for rectangle $ABDC + I$ for the rectangle $EFGH$, the latter being considered negative, *i.e.*,

I for $ABCD - I$ for $EFGH$

Now I for any rectangle is $= \frac{1}{12} bd^3$ round its own neutral axis.

Therefore substituting,

$$I \text{ for } ABDC = \frac{1}{12} kc^3,$$

$$I \text{ for } EFGH = -\frac{1}{12} kb^3.$$

Also the area of rectangle $ABDC \times$ the square of the distance of its *c.g.* from the neutral axis OO of the whole figure

$$= kc \times 0$$
$$= 0,$$

and the area of rectangle $EFGH \times$ the square of the distance of its *c.g.* from the neutral axis of the whole figure

$$= kb \times O$$
$$= O.$$

Therefore I for the portion of the section under consideration round the axis OO

$$= \frac{1}{12} kc^3 - \frac{1}{12} kb^3 + O - O$$

$$= \frac{1}{12} k (c^3 - b^3).$$

This is the expression which gives I for the top and bottom flanges of a girder, and is of common occurrence, *vide* Table VIII.

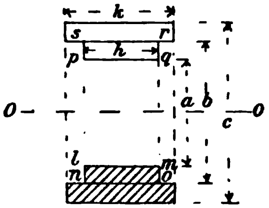

FIG. 95.

This can be further elaborated for a section as in Fig. 95.

Here the section consists of two rectangles above and below the neutral axis OO.

Then the moment of inertia of the whole section under consideration is the sum of the moments of inertia for the component parts round OO.

This has been found above, for the outer rectangles; it remains only to find I for the inner rectangles *lmon*, and *pqrs*.

This is done in the same way as before, by subtracting I for *lmqp* from I for *nors*.

Therefore I for the inner rectangles

$$= \frac{1}{12} hb^3 - \frac{1}{12} ha^3 \times O - O$$

$$= \frac{1}{12} h (l^3 - a^3).$$

Therefore the required I for the whole section shown shaded is

$$= \frac{1}{12} k (c^3 - b^3) + \frac{1}{12} h (b^3 - a^3).$$

This may be expanded to any number of rectangles which may make up any shape required.

Now let Fig. 96 be a section of a steel I beam.

It is required to find a formula for I.

Draw Fig. 97, building it up of rectangles which contain the same area or cross-section as the beam in Fig. 96.

Lettered dimensions are marked on the figure.

Then I for the whole section is the sum of the moments of inertia for all the component sections plus the areas of these sections multiplied by their *c.g.* distances from OO; this latter term vanishes since the *c.g.* of the rectangles considered are on the

N.A., and the total moment of inertia is moment of inertia of the web or inner rectangle $\left(= g\,\frac{a^3}{12}\right)$ plus the moment of inertia of a

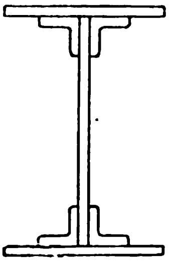

FIG. 96.

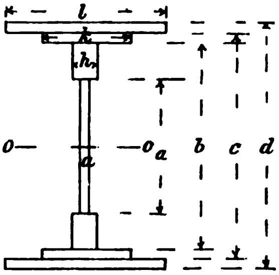

FIG. 97.

rectangle, length b width h, – the moment of inertia of a rectangle length a width h $\left(= \frac{h}{12}(l^3 - a^3)\right)$ plus the moments of inertia of the two outer rectangles

$$\left(= \frac{l}{12}\,(d^3 - c^3) \text{ and } \frac{k}{12}\,(c^3 - l^3) \text{ respectively}\right)$$

which are obtained similarly.

$$\text{Therefore } I = \frac{1}{12}\left\{ga^3 + h\,(b^3 - a^3) + k\,(c^3 - b^3) + l\,(d^3 - c^3)\right\}.$$

This is the general formula for I for all built up steel girders and beams.

The general rule can also be applied to a T section, such as is shown in Fig. 98.

I for the area $ABCD$ about the neutral axis is

$$= \frac{1}{12}\,bx^3 + bx\left(\frac{x}{2}\right)^2 = \frac{bx^3}{3}$$

I for the area $EFCD$ is similarly

$$= -\frac{bz^3}{3}$$

I for the area $GHLK$ is :

$$= t\,\frac{z^3}{3}$$

I for the area $KLNM$ is :

$$= \frac{t\,y^3}{3}$$

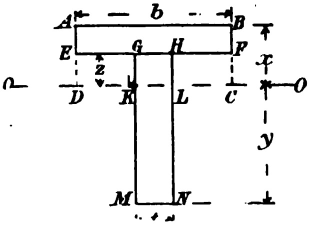

FIG. 98.

Therefore I for the whole is—

$$= b\frac{(x^3 - z^3) + t(y^3 + z^3)}{3}.$$

The general equation for the moment of resistance of any section has been found to be

$$M_R = \frac{I}{y}f,$$

where f is the resistance of the material composing the section to fracture, I is the moment of inertia of the section, and y is the distance of the neutral axis to the extreme fibre.

The moment of resistance must be at least equal to the moment of bending in order to ensure safety against failure, and so we obtain the equation

$$M_R = M_B.$$

FACTOR OF SAFETY.

In members composing a structure it is not sufficient that the moment of resistance shall be only equal to the bending moment. Some factor of safety must be introduced. Materials do not retain for ever their original strength; timber may rot, iron and steel may rust, there may be some slight subsidence, or many other disturbing factors may alter the original state of affairs. It is impossible to foresee all these things, and without some margin of strength over and above the actual strength required any such unforseen alteration in the actual conditions would necessitate some strengthening or renewal. To avoid this necessity and to meet the uncertainty as to the exact strength of the materials a factor of safety is introduced.

The factor of safety should be introduced in the value assigned to f.

If R is the ultimate resistance of the material to cross breaking, f should be equal to $\frac{R}{\text{factor of safety}}$.

The value of the factor of safety is largely a matter of experience and expediency, and will vary from 2 in an extreme case, where the conditions warrant such a bold course, to possibly 10 where a large margin is considered necessary. Broadly speaking, in building work the ordinary factors of safety may be taken as follows:—

Timber...	5
Steel	4
Wrought iron	4
Cast iron	6

It is impossible to lay down any definite figures, but the above may be considered as a guide.

These factors of safety, of course, are entirely independent of any addition to actual live load which may be considered necessary to compensate for shock, impact, etc.

Various values for f can be found in many places, and the following table gives some values taken from many tables:—

TABLE I.

SAFE WORKING STRESSES ON TIMBER.

Factory of Safety of 5.

Name of Timber.	Transverse stress. f	Tension. f_t	Compression. f_c	Bearing. f_b	Shearing across grain. f_s	Shearing along grain. f_d	Modulus of Elasticity. E
	Safe stresses in lbs. per square inch.						
Baltic Fir	1,200	1,400	1,000	1,340	360	140	1,440,000
Pitch Pine	1,400	1,120	1,120	1,400	—	—	1,500,000
British Oak	1,600	1,750	1,450	2,800	560	224	1,200,000
Deodar (India) ...	1,008	700	700	700	—	—	1,500,000
Teak (Burmah) ...	1,530	1,210	1,340	1,340	—	—	1,600,000
Sal (India)	1,640	1,210	1,210	1,210	—	—	1,800,000

SAFE WORKING STRESSES ON IRON AND STEEL.

Factory of Safety 4.

Material.	Transverse stress. f.	Tension. f_t	Compression. f_c.	Bearing. f_b.	Shearing. f_s.	Modulus of Elasticity E.
	Safe stress in tons per square inch.					lbs. per sq. inch.
Steel	7	7½	7	8	5½ to 6	30,000,000
Wrought Iron	6	6	5	6	4 to 5	29,000,000
Cast Iron	3½	1½	8	10	2½	17,000,000

The formula $M_R = \frac{fI}{y}$ is a general formula and applicable to any section.

For simple sections it can be simplified.

These simplifications only apply to beams of homogeneous materials, that is, whose fibres have an approximately equal resistance to tension and compression caused by bending.

Wood, steel in beams and small girders, and wrought iron are materials of such a kind.

For a rectangular beam of breadth b and depth d

$$I = \frac{1}{12} bd^3, \text{ also } y = \frac{d}{2}.$$

$$\text{then } \frac{fI}{y} = \frac{2fbd^3}{12d}$$

$$= \frac{1}{6} fbd^2.$$

Similarly for a circular section substituting $\frac{\pi d^4}{64}$ for I and $\frac{d}{2}$ for y.

$$\frac{fI}{y} = \frac{\pi fd^3}{32}.$$

It should be noted that a beam of rectangular section is very extravagant as far as resisting bending moment is concerned.

A beam of the section shown in Fig. 99 (shaded portion) is the most economical in theory.

The useless portion of a rectangular beam would, however, have to be cut away, and the labour and waste involved would cost more than any saving of weight would warrant.

Furthermore, in practice, the lateral cohesion of the fibres is such that some of the pressure is transmitted from the shaded to the plain part of the section, increasing the strength as much) per cent. above the theoretical strength of the

ron beams where the sections are rolled to any

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required shape (nowadays standardised) the beam is made of the section shown in Fig. 100, where the most of the metal is grouped in the flanges which are united by a thin web of steel, thus saving weight and material without waste.

Most tables of standard sections and makers' catalogues give for each section the value of $\frac{I}{y}$, generally called the "section modulus."

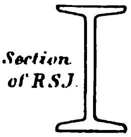


FIG. 100.

Knowing the section modulus, it is only necessary to multiply this by the value of f, in order to obtain the true M_R of the section.

In ordering sections it is best to state the standard number, size, and weight, thus :—

B.S.B. No. 8, 6 inches by 3 inches, at 12 lbs. per foot.

All this information is given in tables of sections.

For larger built-up girders and beams, it is laborious to find the values of I, and an approximate formula is often very valuable.

$$M_R = f\,a\,d,$$

where a = net sectional area of one flange, *i.e.*, deducting area of rivet holes

d = distance between the centres of gravity of the top and bottom flange areas.

NOTE.—In this formula, "a" is really the equivalent area for the flange, *vide* definition of equivalent area below. But in built-up beams where the M_R exerted by the web plate is so small as to be negligible, and where the top and bottom, or tension and compression flanges, are equal and symmetrical with one another, "a" may be taken as the actual net area of either flange.

For joists or small girders "d" can be taken as the full depth.

For larger or built-up girders "d" is usually taken as the depth of the web plate.

For other beams, joists, or girders, where the web is very thick or of considerable sectional area compared with the flanges, "d" must be taken as the actual distance between the centres of gravity of the flanges.

This formula is deducible from the formula

$$M_R = \frac{1}{6} f b d^2$$

for rectangular beams (Fig. 99) or *vice versâ*.

EQUIVALENT AREA.

DEFINITION.

The equivalent area for any section of a beam or member loaded transversely, is that area which, if all its fibres were exerting their maximum stress, would produce a moment of resistance round the neutral axis, equal to that actually produced by the whole section of the beam about the neutral axis; or, in other words, referring to Fig. 99, if the shaded portion above the neutral axis were equally stressed all over up to the utmost permissible value of f, for the material in question, it would produce the same moment round the neutral axis as the whole area *aoo′d*, whose fibres are stressed at a maximum at *ad*, and decreasingly down to zero at OO'. The shaded portion is the equivalent area for the section *aoo′d*.

Some examples on M_R are given in Chapter VII.

Generally, the use of equivalent areas is to find the M_R of a section of a built-up girder, for which I is not known and is difficult to calculate.

MODULUS OF RUPTURE.

A definition is given on p. 6.

As previously stated, it is derived from experiments on different materials, and is given in nearly all tables.

Samples of the different materials are loaded until they break, and f is then calculated from the formula,

$$M_B = \frac{fI}{y}$$

in which the M_B is known.

The values obtained for f are misleading, as the yield point of the material is past before rupture takes place, and consequently the formula for the M_R cèases to be correct.

However, in dealing with any new material not tabulated, it is best to take a sample similar in section to the section required for use, taking care to select a sample rather below than above the average, and test it by cross breaking, calculating the modulus of rupture.

$$
\begin{aligned}
M_B &= M_R \\
&= f\frac{I}{y} \\
&= f.a.d. \\
&= \frac{1}{6}.f\,bd^2.
\end{aligned}
$$

CHAPTER V

SHEAR

So far only tensile and compressive stresses, due to transverse loading of a beam or member, have been considered.

Transverse loading, as has been shown, tends to produce failure by tension or compression in the upper or lower fibres of the beam or member.

There is, however, another stress produced by transverse loading, which is of very great importance. This is called Shearing Stress.

Consider any loaded beam, and draw an M_B diagram as shown in Fig. 101 (for a supported beam). It will be seen that the M_B is a maximum at one point, decreasing to 0 at both points of support.

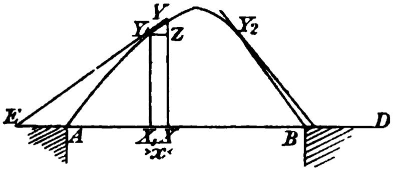


FIG. 101.

There is, therefore, a difference in M_B on the beam at any two adjacent points, such as X and X_1, represented by the portion of ordinate YZ. This difference shows that the statical equilibrium between these two adjacent points or sections must depend on some force or other moment not apparent on this diagram without further investigation, or in other words:—

At the point X the M_B is represented by the ordinate XY, and calls into play a certain M_R from the beam, in order to resist it.

At the point X_1 the M_B is represented by the ordinate X_1Y_1, which is less than XY, and calls into play a lesser M_R from the beam in order to resist it.

That is, the M_R at X_1 is less than at X,

and consequently, since $M_R = f\dfrac{I}{y}$,

the value of the f at X_1 must be less than at X (since $\dfrac{I}{y}$ is a constant), *i.e.*, the tensile and compressive stresses along the fibres of the beam are less at X_1 than at X.

This proves that there must be some other force or moment tending to disrupt the beam, acting between X and X_1, which will take up or meet the reduction in fibre stress between X and X_1, so as to preserve equilibrium. This is termed "shearing stress," and is resisted by the strength of the beam to resist shear.

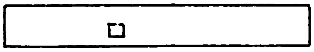


Fig. 102.

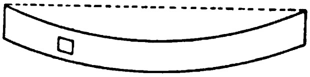


Fig. 103.

If a particle of rectangular form on any longitudinal section of a beam be considered before and after bending, it will be seen that if before bending it is of the form shown in Fig. 102, after bending it will assume the form as shown in Fig. 103, both figures being greatly exaggerated for clearness.

From the shape of the parallelogram in Fig. 103 as compared with the rectangle in Fig. 102 there must be forces acting on the parallelogram in Fig. 103 as illustrated diagrammatically in Fig. 104.

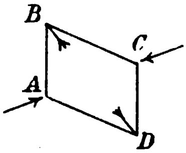


Fig. 104.

This gives an idea of the action of shear.

If the parallelogram were to become so elongated as to become almost one straight line, the beam would have failed by shearing, *vide* Fig. 105.

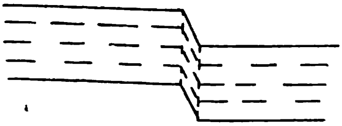


Fig. 105.

The diagonal forces tending to compress the diagonal AC (Fig. 104) necessarily induce a tension along the diagonal BD as shown by the force lines inside the parallelogram.

It is sufficient for simple members to consider this shearing force as a compressive force acting at 45° to the vertical, upwards, towards the centre of gravity of the load, and a tensile force acting at 45° upwards away from the centre of gravity of the load, being induced by the minute deformation, or strain, caused by the compression.

It is only necessary, then, for homogeneous members, in which resistance to tension and compression are the same, to consider the compressive force, acting at 45° to the vertical, upwards towards the centre of gravity of the load.

This force may be looked upon as resolved horizontally and vertically into two forces, a horizontal force tending to separate imaginary horizontal layers one from the other, as in Fig. 107, and a vertical force tending to separate imaginary vertical layers from one another, as shown in Fig. 106.

It is, however, important and necessary to remember in considering shear in complicated non-homogeneous beams, that this is only a simplified point of view, and that shear is a diagonal force, *not* two entirely separate and distinct forces, one vertical and the other horizontal.

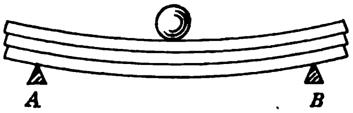


FIG. 106.

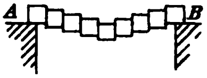


FIG. 107.

Owing to the fact that the diagonal force of shear is regarded as acting at an angle of 45° to the vertical, its components, namely, the vertical and horizontal shear, must be equal.

This enables simple rules to be devised to obtain the value of either the vertical or horizontal shear at any section.

The *intensity* of vertical shear at any section is equal to the *intensity* of the horizontal shear at the same section.

THE SIGN OF SHEAR.

A convention for sign has already been laid down as regards M_B.

Shear being in the nature of a secondary stress to the M_B, the same convention must be followed.

Referring to Fig. 101, the difference in M_B between X and X_1 $= M_B$ at $X - M_B$ at X_1.

This difference is distributed over a length x, the distance between X and X_1.

Then if X and X_1 are sufficiently close, the shear may be assumed to vary uniformly between X and X_1, whatever the M_B curve may be, and the shear at any point between X and X_1 will be

$$\frac{M_{BX} - M_{BX_1}}{x},$$

which is represented graphically by

$$\frac{XY - X_1Y_1}{XX_1} = \frac{YZ}{Y_1Z},$$

or the tan of the angle YY_1Z.

If X and X_1 are very close this will be the same as the tan of the angle made by the tangent to the M_B curve at that point with the horizontal.

If this angle YEB is within the first or third quadrant the tangent is positive, and therefore the shear will be considered positive.

To the right of the apex of the M_B curve, the angle is in the second or fourth quadrant, and the tangent of the angle is negative. The shear will therefore also be considered negative.

This determines the sign of the shear.

RULES FOR SHEAR.

1. For cantilevers, the vertical shear at any section is equal to the sum of the loads between that section and the unsupported end.
2. For supported beams the vertical shear at any section is equal to the reaction at one end minus all the loads between that end and the section in question.
3. The intensity of horizontal shear at any point is equal to the intensity of the vertical shear at the same point.

SHEAR DIAGRAMS (GRAPHIC).

It is frequently of value to plot graphically the shear, vertical or horizontal, throughout a beam, and the following show the methods employed.

Cantilever. Single load.—Fig. 108 shows the shearing stress diagram for a cantilever with a single load. The ordinates are the measure of the shear at any point.

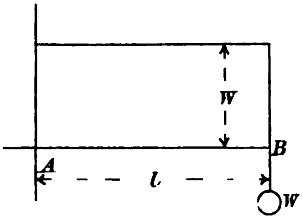


FIG. 108.

The vertical shear at any section is equal to the reaction at the support minus any loads between the support and the section in question.

In this case the reaction is equal to the load (W) and is constant throughout the beam.

The shear is positive, and is plotted upwards and measured upwards from AB. A scale of loads is required as the shear is measured in units of weight.

Cantilever. Several loads.—With several loads on a cantilever, the shearing stress diagram is as on Fig. 109. This diagram requires no explanation, as it is merely a combination of several diagrams similar to that in Fig. 108; or the ordinates can be obtained by the general rule for determining the shear at any vertical section.

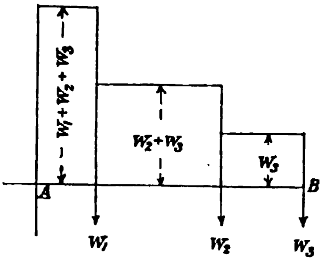


FIG. 109.

Thus, between the support and W_1 the shear is equal to the reaction and is constant throughout the length, *i.e.*, being equal to $W_1 + W_2 + W_3$.

Between W_1 and W_2 the shear is equal to the reaction, minus W_1 again being constant in this length and

$$= W_1 + W_2 + W_3 - W_1$$
$$= W_2 + W_3$$

Similarly between W_2 and W_3 the shear is equal to W_3 and is constant throughout this length.

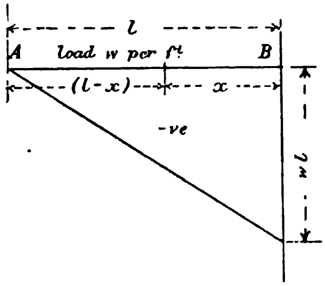


FIG. 110.

Cantilever. Uniform load.—For a cantilever loaded uniformly with w per unit length l, the shear on any section distant x from the support is equal to the load between this section and the unsupported end, *i.e.*, to the load on the length $(l - x)$

$$= w\,(l - x).$$

This is obviously a maximum when $x = o$, so that the maximum shear is at the support and equal to wl.

Further, in the equation $S = w\,(l - x)$, x is the only variable; also the shear is zero at the end of the beam; so that the shearing stress diagram is as on Fig. 110.

In this case it will be noticed that the cantilever is fixed at the right hand end, and therefore the shear is negative and is plotted below the line.

Other loading.—The above diagram can be compounded with the diagrams for single loads, giving a complete diagram as in Fig. 111.

Supported beams.—In a supported beam, the sign of the shear is more important, since it changes at some point in the beam.

Single load.—Taking the case of a single eccentric load (Fig. 112), the shear at any section between the load and the left hand support A is equal to the reaction at A and is positive. Also at any point between the load and the right hand support B, the shear is negative and equal to reaction at B.

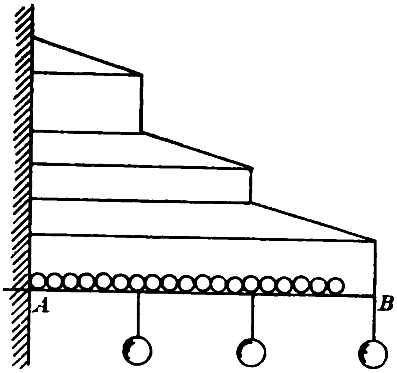


FIG. 111.

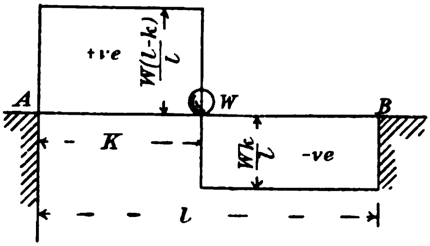


FIG. 112.

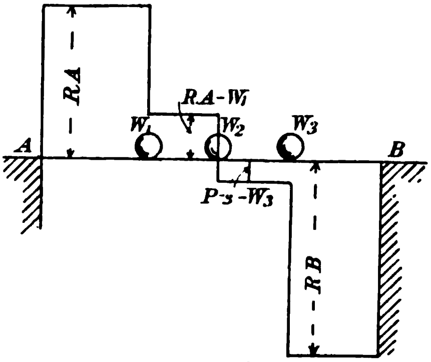


FIG. 113.

Plotting these results we get a diagram as in Fig. 112.

Several loads.—For several loads the shear at each end and under each load can be worked out and plotted as in Fig. 113.

Uniform load.—With a uniform load (Fig. 114) the shear at either support is equal to $\frac{wl}{2}$ having due regard to sign.

The shear at any section distant x from the left hand support is $\left(\frac{wl}{2} - wx\right)$ by rule.

This is the equation to a straight line, the ordinates being a maximum and equal to $\frac{wl}{2}$ at the support, and a minimum and $= 0$ at the centre. The shearing stress diagram is shown in Fig. 114.

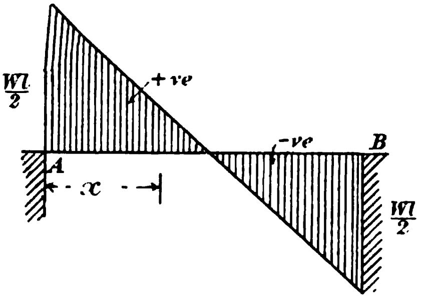


Fig. 114.

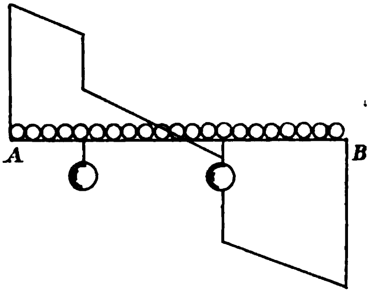


Fig. 115.

Such a diagram is easily combined with other diagrams for other loads as shown in Fig. 115 by the addition of the ordinates, having regard to sign.

M_B FROM SHEAR STRESS DIAGRAMS.

The shearing stress diagram can be applied in the case of simple beams to find the M_B. The M_B at any section is equal to the area of the shearing diagram (or sum of the shears) between the section in question and any other point where the ordinates of the M_B diagram are equal to zero.

In the case of a supported beam, such points are the ends or points of support of the beam.

In the case of a cantilever, the unsupported end is such a point.

In the case of fixed beams, the points of contraflexure are such points.

EXAMPLE.

Take the case of a uniform load w per unit length, acting on a cantilever (Figs. 116 and 117).

Fig. 117 is the M_B diagram.

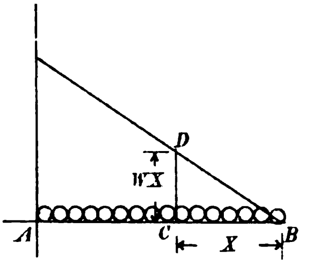


FIG. 116.

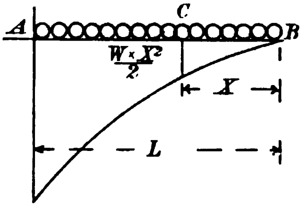


FIG. 117.

The M_B at a point x from the unsupported end

$$= wx \times \frac{x}{2}$$

$$= \frac{wx^2}{2}.$$

Fig. 116 is the shear stress diagram.

The shear at the same point

$$= wx$$

The area of the triangle DCB

$$= \frac{wx \times x}{2}$$

$$= \frac{wx^2}{2}.$$

which is the same as the M_B at that point.

The rule generally stated is :—

The M_B at any section is equal to the sum of the shears between that section and the nearest point of zero "bending moment."

This applies equally to a supported beam, as may be seen on considering the shear and M_B diagrams, for any given condition of loading.

A second useful fact which applies to supported beams, is that the *Max.* M_B occurs at the point where the shear changes sign. If the M_B diagram for any loaded beam is considered, this fact will be apparent (Fig. 101); for at the apex of the M_B diagram the tangent to the curve is horizontal, or indeterminate, and therefore the shear is zero.

These two facts often enable a beam to be solved without finding the M_B either analytically or graphically by the methods given in Chap. II.

Care must be taken, however, only to apply the shearing stress diagram to suitable cases, such as simple cantilevers, supported beams, or fixed beams under stationary loads.

RESISTANCE TO SHEAR.

Nothing is here said about the resistance to shear of various materials or sections.

The practice of testing beams of a simple nature by calculation in order to see whether they are strong enough to resist shear at their abutments or points of support is a useless one, since they are always amply strong enough, due to practical considerations of construction.

Such perfunctory calculation, even when it is carried out, is merely concerned with vertical shear, and by no means assures the engineer of complete safety.

It is, therefore, thought better to avoid any calculations for shear at this juncture, where they are not necessary, since they only serve to put into the mind an entirely wrong conception of the "shearing stress" and the forces resisting shearing.

Simple cleavage by shearing will be treated under the subject of joints, etc. The more complex action of shear in beams, girders, and other composite beams or frames, must be treated at some length in each case.

CHAPTER VI

DEFLECTION

In many structures it is not sufficient that the members shall be strong enough to resist cross breaking, these members must be also *stiff* enough, that is to say, the load must not bend the member more than a certain amount. It is necessary to be able to calculate the amount of this bending or deflection, and for that purpose the following formulæ are given :—

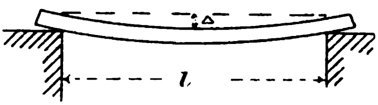


Fig. 118.

If Δ is the deflection measured as shown on Fig. 118.

Then

$$\Delta = \frac{nWl^3}{EI}.$$

n is a constant depending upon the method of fixation or support of the beam and method of loading. Values of *n* will be found on the table below.

W is the total load.

l is the span.

E is the modulus of elasticity of the material to be found on Table I., Chapter IV.

I is the moment of inertia of the section.

Note.—The deflection of a steel beam should not exceed $\frac{1}{360}$ of its span.

Table II.—Values of *n* and *n′*.

Arrangement of Beam and Load	Beam of uniform cross section throughout its length.	
	Value of *n*.	Value of *n′*.
Supported at both ends and loaded uniformly	$\frac{5}{384}$	$\frac{5}{48}$
Supported at both ends and loaded at centre	$\frac{1}{48}$	$\frac{1}{12}$
Fixed at both ends and loaded uniformly	$\frac{1}{384}$	$\frac{1}{16}$
Fixed at both ends and loaded at centre	$\frac{1}{192}$	$\frac{1}{24}$
Fixed at one end and loaded at the other (cantilever)	$\frac{1}{3}$	$\frac{1}{3}$
Fixed at one end and loaded uniformly (cantilever)	$\frac{1}{8}$	$\frac{1}{4}$

In using this formula the greatest care must be taken not to confuse units.

If W is in tons E must be in tons per unit area.

The deflection Δ is usually measured in inches so the span must be in inches.

I is usually inch^4 units.

Any alteration in units must be very carefully watched so as to prevent any chance of such an error.

There is a second formula or modification of the previous formula suitable for use with a proof load or where it is desirable to test the safety of a structure by measuring the deflection.

The formula is

$$\Delta = \frac{n'fl^2}{Ey},$$

n' is a factor as before to modify the result according to the loading, etc., and various values of n' will be found on the table given previously.

f is the resistance per square inch of the material to rupture.

l is the span.

E is the modulus of elasticity.

y is the distance from the neutral axis to the extreme fibre.

Great care must be taken as before to observe the proper units.

This formula is somewhat easier to work out as the arithmetic is simpler, but is more limited in its application. Its chief use is, as previously stated, to determine the maximum fibre stress, f, under a test load from the observed deflection.

It is worthy of note that the strength of a beam varies as the breadth and square of the depth, but the deflection varies as the breadth and the cube of the depth.

Take as an example a rectangular homogeneous beam.

Then $$M_R = \frac{1}{6}fbd^2,$$

and deflection $$\Delta = \frac{nWl^3}{EI}.$$

For a given span and load and material I is the only variable, and as the increase of deflection means decrease of stiffness, the stiffness can be considered as the reciprocal of the deflection, that is to say, the stiffness varies as $\frac{1}{\Delta} = \frac{EI}{nWl^3}$.

I being the only variable the stiffness varies directly as I. The moment of inertia of a rectangular homogeneous beam is $\frac{1}{12} bd^3$.

So that the stiffness varies as the breadth and the cube of the depth, while it is clear from the formula $M_R = \frac{1}{6} fbd^2$ that the strength varies with the breadth and square of the depth.

This naturally leads to the use of beams of considerable depth compared to their breadth, but practical reasons, as well as the restrictions entailed by the necessary resistance to shear, limit the depths which can profitably be employed. In timber it is difficult to secure thin beams placed on edge; in steel and iron the web of the beam will buckle and fail, due to shear, unless made extravagantly heavy.

CHAPTER VII

GENERAL AND RECAPITULATORY WITH EXAMPLES OF BEAMS

EVERY loaded beam has to resist:—

(a) M_B,

(b) Shearing stress,

(c) Deflection.

(a) The M_B is resisted by a moment of resistance or M_R which is produced by the strength of the material itself, and in a homogeneous material:—

$$M_R = \frac{fI}{y} \text{ for any section}$$

$$M_R = fad \text{ for any symmetrical section}$$

$$M_R = \frac{1}{6} fbd^2 \text{ for any rectangular section}$$

(b) Shear is resisted by the cohesion of the particles of the whole cross-section of the beam.

(c) Deflection is resisted by the stiffness of the section due to its form, and by the elasticity of the material.

Beams may either be of uniform cross-section throughout, or may be of increasing section towards the point of greatest strain. The former are the most common.

Consider Fig. 21.

The *Max.* M_B is produced at C, and the beam must be strong enough at C to meet this *Max.* M_B.

But at either side of C the M_B decreases towards the ends, and the beam need not theoretically be so strong. In fact, the cross-section and depth of the beam might be greatly reduced towards the ends, even down to zero at the actual points of support, if it were not for considerations of shearing stress, and also practical economy of labour. In the case of timber beams it is not economical to cut away the useless portions of the beams towards the ends, since the labour would be great and there would be no saving. It is consequently never done.

In the case of rolled steel beams, of I section, the same theoretical considerations apply.

But here again it is more economical to roll these beams of uniform section throughout, the saving in labour more than compensating for any waste of metal and slight increase of weight.

In built-up beams of steel or iron, however, an attempt is always made to reduce metal and weight wherever possible, so as to accord most closely to theoretical considerations.

Such beams are therefore made of light section near the supports or points of least stress, and are strengthened by the addition of extra plates at the points of greatest stress.

Fish-bellied girders and beams are instances of this; also all larger girders in which the flange area is increased towards the centre of the span, so as to give increased strength where the M_B is greatest.

Practical considerations usually determine the width of any beam, and it is very rarely that the width can advantageously be increased or decreased at different portions of the beam, *i.e.*, it will nearly always be a constant throughout the beam.

In practice the only two variables in the design of a beam are the

(1) Flange area;
(2) Depth.

Of these two the depth is the most useful for variation in the design of small beams, and the area of the flange in the design of larger girders.

For ordinary sections of steel usually rolled, the British Standard Sections should always be employed. The moment of inertia of all these sections has been most carefully worked out and can be found in the publications of the British Standards Committee, or in the catalogues of firms who roll and supply these sections.

It is usually most injudicious and extravagant to use any but a standard section, and preferably stock sections in common use. Any economy in material which may result from a specially designed section, which has to be of rolled steel, will be more than swamped by the cost of the rolls, etc., required to produce such a section.

Where light loads and small spans are involved (up to 10 feet or 12 feet) timber beams or small metal beams of rectangular section are generally met with.

In this case only the shaded portion shown in Fig. 99 is theoretically used in producing M_R, the unshaded portion being wasted except for resisting shear.

This shaded portion is known as the equivalent area of the section, *i.e.*, that area over which f may be assumed to act with its full intensity in order to produce the same M_R as the beam actually does.

It is not economical for small spans or loads to cut away this useless portion.

For larger loads and spans (10 feet to 20 feet) rolled steel joists and I section beams can be used.

Their $M_R = \frac{fI}{y}$, I and y being easily found graphically or from standard tables.

They are not very economical for the longer spans, for they are of uniform section throughout.

The M_R can also be got from the "*fad*" formula, but "*a*" and "*d*" are not always easily obtainable for rolled sections.

Makers' catalogues nearly always give the values of I and y.

For still larger loads and spans (20 feet to 80 feet) built up girders must be used, the M_R or strength of the girder being accurately or closely designed to meet the actual M_B at every point.

The simplest form of a built-up beam is a *plate girder*, consisting of a top and bottom flange and a connecting web plate.

The flanges increase in thickness towards the centre to meet the increase in M_B. The web may decrease towards the centre proportionately with the shear. Such a girder is nearly universal for spans of 20 feet to 50 feet in permanent work.

The M_R of such a section is most easily found by the formula $M_R = fad$ where f is the safe resistance to compression or tension in tons per square inch. M_R is the moment of resistance in foot-tons or inch-tons, according to the unit of d.

Table VIII. gives the moment of inertia for additional top and bottom plates on such a girder.

NOTE.—y must always be taken as the distance from the neutral axis to the *extreme* fibre.

E.g., in the case of a T beam Fig. 119, y is greater than j. j is usually given in tables, but should not be used; $y = d - j$.

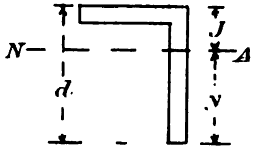


FIG. 119.

EXAMPLE.

To find the moment of resistance or M_R of an I section beam, using makers' tables.

Referring to tables, a section that appears suitable for the particular purpose can be selected.

Let us assume that a beam $19\frac{11}{16}$ inches by $11\frac{13}{16}$ inches in size (nominal 20 inches by 12 inches) weighing 138 lbs. per foot appears suitable.

The makers' catalogue or tables state:—

Greatest moment of inertia (longer axis) = 2,674 inches^4.

Least moment of inertia (shorter axis) = 282 inches^4.

Let us assume that the beam is to be used with its greatest dimension upright. (It would not usually be used on its side.)

Now $$M_R = \frac{fI}{y}.$$

Here $$f = 7 \text{ tons for steel.}$$
$$y = 19\tfrac{11}{16} \text{ inches} \div 2 = \tfrac{315}{32} \text{ inches,}$$
$$I = 2674,$$
$$\therefore M_R = \frac{7 \times 32}{315} \times 2{,}674 \cdot \text{ inch-tons,}$$
$$= 1{,}902 \cdot 75 \text{ inch-tons.}$$

To find the M_R when the beam is on its side, use the least moment of inertia.

EXAMPLE.

M_R of a beam by finding I.—Find the strength of an iron beam, Fig. 120, by finding I and allowing for rivets.

The formula is $M_R = \frac{fI}{y}$.

and $I = \frac{1}{12}\ [ga^3 + k\ (b^3 - a^3) + l\ (c^3 - b^3) + \text{etc.} \ldots]$ (Chap. IV., p. 53).

In this example the flange section is cut up into imaginary rectangles, *abdc*, *cdfe*, *ejhg*, *kimn*.

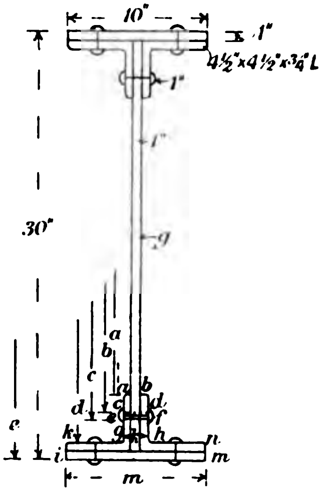


FIG. 120.

The rectangle *cdfe*, representing a rivet hole, is merely omitted.

Hence there is no term containing $(c^3 - b^3)$ in the above formula.

The rivets in the top rectangle *kimn* are allowed for by subtracting them from *m* (viz. 10 inches − 2 inches = 8 inches).

The values *a*, *b*, *c*, *d*, *e*, *k*, *m* are easily obtainable from the dimensions of the beam.

Referring to the lower portion of the beam in Fig. 120 and substituting, we get:—

$$I = \tfrac{1}{12}\ [1 \times 19^3 + 2\tfrac{1}{2}\ (21\tfrac{3}{4}^3 - 19^3) + 2\tfrac{1}{2}\ (26\tfrac{1}{2}^3 - 23\tfrac{3}{4}^3) + 8\ (30^3 - 26\tfrac{1}{2}^3)].$$

$$= \tfrac{1}{12}\ (6{,}857 + 8{,}607{\cdot}5 + 10{,}425 + 67{,}120).$$

$$= 7{,}751.$$

Then since $M_R = \frac{fI}{y}$,

Therefore $M_R = 6 \times \frac{7{,}751}{15} = 3{,}100$ inch-tons.

assuming $f = 6$ tons per square inch for iron.

In practice all these rivets would not occur at the same section of the beam, but those in the flange would break joint with those in the web.

EXAMPLE.

To find the M_R of a beam by using the formula

$$M_R = fad.$$

Assume the same section of a beam as in the previous example (Fig. 120).

Divide it up as before into convenient rectangles.

Then $M_R = \Sigma fad$.

$= f\Sigma ad$, where d = distance between the c of g of rectangles respectively, and a = the areas of rectangles after deducting for rivets.

= 6 tons [(10 inches − 2 inches) × 1 inch × 29 inches + (10 inches − 2 inches) × ¾ inch × 27¼ inches + 1⅜ inches × 2½ inches × 25⅛ inches + 1⅜ inches × 2½ inches × 20⅝ inches].

= 6 (232 + 163·6 + 86·4 + 70·9).

= 3,317·4 inch-tons.

In the above, the rectangle *kimn* has been divided into two to illustrate the principle involved. It does not always happen that the angle irons come exactly flush at their edges with the edge of the iron plate.

The rectangle *cdfe* is omitted as before to allow for rivets.

The result is large as it includes a portion of the web. Compare with the previous result.

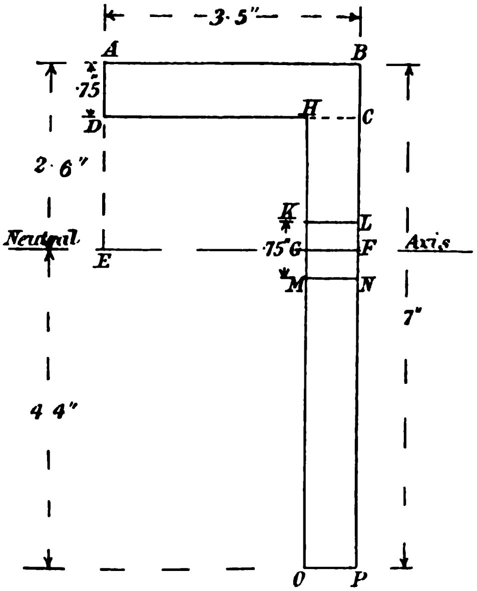


FIG. 121.

EXAMPLE.

To find the M_R of an L section steel beam by finding I.

The section is 8½ inches by 7 inches and is ¾ inch thick. There is a hole through it for a ¾-inch rivet at the neutral axis, which is 2·6 inches from the edge of the table of the *L*.

Fig. 121.

It is first necessary to find I.

Refer to p. 53 and to Fig. 98, where the principle is given at length, also the general formula.

Here :—

$$I \text{ for rectangle } AEFB \left(=\frac{bx^3}{3}\right) = \frac{3{\cdot}5 \times (2{\cdot}6)^3}{3}.$$

$$\text{,, ,, } DEFC \left(= -\frac{bz^3}{3}\right) = -\frac{3{\cdot}5 \times (1{\cdot}85)^3}{3}.$$

$$\text{,, ,, } HCFG = \frac{{\cdot}75 \times (1{\cdot}85)^3}{3}.$$

$$\text{,, ,, } KLFG = -\frac{{\cdot}75 \times ({\cdot}375)^3}{3}.$$

$$\text{,, ,, } GOPF = \frac{{\cdot}75 \times (4{\cdot}4)^3}{3}.$$

$$\text{,, ,, } GMNF = -\frac{{\cdot}75 \times ({\cdot}375)^3}{3}.$$

Therefore
I for whole section.

$$= \frac{3{\cdot}5}{3}[(2{\cdot}6)^3 - (1{\cdot}85)^3] + \frac{{\cdot}75}{3}[(1{\cdot}85)^3 - ({\cdot}375)^3 + (4{\cdot}4)^3 - ({\cdot}375)^3]$$

$$= \frac{3{\cdot}5}{3} \times 11{\cdot}25 \qquad + \frac{{\cdot}75}{3} \times 91{\cdot}42$$

$= 35{\cdot}98$ inch^4 units.

Now the M_R for the beam is $\frac{fI,}{y}$ where $f = 7$ tons per square inch.

$$\text{Hence } M_R = \frac{7 \times 35{\cdot}98}{4{\cdot}40}$$

$$= 57{\cdot}22 \text{ inch-tons.}$$

NOTE.—The value found for I is a little too large since no allowance has been made for rounded corners or angles. The error is about 1 per cent.

EXAMPLE.

To find I for a rolled section, without the use of a formula, from first principles.

The same section of beam will be considered as in the previous example, Fig. 120.

The section should be drawn out to a suitably large scale and then be divided up into a convenient number of rectangles as before.

Now, for extreme accuracy, the rectangles chosen should be very small, or else their own moment of inertia round their neutral axes would have to be taken into account, *vide* rule p. 51.

But if the areas are small, it is generally sufficiently accurate and on the safe side to ignore their moments of inertia round their own axes, and simply to work to the definition:—

$I =$ sum of the products of areas into the squares of their *c.g.* distances from the neutral axis of the whole figure.

Now the neutral axis passes through the *c.g.* of the section. Assume this to be half-way down.

I for top plate and corresponding bottom plate $\Big\} = 2\left[10'' \times 1'' \times \left(\frac{29}{2}\right)^2\right]$

$= 4{,}205$

I for rest of rectangle *kimn* and corresponding rectangle at bottom $\Big\} = 2\left[10'' \times \frac{3}{4}'' \times \left(\frac{27\frac{1}{4}}{2}\right)^2\right]$

$= 2{,}785$

I for rectangle *aghb* and lower corresponding rectangle $\Big\} = 2\left[2\frac{1}{2}'' \times 3\frac{3}{4}'' \times \left(\frac{22\frac{3}{4}}{2}\right)^2\right]$

$= 2{,}425$

Total I $= 9{,}415$

The moment of inertia for the rivet holes must be found and deducted from the total.

I for rivet holes in top plate $\Big\} = 2\left[2'' \times 1'' \times \left(\frac{29}{2}\right)^2\right]$

$= 841$

I for two rivet holes in the rest of *kimn* $\Big\} = 2\left[2'' \times \frac{3}{4}'' \times \left(\frac{27\frac{1}{4}}{2}\right)^2\right]$

$= 557$

I for one rivet hole $= 2\left[2\frac{1}{2}'' \times 1'' \times \left(\frac{22\frac{3}{4}}{2}\right)^2\right]$

$= 646{\cdot}5$

Total for rivets $= 2044{\cdot}5$

Therefore deducting for rivets, net value of I

$= 7{,}370{\cdot}5$

Therefore $M_R = \frac{fI}{y} = \frac{6 \times 7{,}370{\cdot}5}{15}$

$= 2{,}948$ inch-tons (nearly).

The less the thickness of the flange and the smaller the rectangular areas, the more accurate this formula becomes. On the other hand, it is only an approximation and would be quite inaccurate for a beam I section (say) 4 inches wide and 6 inches deep, where the areas would become very large in proportion to y or the depth of the beam.

This method of finding I is not as accurate as either of the preceding methods given.

EXAMPLE.

M_R by the use of Equivalent Areas.—To find the strength of a beam of unsymmetrical section, *e.g.*, a steel trough floor.

The trough itself is symmetrical, but has a cover plate on top.

This is shown in Plate I, being 2′.8″ wide, 1′.1″ deep, 51.8 lbs. per foot of covered area.

The trough is strengthened with a cover plate on top, 9″ × ¼″.

NOTE.—This may sometimes be necessary for joining the troughs together as well as for strength.

Consider one half of the trough only; *i.e.*, one half corrugation.

Use the formula $M_R = fad$.

To do this we must first find "a," or the equivalent flange area, either above or below the neutral axis; either will suffice for our purpose.

Therefore first find the position of the centre of gravity, which will give the neutral axis.

In order to do this, it is necessary to take moments of the component areas of the section round the bottom edge of the section, or round any other convenient line.

The sectional area of half the trough without the cover plate on top = 18·84 square inches (found from tables or planimeter).

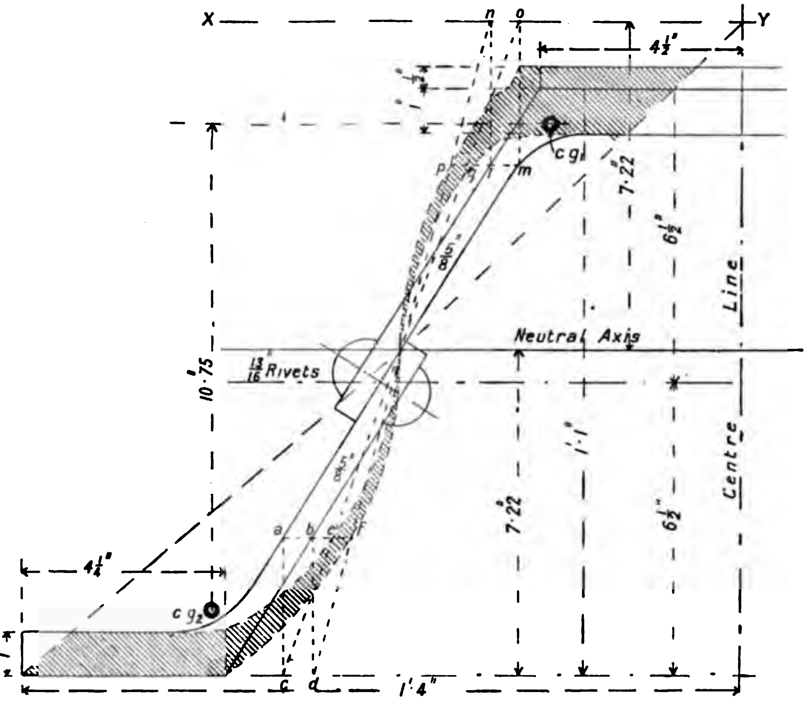
The sectional area of the cover plate is

= 2·25 square inches.

Taking the moment of these round the bottom edge of the trough.

$$\text{The moment of the trough section} = 18{\cdot}84 \times \frac{13}{2}.$$
$$= 122{\cdot}46$$
$$\text{The moment of the cover plate} = 2{\cdot}25 \times 13{\cdot}25.$$
$$= \underline{29{\cdot}81}$$
$$\text{Total moment} = 152{\cdot}27.$$

PLATE I.—SECTION OF STEEL TROUGH.



Then divide this moment by the total area, and obtain the distance of the centre of gravity of the whole figure from the bottom edge:—

$$\text{Required distance} = \frac{152{\cdot}27}{18{\cdot}84 + 2{\cdot}25}$$
$$= 7{\cdot}22 \text{ inches.}$$

This fixes the neutral axis.

Where the troughs overlap and are riveted together, only one thickness is considered as operative and useful.

It is now required to find "*a*" the equivalent area.

Consider any fibre of the trough, in the lower half of the section, *e.g.*, fibre *ab*.

From *a* and *b* draw perpendiculars to the bottom line of the trough, produced, meeting this line at *c* and *d*, and join *c* and *d* to *O*, the point where the neutral axis cuts the centre line of the trough section.

These lines will intercept some length *ef* on the line *ab* or *ab* produced.

Plot these two points *e* and *f* as points on the equivalent area diagram.

A similar process must be gone through for all fibres in the lower half of the section, obtaining a number of points similar to *e* and *f*, which will give the equivalent area. This area thus found is shown shaded in the figure; it has been drawn by constructing for fibres similar to *ab* about ¼ inch apart.

The area of this shaded figure must be measured and is found to be (by planimeter) = 7·15 square inches.

The same process must be applied to the upper half of the trough, but since the upper edge of the trough section is nearer to the neutral axis than the lower edge, a line of reference *XY* must be drawn, before making the projections of fibres, which is the same distance from the neutral axis as the lower edge of the trough, viz., 7·22 inches.

When this new line of reference has been drawn the construction is the same as for the lower portion.

We have now found the equivalent area both for the top and the bottom portion of the trough section, and if our construction is correct these two areas should be equal and each equal to 7·15 square inches.

Hence 7·15 square inches is the equivalent area of the section; "*d*" still remains to be found.

Now *d* is the distance between the centres of gravity of the upper and lower equivalent areas.

These *c* of *g* must therefore be found; and this is most easily done by cutting out these areas accurately in cardboard or tin to scale, and suspending them against a vertical drawing board or other vertical surface, by a pin, stuck through a very small hole in the tin template so that it can swing freely.

A thin string or line with a weight fastened to the pin or point of suspension, must pass through the *c* of *g* of the section or template.

The vertical line so found should be marked on the template, and the template should then be suspended from another point and another line drawn upon it in the same way.

The intersection of these two lines thus found is the *c* of *g* of the template.

This method is not very accurate, but is far the simplest, analytical solution being out of the question.

The *c* of *gs* of the shaded areas in this example have been found thus and are found to be 10·75 inches apart.

Now $M_R = fad$

$= f \times 7{\cdot}15$ inches $\times\ 10{\cdot}75$ inches

assuming $f = 7$ tons per square inch

$= 538{\cdot}04$ inch-tons.

DEFLECTION.

There are two deflection formulæ:—

(a) $$\Delta = \frac{nWl^3}{EI}.$$

This gives the actual deflection under any specific load. *W* and *E* must be in the same units.

(b) $$\Delta = \frac{n'fl^2}{Ey}.$$

This gives the deflection when the outside fibres of the beam are fully stressed up to the maximum permissible value of f, and is independent of the shape of the section, or the method of support or loading of the beam.

EXAMPLE.

A rolled steel I beam, 8 inches by 4 inches, 18 lbs. to the foot, is supported at both ends over a span of 14 feet, and is loaded with a total distributed load of 4·64 tons.

To find the deflection produced.

It is assumed that the beam has been so selected that it will just produce sufficient M_R to safely resist the M_B produced by the load.

In this case the extreme or outer fibres will be fully stressed to 7 tons per square inch under the load, and the formula that introduces f, but not I or W may be used.

$$\Delta = \frac{n' f l^2}{E y}.$$

Here $l = 168$ inches.

$f = 7 \times 2{,}240$ lbs.

$n' = \frac{5}{48}$ from Table II.

$E = 30{,}000{,}000$ lbs. per square inch for steel.

$y = 4$ inches.

$$\therefore \Delta = \frac{5}{48} \times \frac{7 \times 2{,}240}{30{,}000{,}000} \times \frac{168^2}{4}$$

$$= \cdot 385 \text{ inches.}$$

This deflection is not excessive, as it does not exceed $\frac{1}{360}$ of the span.

The value of f is taken in pounds so as to be in the same unit as E. Both might equally well be in tons.

EXAMPLE.

A steel I beam, 9 inches by 4 inches, 21 lbs. per foot run, is supported at both ends over a span of 20 feet.

It is loaded with a central concentrated load.

The deflection under this load is observed to be ·717 inch.

To find whether the steel beam is safe.

Use the formula $$\Delta = \frac{n'fl^2}{Ey}.$$

Here $n' = \frac{1}{12}$

f is unknown.

$l = 240$ inches.

$y = 4\frac{1}{2}$ inches.

$E = 30{,}000{,}000$ lbs.

$\Delta = \cdot717$ inch.

Therefore $$\cdot717'' = \frac{1}{12} \times \frac{f}{4\frac{1}{2}} \times \frac{240^2}{30{,}000{,}000},$$

and $f = 20{,}160$ lbs.

$= 9$ tons per square inch.

This is excessive and the beam is unsafe.

Example.

A B. S. B.,* 8 inches by 5 inches, 28 lbs. per foot run, is fixed as a cantilever, with a span of 10 feet.

A load of 1·4 tons is resting on its end, *i.e.*, 10 feet from the point of fixing.

What will the deflection be?

Here it is required to find the actual deflection under a certain definite load, and not the maximum deflection when the beam is stressed to its maximum limit.

The formula is therefore

$$\Delta = \frac{nWl^3}{EI},$$

where $n = \frac{1}{3}$, *vide* Table II.

$W = 1\cdot40$ tons.

$l = 120$ inches.

$E = 30{,}000{,}000$ lbs.

$I = 89\cdot4$ inches from B. S. S. Tables.

$$\therefore \Delta = \frac{1}{3} \times \frac{1\cdot4 \times 2{,}240}{30{,}000{,}000} \times \frac{120^3}{89\cdot4}$$

$$= \cdot6735''.$$

---

* British Standard Beam.

CHAPTER VIII

STRUTS AND TIES.—TENSION AND COMPRESSION MEMBERS

GENERAL.

In the ensuing discussion every material is supposed to be uniform and homogeneous throughout its cross-section. Any error in this assumption is allowed for in the usual factor of safety.

It has been proved that when a bar is subjected to either tension or compression, the stress distributes itself uniformly over the whole section of the member or bar.

TENSION.

Tensile stress is independent of the length of a bar or member.

Neglecting the weight of the bar itself, let —

F_t = total safe pull that the bar will stand.

a = cross section of the bar in square inches.

f_t = safe resistance to tension of the material of the bar or member.

Then

$$F_t = a \times f_t.$$

Hence F_t is known for any bar.

Example.

A steel bar has an average ultimate intensity of tensile stress of 32 tons per square inch (Table III.).

Assume a factor of safety of 4.

Suppose the bar to be $\frac{7}{8}$ inch diameter.

Then $F_t = \text{cross-section} \times \frac{32}{4}$ tons.

$= 4{\cdot}8$ tons.

UNSYMMETRICAL STRESS.

It is obvious that Figs. 122 and 123 must represent to some extent the state of stresses in a beam when symmetrically and unsymmetrically strained.

Failure would take place in a beam unsymmetrically strained before the whole tensile strength of the beam was fully developed.

The necessity for careful design of joints and connections is therefore obvious.

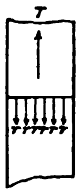


Fig. 122.

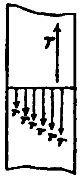


Fig. 123.

HOLES, NOTCHES, ETC.

The strength of a bar is that of its weakest section. Therefore in considering any area "*a*" over which f_t acts, the minimum effective section must be taken.

If a hole be bored in a bar this reduces its effective sectional area (see Fig. 124).

If two or more holes be made in line with each other (Fig. 125) only one need be deducted.

If two holes are side by side (Fig. 126), two must be deducted.

If there are three holes, (Fig. 127), breaking joint, experiment proves that the distance *cc* must be greater than *b*, and in that case only two holes need be deducted from the cross-section.

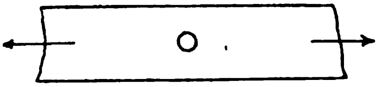


Fig. 124.

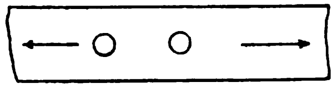


Fig. 125.

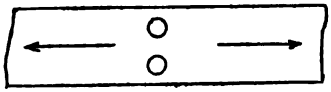


Fig. 126.

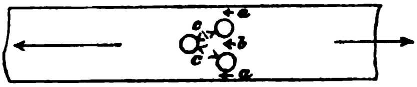


Fig. 127.

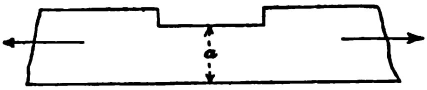


Fig. 128.

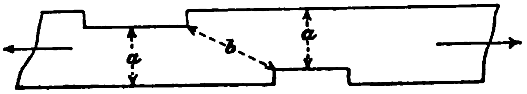


Fig. 129.

A simple notch (Fig. 128), reduces the effective breadth of "*a*."

Another notch (Fig. 129) will not decrease the strength, provided that b is greater than a.

If b is less than a, then b is the effective breadth.

Example.

Find dimensions of a flat steel bar for the tie rod of a roof. The stress to be carried is 7·2 tons, and the bolts for connecting the ends are ⅞ inch.

Assume f_t for steel = 7 tons per square inch.

Then since $F_t = a \times f_t$.

$$\therefore a = 7{\cdot}2 \div 7 \text{ square inches.}$$
$$= 1{\cdot}03 \text{ square inches.}$$

Assume a thickness for the bar of ½ inch.

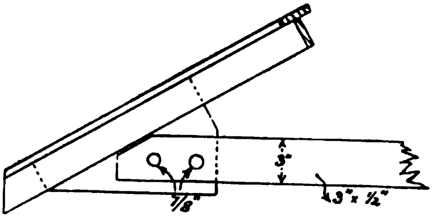


Fig. 130.

Then to give the required section, the breadth must be

$$= 1{\cdot}03 \div \tfrac{1}{2}$$
$$= 2{\cdot}06 \text{ inches.}$$

Assume that the ends of the tie bar are secured by ⅞-inch pins or bolts.

Then the diameter of the bolt hole must be added to this, so that the breadth comes to 2·06 + ⅞ inch = 3 inches (nearly).

The joint would be arranged as in Fig. 130. and the tie bar would be 3 inches × ½ inch flat.

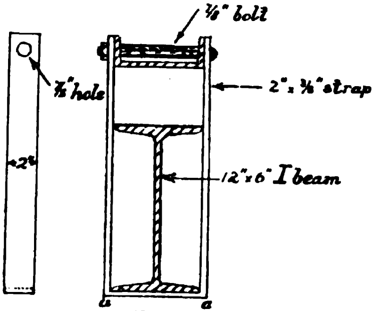


Fig. 131.

Example.

A flat steel bar, 2 inches × ⅜ inch, is carried as a supporting strap under an I steel beam (Fig. 131).

What weight may safely be put on the beam?

The upper ends of the strap are fixed by a ⅞-inch bolt, placed centrally, and their attachment may be considered as safe.

Allowing for the ⅞-inch hole, by which the strap is hung, the effective cross-section = (2 inches − $\frac{7}{8}$ inch) × $\frac{3}{8}$ square inch

= $\frac{27}{64}$ square inch.

Taking f_t = 7 tons for steel.

Safe pull = $\frac{27}{64}$ × 7 × 2 tons (since strap is double)

= 5·9 tons.

Therefore, as far as tension is concerned, the steel I beam and its load may weigh 5·9 tons.

This strap, however, should be tested for shearing at *a*, *a*.

Cross-section there = 2 × 2 × $\frac{3}{8}$ square inches

= 1·5 square inches.

Taking f_s (safe resistance to shear) = 4·8 tons.

Safe stress = 1·5 × 4·8

= 7·2

5·9 tons is therefore the greatest permissible load.

EXAMPLE.

The lower flange of a plate girder has to withstand a tension of 67 tons. To find of what sections of steel it is to be composed.

The flange is to be made of a plate riveted to two angle steels, as shown in Fig. 132, and these angle steels are separated by a $\frac{1}{2}$-inch web plate, which will be omitted from this calculation.

Suppose the angle steels to be 4 inches by 4 inches by $\frac{1}{2}$ inch. And suppose that the holes for the rivets joining the angle steels to the plates are to be $\frac{7}{8}$ inch in diameter.

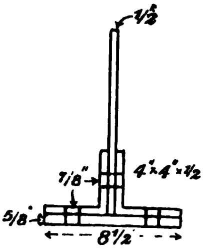


FIG. 132.

Then each angle will have two $\frac{7}{8}$-inch holes through it.

These must be deducted from the sectional area in arriving at the section available for resisting tension.

The net sectional area of one angle steel will be

= (8 inches − $\frac{1}{2}$ inch − 2 × $\frac{7}{8}$ inches) $\frac{1}{2}$ inch

= (5$\frac{3}{4}$ inches) $\frac{1}{2}$ inch.

And for the two will be

= 5·75 square inches.

The safe stress on these two angles is

$= 5{\cdot}75 \times 7$ tons, taking 7 tons per square inch as the safe working load for steel in tension

$= 40{\cdot}25$ tons.

This leaves 26·75 tons to be taken by the plate which is to be riveted to the angles.

Now the safe working load is 7 tons per square inch.

Hence the area required for the remainder of the tension, 26·75 tons, is

$$= \frac{26{\cdot}75}{7}$$

$= 3{\cdot}82$ square inches.

This is therefore the area of the plate required.

Assume the plate to be $8\frac{1}{2}$ inches wide.

This plate will also have two rivet holes through it, $\frac{7}{8}$ inch diameter each.

Hence whatever the thickness of the plate is to be, the net width will be

$$= 8{\cdot}5 \text{ inches} - 2 \times \tfrac{7}{8} \text{ inches}$$
$$= 6{\cdot}75 \text{ inches.}$$

Then if t be the thickness required,

$$t \times 6{\cdot}75 \text{ inches} = 3{\cdot}82.$$

Therefore $t = 0{\cdot}57$ inches nearly

$= \frac{5}{8}$ inch.

Therefore the plate will be $\frac{5}{8}$ inch thick, and the sizes required are as shown in Fig. 132, viz.:—

Two angles, 4 inches by 4 inches by $\frac{1}{2}$ inch.
One plate, $8\frac{1}{2}$ inches by $\frac{5}{8}$ inch.

COMPRESSION MEMBERS.

The ultimate resistance to tension and compression of various materials is given in Table III.

For compression these values are obtained by experiment on small cubes of the material and they are useful only for the following :—

(a) For computing the dead load that a block of the material will carry without disintegration or being crushed.

(b) For assessing the maximum intensity of stress that a fibre of the material may be allowed to take.

The safe resistance to crushing per square inch is usually known as f_c.

f_c is not directly applicable to struts or columns of any considerable length in calculating the force that will produce failure, because a strut finds relief from the compressive stress by lateral bending, and the load causing failure on any strut is found by experiment to vary inversely as the square of the length of the column (approximately).

SYMMETRICAL APPLICATION OF STRESS.

An unsymmetrical application of the thrust is obviously bad as it assists the column to buckle. Every effort must, therefore, be made to communicate the load symmetrically to the member.

If this is not possible then the bending moment in the strut caused by the load must be allowed for.

HOLES.

There is no need to deduct holes at the ends of compression bars when ascertaining their effective cross-section.

A consideration of Fig. 133 will show this clearly, there being really no need for the material at the back of the bolt.

In the case of riveted work, rivets completely fill their holes, being put in hot and squeezed in by pressure; there is, therefore, no real need to deduct such holes from the effective section of a compression bar if the holes are completely filled. It is, however, usually done.

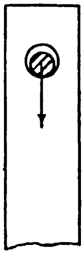


FIG. 133.

ENDS OF STRUTS.

It is obvious that a strut with rounded or hinged ends (Fig. 134) will bend more easily than a strut with fixed ends, *i.e.*, where the ends are held firmly in position.

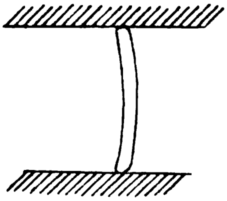


FIG. 134.

It is necessary, therefore, to divide struts up into three classes:—

(a) Ends rounded or hinged.

(b) Ends fixed.

(c) One end fixed and one end rounded or hinged.

(a) and (b) are dealt with in the formulæ and tables referred to below. (c) will have to be obtained by interpolation.

In steel structures, members with pin and link joints or light riveted joints are considered as hinged or free at their ends.

Heavy riveted joints are considered as fixing the ends of a member if the part of the structure to which they are so joined is strong enough.

FIXITY.

It must be remembered that merely fixing the ends of a strut is not always good enough.

The member, foundation, or wall, etc., to which the strut is fixed must be capable of exerting the necessary moment to prevent the strut bending. If it be unable to do this no advantage is gained by fixing.

In some cases the fixity of a strut may be a real disadvantage, as the member to which the strut is fixed may itself be liable to bending, and may thus cause the strut to deflect out of the straight.

It is, therefore, not the custom to consider members as fixed unless the fixing is beyond question or shadow of doubt.

GENERAL METHOD OF CALCULATION.

The struts to be considered here may be of steel, wrought iron or cast iron.

They are made of different sections, the most usual of which are shown on Table IV.

An attempt is made in designing the form of these sections to group the metal of the member as far out from the centre of gravity or "centroid" of the section as possible, so as to be able to resist bending.

The formula for the solution of simple struts is given in Table V., viz.:

$$\frac{l}{k} = n\frac{l}{h}$$

Where l = length of the strut in inches.

k = least radius of gyration of the section.

f_c = safe resistance to compression per square inch.

h = least dimension of the strut.

n = a constant for this formula depending on the section of the strut.

It is the ratio of the least dimension of the strut to the radius of gyration, and is given for all common sections of struts in Table IV.

This formula is ready for use and it is a very easy one for simple struts and all ordinary cases.

It is an adaptation from others.

It will be illustrated by examples.

It should be noted that the radius of gyration of the sections has not been previously alluded to, and there is no need to know it for the use of this formula, as it is contained in the calculations for the table only.

USE OF FORMULA FOR STRUTS.

Choose a trial section to suit the particular case, *e.g.*, a 3-inch by 3-inch by ½-inch T bar. Refer to Table IV. and find—

$n = 4{\cdot}9$.

$h = 3$ inches.

Suppose strut to be 10 feet long (120 inches).

Apply formula, $\frac{l}{k} = n\frac{l}{h}$, and find $\frac{l}{k}$.

Then $\frac{l}{k} = 4{\cdot}9 \times \frac{120}{3}$ inches.

$= 196$.

Look for 196 in the first column of Table V., and obtain a value of f_c for steel, ends fixed;

$f_c = 1{\cdot}9$ tons per square inch.

Then total safe thrust on the strut—

$F_c = 1{\cdot}9 \times$ net sectional area of the T bar.

$= 1{\cdot}9 \times \frac{1}{2}$ inch $\times$ (3 inches + 3 inches − ½ inch).

$= 5{\cdot}225$ tons.

Table X. gives the safe loads on simple T struts of light section and short length, suitable for light framed structures.

EXAMPLE.

A Plain T Strut.—Find maximum safe load for a 10-foot strut of T steel 9 inches × $4\frac{1}{2}$ inches × $\frac{3}{4}$ inch, ends rounded.

From Table IV., $n = 4·2$

$h = 4·5$ inches.

Then $\frac{l}{k} = n\frac{l}{h}$

$= 4·2 \times \frac{120}{4·5}$ inches

$= 112.$

Then from Table V. steel, ends rounded.

$f_c = 2·02$ tons per square inch $= 2$ tons, say.

Area of section $= (9 + 4\frac{1}{2} - \frac{3}{4}) \times \frac{3}{4}$ inch

$= \frac{153}{16}$ square inches.

$\therefore$ safe thrust $= 2 \times \frac{153}{16}$ tons.

$= 19·12$ tons.

Note.—It will be seen that f_c would be twice as large, if ends were fixed.

Example.

A Cast-Iron Pillar.—Find the diameter of a cast-iron pillar 12 feet high to bear a weight of 10 tons.

Note.—Since a *C.I.* pillar has good broad, thick ends, it may be considered as having ends fixed when loaded with a heavy dead load.

Proceed by trial and error.

Assume a 4 inches diameter column.

From Table IV., $n = 3·1$

$h = 4$ inches.

$\therefore \frac{l}{k} = n\frac{l}{h} = 3·1 + \frac{144}{4}$

$= 111·6.$

Therefore from Table V.,

$f_c = 2·8$ tons per square inch (approx.).

The metal thickness of a column of this nature is one-tenth of the diameter.

Therefore the sectional area of the column

$$
\begin{aligned}
&= \pi \times (R^2 - r^2) \\
&= \frac{22}{7} \times (4 - 1{\cdot}7^2) \\
&= \frac{22}{7} \times 1{\cdot}11 \\
&= 3{\cdot}49 \text{ square inches.} \\
\text{Total safe load } &= 2{\cdot}8 \times 3{\cdot}49 \\
&= 9{\cdot}772 \text{ tons.}
\end{aligned}
$$

A larger section will therefore be required, say 4½ inches diameter.

Example.

Double Strut.—Design a wrought iron strut of the form shown in Fig. 135 for a roof truss; it must take a stress of 1·69 tons, the length being 2 feet 11 inches.

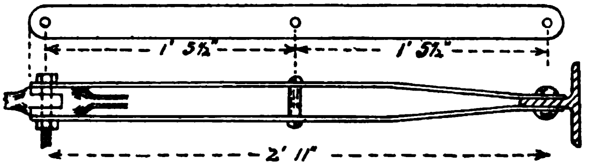


FIG. 135.

Struts of this form must be tested, firstly, in portions carrying portions of the stress; secondly, as a whole carrying the whole stress.

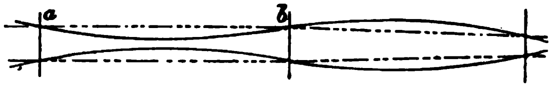


FIG. 136.

Firstly, when composed of two flat bars, stiffened by one or more ferrules, as shown in Figs. 135 or 137, the strut will tend to fail as in Fig. 136.

The simplest process for calculation is to assume a size for these flat bars, and then calculate the number of distance pieces or ferrules.

The portion of flat bar, *ab*, between ferrules (Fig. 136) may be considered as a separate strut carrying its own proportion of the whole load on the compression bar.

For instance, assume $1\frac{3}{4}$ inches × $\frac{3}{8}$ inch as the cross-section of each side bar. Then, as each has to bear half the stress or 0·85 ton, the intensity of stress will be

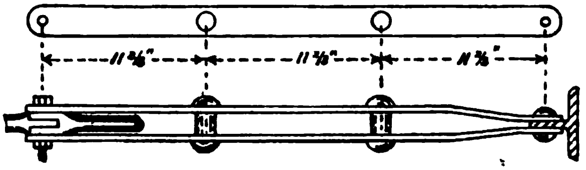


FIG. 137.

$$= \frac{0{\cdot}85}{1\frac{3}{4} \times \frac{3}{8}} = 1{\cdot}3 \text{ tons per square inch.}$$

On referring to Table V. it will be seen that this intensity of stress, ends rounded, corresponds to a ratio of $135 = \frac{k}{l}$. Hence, since $n = 3{\cdot}5$ for a rectangular cross-section, the maximum length of such a strut or in other words the safe distance between distance ferrules

$$l = \frac{3}{8} \times \frac{135}{3{\cdot}5}$$

$$= 14{\cdot}5 \text{ inches.}$$

One distance piece is, therefore, not sufficient, as the strut is 35 inches long, and it would be necessary to use two, as shown in Fig. 137.

Secondly, considering the strut as a whole, and again assuming the flat bars to be $1\frac{3}{4}$ inches × $\frac{3}{8}$ inch.

Then stress intensity is 1·3 tons per square inch, as above, and from Table V. $\frac{l}{k} = 135$.

But in this case $n = 3$ (fourth case, Table IV.)

$$\therefore \frac{l}{h} = \frac{l}{k} \times \frac{1}{n} = \frac{135}{3}$$

$$= 45,$$

Now $l = 2$ feet 11 inches $= 35$ inches, hence the size of the distance ferrule,

$$h = \frac{35}{45}$$
$$= \tfrac{3}{4} \text{ inch.}$$

∴ ferrule should be *at least* $\frac{3}{4}$ inch — the thickness of the flat bars.

In this case, for practical reasons, a $1\frac{1}{4}$-inch ferrule is used.

The flat bars should also be tested for bending the other way, viz., across their breadth. Here n would be 2 (second case, Table IV.), and l would be 35 inches.

There is no need to bother about the bolt holes where ferrules occur, for there will be no buckling tendency in the flat steel bars at these points, and the full value of f_c can therefore be counted on for resisting the thrust.

Example.

To design a strut in a braced girder composed of angle steels. The stress is 40·7 tons.

The length of the strut is $= 8\sqrt{2}$ feet.

The arrangement of the angles for this member is shown in Fig. 295 and Plate XI.

The angles are arranged in pairs back to back, and they are all joined into one member by a plate $\frac{3}{8}$ inch thick riveted between them.

The angles are $3\frac{1}{2}$ inches × 3 inches × $\frac{1}{2}$ inch.

When joined together as shown in the figure they form one strut 16 inches × $7\frac{3}{8}$ inches, the larger dimension being at right angles to the plane of the girder.

It is in the direction of the smaller of these two dimensions that the strut appears likely to fail first; this will mean bending in the plane of the girder, and in this direction the ends of the strut may be considered as "fixed."

Each pair of angles back to back is $7\frac{3}{8}$ inches across and will take one quarter of the total stress, *i.e.*, $\frac{40 \cdot 7}{4}$ or 10·17 tons.

Now the net sectional area of each pair is $3\frac{3}{4}$ square inches.

Therefore the intensity of pressure will be

$$= \frac{10{\cdot}17}{3\frac{3}{4}} \text{ tons.}$$

$$= 2{\cdot}71 \text{ tons per square inch.}$$

Referring to Table V., it will be found that for a load of 2·71 tons per square inch on steel struts with ends fixed

$$\frac{l}{k} = 150 = \frac{nl}{h}$$

Now $n = 4{\cdot}2$ from Table IV.

And $h = 7\frac{3}{8}$ inches.

Therefore $$l = 150 \times \frac{7\frac{3}{8}}{4{\cdot}2}$$

$$= 263 \text{ inches or } 22 \text{ feet.}$$

That is to say that the strut may be as long as 22 feet.

This is therefore amply safe, and these sizes will be used.

The finished strut may be seen in Plate XI., lettered *ld*.

Ideal Columns.

Consider a bar which is perfectly straight, of material that is perfectly homogeneous throughout, and where the axis coincides with the line of pressure; such a bar is "an ideal column." Suppose it to be subjected to a compressive force F_c (Fig. 138), then there will be excited in the bar a certain resilient force R, which corresponds to the spring of a bow. The behaviour of the bar will be as follows :—

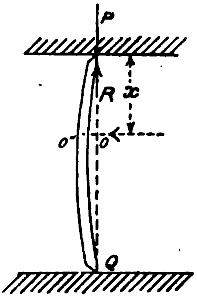


Fig. 138.

(1) As long as F_c is less than R there will be no deflection. If lateral pressure is applied to the bar it will deflect, but if that lateral force is withdrawn, the bar will straighten itself.

(2) If F_c is increased until it is equal to R, the load will not produce deflection, but any, even the smallest, force applied to the middle of the strut will be sufficient to cause it to bend, and if such force be withdrawn, the column will not now be able to straighten itself.

(3) If the force F_c be still further increased the column will be in a state of unstable equilibrium ; it will bend in *some*

direction without the application of any lateral force, and will continue to bend, or break.

What we want to find is the value of the resilient force R.

This value, for an ideal column, is given by:—

Euler's Formula.

$$R = EI\frac{\pi^2}{l^2}$$

It is sometimes written

$$p = \frac{\pi^2 E}{\left(\frac{l}{k}\right)^2}$$

where R = resilience of strut in lbs. or tons.

E = modulus of elasticity of the material in lbs. or tons.

p = pressure per square inch, *i.e.*, $\frac{R}{A}$, in lbs. or tons.

l = length of column in inches.

k = least radius of gyration of the cross-section, *i.e.* $\sqrt{\frac{I}{A}}$ in inch units, I being the least moment of inertia, and A the area, of the cross-section.

Explanation of Euler's Formula.—The proof of this formula appears unnecessary here. It may be found in most treatises on this subject.

The formula was first produced in the form $R = EI\frac{\pi^2}{l^2}$.

Now $R = F'_c$ (total pressure on the strut) in order to produce equilibrium.

$\therefore R = p$ (pressure intensity per square inch) × A (area of cross-section).

$\therefore$ substituting in above formula we get:—

$$p = E \times \frac{I}{A} \times \frac{\pi^2}{l^2}.$$

For simplicity I and A are combined, thus:—

$$\text{Let } k = \sqrt{\frac{I}{A}}.$$

Then k is called the radius of gyration of the section, and Euler's formula may be expressed thus:—

$$p = \frac{E\pi^2}{\left(\frac{l}{k}\right)^2}.$$

Radius of Gyration—Definition.—Many books give definitions of varying clearness, but the simplest is this:—

$$\text{Radius of gyration of an area} = \sqrt{\frac{I}{A}}.$$

$$\text{Radius of gyration of a mass} = \sqrt{\frac{I}{M}}.$$

Mathematics have shown the necessity for a symbol π to denote a certain ratio; so also do mechanics require symbols I and k to denote inherent properties of masses and areas.

The values of I and k for all sections in which steel and iron bars are rolled are given in manufacturers' catalogues. Table VI. also gives the values of k for various common sections in terms of their dimensions.

When Table VI. does not suffice, k can be found from the definition $k = \sqrt{\frac{I}{A}}$.

It should be noted that both a plane section of area A, and also a body of mass M, have each a moment of inertia and a radius of gyration.

Radius of Gyration; Explanation: or

I and A for a Plane Area.—We know that the moment of inertia of a plane area about any axis in that plane is the sum of the areas of a very large number of very narrow strips, multiplied by the squares of their distances from the axis.

Now if we can imagine the whole area to be concentrated on one spot, such that the moment of inertia of this whole area at this spot about the axis is equal to the real moment of inertia of the figure, then the distance of this spot from the axis is the radius of gyration,

e.g., $I = \text{sum of (small areas} \times y^2)$.

Then $k^2 \times$ whole area $= I$.

$$\therefore k^2 = \frac{\text{sum of (small areas} \times y^2)}{\text{whole area}}$$

$$= \text{sum of } (y^2).$$

$$\therefore k = \sqrt{\text{sum of } (y^2)}.$$

Illustration.—Suppose that we want to find the distance of the *c.g.* of the whole figure (Fig. 139) from the axis *AB*. The well-known rule is, x (the required distance) × (sum of the three rectangles) = sum of areas of each rectangle × the distances of their *c.gs.* from the axis, viz.:—

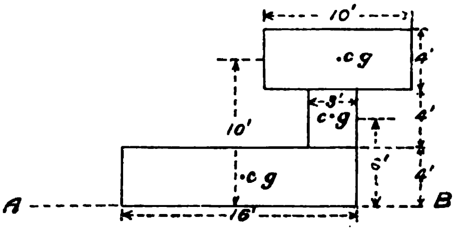


FIG. 139.

$$x \times (40 + 12 + 64) = 40 \times 10 + 12 \times 6 + 64 \times 2.$$

$$\therefore x = \frac{600}{116} = 5{\cdot}17 \text{ ft.}$$

The process for k is nearly similar, viz.:—

k^2 × (whole area) = sum of areas of each rectangle × the squares of the distances of their *c.gs.* from the axis, viz.:—

$$k^2 \times 116 = 40 \times 10^2 + 12 \times 6^2 + 64 \times 2^2.$$

$$\therefore k^2 = \frac{4{,}688}{116}$$

$$= 40{\cdot}41$$

$$\therefore k = 6{\cdot}3.$$

NOTE.—This forms a good illustration of k, but the result obtained is not accurate, because the areas considered should be infinitely small. Their summation of course is easy by the Calculus.

Care must be taken to use the proper values of I and k. In all standard sections two values of I are given. The larger is usually applied in calculating the M_R of beams, whereas the smaller must usually be taken in calculating struts to resist failure by bending.

GORDON'S FORMULA.

This is the formula most usually employed in this country.

It is:

$$F_c = \frac{f_c \times A}{1 + a\left(\frac{l}{h}\right)^2}.$$

Where F_c = total pressure on the column in lbs. or tons.
f_c = safe intensity of resistance to crushing of the material in lbs. or tons (per square inch, etc.)
h = least transverse dimension of the column.
l = length of the bar in the same unit as h.
a = empirical constant.

The value of a varies with the material and with the shape of the column.

Table VII. gives some of these values.

This formula and Euler's approximate very closely together in most ordinary cases, and they are prepared for use in Tables IV. and V.

These tables are widely used and give a great saving in time in calculations.

The practical strength of struts will nearly always exceed the calculated strength, provided care is taken to apply the stresses and loads symmetrically.

COMBINED BENDING AND DIRECT STRESS.

It has been shown that a strut tends to bend when under a heavy thrust.

But if the strut be made strong enough to take the strain there will be no tendency to bend and the thrust will then be distributed evenly over the whole cross-section of the beam.

Now a strut may also be subjected to a definite *transverse* force or load tending to break it.

Simple Solution.

The method of dealing with simple cases of steel or timber struts of this description is to consider the strut, or member, or beam as being divided into two parts and as resisting the two separate stresses separately.

EXAMPLE.

Suppose that an angle steel 7 inches by $3\frac{1}{2}$ inches by $\frac{3}{4}$ inch is subjected to a transverse loading of $1\frac{1}{2}$ tons distributed along it, and is also under direct thrust (Fig. 140).

Its length is 8 feet and its ends are fixed.

What is the greatest direct thrust that it can safely take?

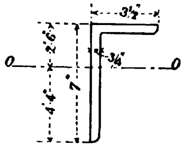


FIG. 140.

The M_B due to the load is

$$= 1{\cdot}5 \times \frac{8 \times 12}{8}$$

$$= 18 \text{ inch-tons.}$$

The M_R of the angle is $= \frac{fI}{y}$.

From tables I or greatest moment of inertia is 35·68, y is the distance from the neutral axis to the extreme fibre of the section, or 4·4 inches.

Hence $$M_R = f \times \frac{35{\cdot}68}{4{\cdot}4} \text{ inch-tons.}$$

$$= M_B$$

$$\therefore f \times \frac{35{\cdot}68}{4{\cdot}4} = 18 \text{ inch-tons (from above).}$$

Solving this we shall find what is the intensity of the stress per square inch on the metal, *i.e.*,

$$f = 2{\cdot}22 \text{ tons per square inch.}$$

This is the maximum amount to which the transverse load alone will stress the metal of the angle.

Now since this member is a strut, we must find out what is the maximum value that may be given to "f" in order that it shall be safe against buckling.

By the strut formula, and Tables IV. and V.,

$$h = 3\tfrac{1}{2}$$

$$n = 3{\cdot}9$$

$$l = 96 \text{ inches.}$$

Therefore $$n\frac{l}{h} = 3{\cdot}9 \times \frac{96}{3\frac{1}{2}}$$

$$= 107$$

$$= \frac{l}{k}.$$

Refer to Table V. and find value of $\frac{l}{k} = 107$.

This gives a value for f_c "ends fixed" of 3·82 tons per square inch.

Now the transverse loading has already stressed the metal to 2·22 tons, and therefore there only remains 3·82 — 2·22 tons, or 1·60 tons per square inch for direct thrust.

Now the sectional area of the angle is 7·31 square inches.

Hence, maximum safe direct stress is

$$= 7{\cdot}31 \times 1{\cdot}60 \text{ tons.}$$
$$= 11{\cdot}696 \text{ tons.}$$

This result will be found to be slightly on the safe side.

The full intensity of pressure, viz., 3·82 tons per square inch, will of course only be exerted on the fibres of the strut on the side on which the lateral force is applied.

On that side of the strut away from the lateral thrust, the fibres of the strut are less stressed, and in this example would actually be in tension, the tensile stress per square inch due to cross bending being greater than the compressive stress due to end loading.

Accurate Investigation of Combined Bending and Direct Stress.—The foregoing method of solution is not accurate enough for any very important structure, since it assumes that the strut does not actually bend at all, and takes no account of that tendency to bend.

In reality the strut has to resist some direct force F (Fig. 138), a bending moment M_1 due to external transverse loading, and also a bending moment due to the direct force F, "*as soon as the strut begins to bend.*" (This moment in Fig. 138 is $F \times 00''$).

Then the formula for transverse stress is this:—

$$f = \frac{M_1 y}{I \pm \frac{Fl^2}{10E}}.$$

Where $M_1 = M_B$ at point of maximum deflection due to external forces.

$F =$ Total direct load on member, tension or compression.

$f =$ Safe stress intensity per square inch.

y, I, E, and l are as usual.

The *negative* sign is for *compression* bars.

The *positive* sign is for *tension.*

This formula gives the f or stress intensity at the extreme fibres, either compression at one side of the bar or tension at the other.

There is no factor of safety in this formula, and the fibre stress, f, found is the actual stress produced by the loading.

CHAPTER IX

RIVETS AND JOINTS

GENERAL.

Broadly speaking the joints between different members of a bridge or other steel structure are made either by rivets or by pin joints.

These two methods are known as "plate and rivet" construction and "pin and link" construction.

A rivet put into a hole and clinched by modern machinery is practically perfect, completely filling the hole into which it is pressed, however uneven the inside edges of the hole may be.

In cutting out rivets from old work, a hand-made rivet can generally be hammered out of its hole, after the head has been cut off; but this is not so with hydraulic pressed rivets.

To do this it is necessary to drill them out, and as a rule a section through work in which a rivet has been properly put in, will not show any visible joint between the fibres of the rivet and the fibres of the surrounding metal.

This perfect contact enables the designer to calculate with more certainty than he could in the past, the bearing and shearing strength of riveted joints.

Imperfect contact is a grave source of weakness, and for this reason should be avoided as much as possible.

Riveted joints are much easier and cheaper to make than pin joints; for rivets, as stated, completely fill their holes and ensure perfect contact without further trouble, whereas the pins for pin joints have to be most accurately turned and fitted, as also have the link ends in order to secure good and proper bearing surface.

Riveted joints are, however, impracticable in the class of work in which pin and link joints are used.

It is impossible to lay too much stress on the need for care in designing riveted joints.

A large number of bridges erected in the past have now proved to be too weak for modern loads, not so much in the plates and members, as in the rivets and joints.

A rivet must never be used in tension, *i.e.*, lengthwise. There is no strength in the head of a rivet to resist such a pull.

Torsion also must be avoided.

The only two stresses which a rivet is intended to resist are—

(a) Shearing stress.

(b) Bearing stress.

These two are closely allied, and failure of a riveted joint would generally be due to a combination of the two.

Resistance to bearing and resistance to shearing undoubtedly help one another, although no allowance is made for this.

A riveted joint must always be designed to take the maximum safe stress which the plates or members which it connects are able to carry, with safety.

If, for example, there is to be a pull of 8 tons in a member, and it is convenient to use steel plates for this member which are capable of taking a pull of 10 tons, and are to be joined together end to end by rivets, the rivets in the joint must also be designed to take a pull of 10 tons, and not only of 8 tons.

STANDARD SIZES OF RIVETS.

The usual sizes for rivets are:—

$\frac{1}{2}$ inch, $\frac{5}{8}$ inch, $\frac{3}{4}$ inch, $\frac{7}{8}$ inch, and 1 inch diameter. They can be obtained up to $1\frac{1}{4}$ inches diameter, but such rivets are not often used.

Rivets are inserted into the holes prepared for them red hot, at what is best described as "cherry heat."

At this temperature they expand considerably in diameter and therefore require a slightly larger hole than they would when cold, in order that they may fit in easily.

Holes are therefore made $\frac{1}{16}$ inch larger than the diameter of the rivet.

Thus while standard rivet sizes are in eighths, viz. $\frac{5}{8}$ inch, $\frac{3}{4}$ inch, $\frac{7}{8}$ inch, their corresponding holes are $\frac{1}{16}$ inch larger, viz., $\frac{11}{16}$ inch, $\frac{13}{16}$ inch, $\frac{15}{16}$ inch. This must be borne in mind when making deductions for rivet holes in calculating the strength of plates, etc. The holes are, however, called by the diameter of the rivet, *i.e.*, a 1-inch hole means a hole $\frac{17}{16}$ inches diameter.

NOMENCLATURE.

In Fig. 141 the portion *a* of the rivet is called the "tail" of the rivet.

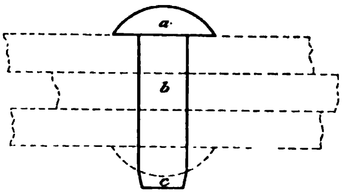


FIG. 141.

The stem or portion *b* which is to be within the hole is called the "shank," or "grip."

The portion *c* which is left projecting out, and which is to be hammered over to form the head, is called the "clink."

The clink when pressed or hammered down into shape, becomes the "head."

SAFE STRESSES.

The stresses "shearing" and "bearing" have been defined in Chapter I.

Rivet steel is softer than plate steel and very ductile. It is usually specified to stand a tensile stress of 26 to 30 tons per square inch, and is subjected to very severe bending tests.

The safe resistances are:—

Bearing, 8 tons per square inch.

Shearing, 6·5 tons per square inch.

The safe resistance to bearing is sometimes taken as 10 tons per square inch; but this value is high and only allows a factor of safety of just over 3.

It is not considered sufficiently safe for common practice, although there are some few cases, such as dead loading or similar, in which this higher stress might possibly be allowed. (*vide* Table IX.).

FORM OF JOINTS.

Single lap joint.—The simplest joint is that in which one rivet connects two flat bars in tension (Figs. 142, 143), the one bar being lapped over the other.

Fig. 144 shows the manner in which this joint tends to deform. This tendency is very undesirable as it brings an unfair stress on the rivet, and if any deformation takes place the rivet is put in tension.

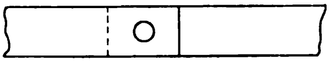


Fig. 142.

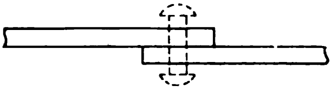


Fig. 143.

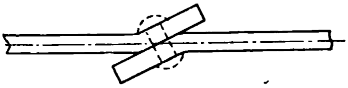


Fig. 144.

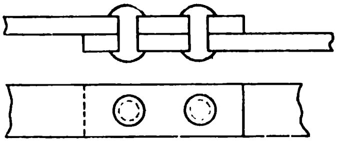


Fig. 145.

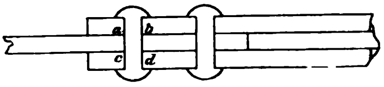


Fig. 146.

A single lap joint with two rivets is shown in Fig. 145. This is an improvement on that in Fig. 144 but is still liable to slight deformation.

It is, however, often used in joining tension bars in light structures, or joining plates together merely to keep them in close contact.

Double lap joint.—A double lap joint is shown in Fig. 146.

Here two plates are lapped over one plate, one on either side of it. This avoids any tendency to deformation, and the pull is transmitted perfectly symmetrically from the two plates to the one plate.

The obvious drawback to this joint is that one member must be a double one, and the other a single one, so as to allow of the lapping arrangements.

It is often used in joints of a steel roof truss, or similar truss, where a third member is also taken in at the same joint, and where the stress carried by the double member is larger than that carried by the single member.

All the above joints are used chiefly for tension members in light structures.

SINGLE COVER PLATE JOINT.

In this joint the ends of the plates, bars, or members to be joined are butted against one another, and a small plate, called a cover plate, is placed against them and riveted to both of them.

This cover plate transmits the stress from one to the other (Fig. 147).

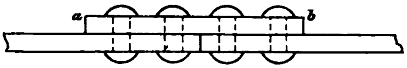


Fig. 147.

The cover plate must of course have the same net cross-section as the members which it unites.

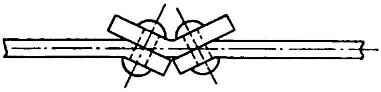


Fig. 148.

Fig. 148 shows the tendency to deformation in such a joint when only a single cover plate is used. Were it not for this deformation, such a joint would be very satisfactory, as it keeps the two plates which are being joined in the same plane.

DOUBLE COVER PLATE JOINT.

This joint is shown in Fig. 149.

The deforming tendency is here overcome by using two cover plates. These cover plates need not be so thick as the single plate; all that is required is that their combined section should be equal to the net section of those that are being joined.

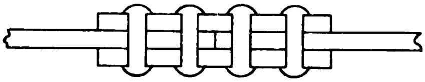


Fig. 149.

Cover plate joints are used for all heavy work, and when there is any possibility of deformation double cover plates must be used.

They necessitate using twice as many rivets as lap joints, as there are two separate transmissions of the stress. All these joints apply both to tensile and compressive stresses.

EFFECTIVE SECTION.

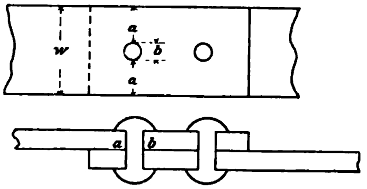


Fig. 150.

The effective section of a plate, bar or member is its cross-section at its narrowest part, *i.e.*, its cross-section after any rivet hole or holes in it have been deducted.

In Fig. 150 the effective section is $= a + a$

$$= 2a$$

or $= w - b$.

It should be noticed that since the two rivet holes are placed one behind the other, the effective section of the bar is only reduced by one rivet hole, not two.

CALCULATION OF STRENGTH OF RIVETS IN A JOINT.

Shearing.—The cross-section of any rivet, of radius r inches,

$= \pi r^2$ square inches.

The safe resistance to shearing

$= 6{\cdot}5$ tons per square inch.

Therefore the total resistance to shear of a rivet is

$= \pi r^2 \times 6{\cdot}5$ tons.

Furthermore, if there be more than one rivet, say n rivets properly arranged in one joint, then the total resistance to shearing is

$= n \times \pi r^2 \times 6{\cdot}5$ tons.

This holds good for all rivets in single shear.

SINGLE SHEAR.

The term single shear means that the stress at the joint tends to shear the rivet or rivets across at one single section, and that that single cross-section of the rivet or rivets has to resist the shearing.

Figs. 143 and 150 show joints in which each rivet is subjected to single shear, viz., at *ab* (Fig. 150).

DOUBLE SHEAR.

Fig. 146 shows a joint arranged differently.

In the first place there is no tendency for the joint to deform, as shown in Fig. 144; but also, in order that the joint may fail by shearing, each rivet must be sheared through at two sections, viz., *ab* and *cd* (Fig. 146).

Now the resistance to shearing at each section is $= \pi r^2 \times 6{\cdot}5$ tons, as before.

Hence this joint is much stronger than a single shear joint.

In practice it is found unsafe to rely on its being twice as strong as single shear, and a double shear joint is taken as 1¾ times as strong as a single shear joint, viz,, the joint in Fig. 146 for two rivets has a resistance to shearing $= 1\frac{3}{4} \times \pi r^2 \times 6{\cdot}5 \times 2$ tons.

Rivets in double shear may therefore be considered 1¾ times as strong as rivets in single shear, and not only has the joint so formed no tendency to deformation, but the tensile or compressive stress in the members is more symmetrically transmitted from one to the other than in a joint such as Fig. 143.

BEARING.

The portion of the shank of a rivet which is in contact with the inside of the hole in a plate or other member, is known as its bearing area; *i.e.*, the surface of contact between the rivet and the plate or member through which the stress is transmitted.

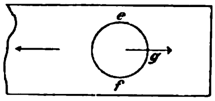


FIG. 151.

In Fig. 151 the plate is pulling to the left and the rivet is preventing it from moving; the rivet then bears against the plate along the curved surface *e g f*, and the stress is transmitted from the plate to the rivet, or *vice versâ*, through that surface.

This surface *egf*, however, has a breadth equal to the thickness of the plate, viz., *hk* in Fig. 152.

Hence the whole area of the rivet which is bearing against the plate is the area *egf* × the thickness of the plate *h k*.

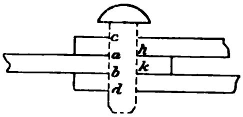


FIG. 152.

Now the pressure will not be equally distributed over the curved area *egf*, but will be a maximum at *g* and a minimum at *e* and *f*.

Hence it is the custom to assume a reduced value for the area *egf*, and to consider the bearing area as the projection of the area *egf*, or *ef*, the diameter of the rivet × *t*, the thickness of the plate.

Over this reduced area the pressure is assumed to be uniform and equal to the pressure at *g*.

Therefore the resistance to bearing of a rivet of radius *r* in a plate of thickness *t* will be

$$2r \times t \times 8 \text{ tons.}$$

There is no such thing as double bearing, and for this reason nearly all joints will be more troublesome to design with regard to resistance to bearing than to shearing.

For example, in Fig. 152 the rivet receives a stress, or pull, from the outer plates through the bearing surfaces *ca* and *bd*, and transmits this stress to the centre plate through the single bearing area *hk*. The bearing area *hk* is therefore the weakest part of this joint, and no advantage is gained by having the double bearing area at *ac* and *bd*. Another reason why the bearing is often more trouble than the shearing is, that the bearing area varies directly as the diameter of the rivet, whereas the shearing area varies as the square of the diameter of the rivet.

Table IX. contains the values of rivets with various constants, both in shear and bearing.

EXAMPLE.

To make a tension joint between two steel plates 16 inches wide by $\frac{1}{2}$ inch thick, to take a pull of 42 tons (Fig. 153):

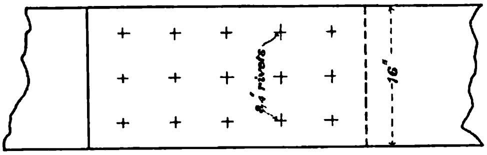


FIG. 153.

The two plates are to overlap each other.

Assume that $\frac{3}{4}$-inch rivets are to be used.

Let n be the number required.

Then the resistance to shear (single) $= n \times \pi \left(\frac{3}{8}\right)^2 \times 6{\cdot}5$ tons.

Equating this to 42 tons,

$$n = 14{\cdot}6$$
$$= 15 \text{ rivets.}$$

Also for bearing, the resistance to bearing on the $\frac{1}{2}$-inch plate is $= n \times \frac{3}{4}' \times \frac{1}{2}$ inch $\times$ 8 tons,

Equating this to 42 tons,

$$n = 14 \text{ rivets.}$$

15 rivets must therefore be used.

Suppose them to be arranged in rows of three (*vide* Fig. 153). Then there will be five rows.

Now the net width of the plate will be 16 inches — three rivet holes, since there are three rivet holes in a row across the plate.

$$= 16 \text{ inches} - 3 \times \tfrac{13}{16} \text{ inches.}$$
$$= 13\tfrac{9}{16} \text{ inches.}$$

Therefore the effective section of the plate is:

$$= 13\tfrac{9}{16} \text{ inches} \times \tfrac{1}{2},$$

and the plate can carry a tensile stress

$$= 13\tfrac{9}{16} \text{ inches} \times \tfrac{1}{2} \times 7 \text{ tons.}$$
$$= 47{\cdot}92 \text{ tons.}$$

This is more than the 42 tons originally stated to be carried by the joint.

But the extra size of the plates and their extra strength is all wasted unless the rivet joint be made equally strong. Therefore the rivet joint should be made strong enough to take 47·92 tons, *i.e.*, the whole safe load of which the plates are capable. This will necessitate adding an extra row of rivets to those shown in Fig. 153 and using 18 in all.

ARRANGEMENT OF RIVETS IN JOINTS.

There are five general principles to be remembered in arranging joints:—

(1) Rivets must be sufficient in number or size to be able to develop the full strength of the members which they are joining, irrespective of the actual stress which is expected to come upon such members.

(2) Rivets should be grouped symmetrically round the "lines of mean fibre" of the members to be joined.

(3) The effective section of the members to be joined should be reduced as little as possible.

(4) Rivets should never be stressed in tension or torsion.

(5) The pitch of rivets adopted for the structure should not be broken or changed if it can be avoided.

(1) has been alluded to before, but is of the utmost importance.

(2) Rivets should be symmetrically arranged with reference to the centre line, or mean fibre, or neutral line of the members which they join. In Fig. 154 position *a* is bad, and position *b* is

good for a rivet, as it distributes the stress more uniformly on to the cross-section of the plate than would be the case at *a*.

In the case of several rows of rivets, as in Fig. 153, these should be symmetrically arranged about the centre line.

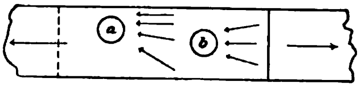


FIG. 154.

Angles and tees are more difficult to arrange for, since the rivet can only very rarely be placed upon the "neutral line," or the line passing longitudinally through the neutral axes of any number of cross-sections.

The neutral line in Fig. 155 is situated in some such position as *OO*, and rivets must be brought as near to that as is practicably possible.

In the case of equal angles the neutral line or mean fibre is very close to the angle.

Hence the best that can be done is to keep rivets as near to the angle as possible.

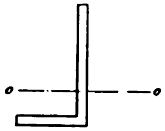


FIG. 155.

The same applies to tees and channels.

(3) As any rivet hole on any one cross-section of a member must be deducted from that section, in order to arrive at its net useful effective section, the arrangment of rivets is important.

In a simple joint such as Fig. 145 there are two rivets, but by placing one behind the other, the effective section of the bars is only reduced by one rivet hole.

Again, suppose that six rivets are required in a joint; it is possible to arrange them so as to have two rows of three at right angles to the general direction of the members, reducing the effective section by three holes, or three rows of two, reducing the effective section by two holes only. This latter enables smaller plates to be used for the given stress in the girder, or structure.

In arranging rivets for any joint, much must depend on the special circumstances of the case.

Fig. 156 represents a portion of the bottom of a girder having four rows of rivets along it; but they are arranged so that there are only two rivet holes at any one section of the plate, viz., *aa* or *bb*.

The net effective section of the plate is therefore only reduced by two rivet holes instead of four.

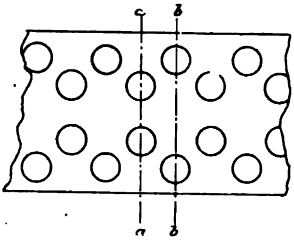


FIG. 156.

This is not, however, always a convenient arrangement to make fit in with other things.

Fig. 157 shows a similar arrangement for an angle steel, which is connecting two plates at right angles to one another. This affords a saving in effective area and strength of the angle steel. It has another great practical advantage in that it enables the rivets to be more easily got at by the riveting tool when they are put in. If the rivets were not spaced to break joint as shown, but were both placed at the same section of the angle steel, the head of one might be in the way of the tool for forming the head of the other. Here again, however, this arrangement is not always possible or convenient.

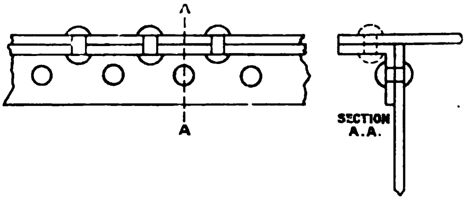


FIG. 157.

LEADING RIVETS.

A good arrangement of rivets in a tension joint is to make one rivet lead the others (Fig. 158). Here the effective section of the plate may be considered as being only reduced by one rivet hole, viz., the leading one. For example, assuming this to be admitted, in Fig. 158 suppose the plates to be 10 inches × $\frac{1}{2}$ inch in section.

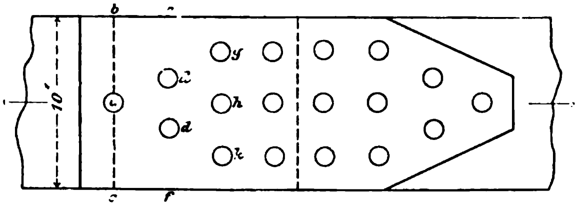


FIG. 158.

If one 1-inch rivet hole be deducted from the plate the effective section will be

$$= (10 \text{ inches} - 1 \text{ inch}) \times \tfrac{1}{2} \text{ inch.}$$
$$= 4{\cdot}5 \text{ square inches.}$$

and at 7 tons per square inch, the safe tensile load on the plate will be

$$= 4{\cdot}5 \times 7 \text{ tons} = 31{\cdot}5.$$

Assume that there are to be nine rivets in the plate and the cover plate.

Let r be the radius of rivet required.

Then for shear:

$$9 \times \pi r^2 \times 6{\cdot}5 = 31{\cdot}5$$
$$r = \tfrac{7}{16} \text{ inch, approximate.}$$

For bearing:

$$9 \times 2r \times \tfrac{1}{2} \times 8 = 31{\cdot}5$$
$$\therefore r = \tfrac{7}{16} \text{ inch nearly.}$$

Therefore $\frac{7}{8}$-inch rivets will exactly suffice, and each takes $\frac{1}{9}$ of the whole stress, or 3·5 tons.

Now consider the leading rivet "*a*" on the left of Fig. 158. The hole for this has already been deducted from the total effective section; hence the remaining portions of the cross-section, viz., *ab* and *ac*, are both fully stressed. Now the rivet "*a*" takes up 3·5 tons of the stress, or $\frac{1}{9}$ of the whole stress in the plate, so that beyond the leading rivet, or to the right of it, in the figure, the total stress in the plate has been reduced by $\frac{1}{9}$ or reduced to $\frac{8}{9}$ of what it was, at the section at *a*.

This reduced stress can be carried by a reduced cross-section, or in other words the cross-section of this plate may be reduced on the right of "*a*" by a ninth of the original cross-section, or by one extra rivet hole.

Hence the cross-section of the plate at *eddf* is sufficient for this reduced stress, when two rivet holes are introduced at *d*, *d*.

Now these two rivets take off a further stress of 3·5 tons each, or 7 tons together, and consequently to the right of them the cross-section of the plate may be reduced still further by another $\frac{2}{9}$, or, stated differently, to the right of the section *eddf*, $\frac{3}{9}$ of the total stress in the plate have been given up, and only $\frac{6}{9}$ remain. Hence the original effective section of 9 inches × $\frac{1}{2}$

inch at "*a*," where there is one rivet, may be reduced to 6 inches × ½ inch, at *ghk*, which allows four rivet holes in the 10-inch plate if required. Only three have been shown as this is more convenient.

It should be noted that the converse applies to the cover plate, and no saving is effected in it.

In the main plates, one rivet hole leads, where the stress is a maximum, and two and three holes are introduced as the stress is gradually transferred to the cover plate, and becomes less. But the maximum stress in the cover plate occurs immediately over the butt joint of the two plates, and here the section of the cover plate is cut into by three rivet holes, thus reducing the effective section by three inches, whereas there is only one rivet towards the end of the cover plate where the stress is least.

To meet this the cover plate would have to be made of greater thickness so as to provide the same effective section as the two main plates.

The cover plate would be cut off or trimmed off to the shape shown on the right of the joint.

This principle is a very important one, but it is not of very much use in ordinary girder work, since it generally happens that the tension plates that are to be joined together are already cut into by rivet holes throughout the whole of their length, for purposes of riveting them to other plates or angles, etc., and in that case of course there is no economy effected by special arrangements for joints.

(4) Rivets must on no account be placed in tension, and it is frequently necessary to go to great trouble to avoid doing so. Where tension must be taken by a joint, bolts and nuts must be calculated for, and used.

(5) The pitch of rivets in use in the rest of the work should not be altered if it can be avoided.

This greatly facilitates plating up and general construction.

PITCH.

The distance between the centres of rivets is called their pitch.

In spacing rivets and rivet holes in any plate, a certain amount of plate is required behind a rivet to give effect to it, or in other words, to take up the stress from the rivet and spread it over the fibres of the plate. This space or distance should not be less than one and a half times the diameter of the rivet.

Thus the hole for a 1-inch rivet must not have less than 1½ inches between it and the edge of the plate or an adjacent rivet hole.

This decides the minimum pitch.

The maximum is 4 inches, for it is found that if rivets are further apart than 4 inches, close contact between the plates or other surfaces joined together is not obtained, and rust action sets up between the plates or members.

The most usual pitch of rivets is 4 inches, but 3½ inches and 3 inches are sometimes used, though rarely for large rivets ; they are, however, quite common in light structures.

The pitch to be used is generally decided by the dimensions of some of the principal members of a structure, and is chosen so as to fit in with these dimensions conveniently, without having to be constantly varied.

In fact the 4-inch pitch is so nearly universal, that nearly all standard sizes are based on this pitch : *e.g.* Troughing is made 1 foot, 1 foot 4 inches, or 1 foot 8 inches, etc., in width from one rise to the next, which fits in with a 4-inch pitch of rivets perfectly.

COVER PLATES.

It sometimes occurs that a plate or other member can be utilised as a cover plate by prolonging it. This generally makes a neat job, and saves a separate cover plate.

For instance, in Fig. 159, *dc* and *ec* represent two plates that are to be joined together, such as plates in the flange of a girder ; and *bf* represents the end of another plate which is riveted to *ce* (to increase the strength of the flange).

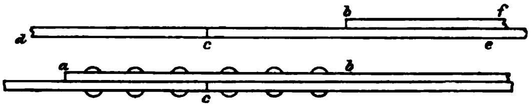


FIG. 159.

There is to be a joint at *c* requiring a cover plate.

The most convenient way of providing this is to lengthen the plate *fb* to *a*, and use the portion *ab* as a cover plate.

The calculations for the rivets for this are the same as those for any ordinary single cover plate joint.

There remains one other principle to be considered, which is most useful in making heavy joints.

Consider Fig. 160, showing a single cover plate joint.

The effective width of the plate at *aa* is its width – 3 rivet holes.

To the right of the section *aa*, it has given up half its stress to the three rivets at *aa*, while the effective section of the plate at *bb* remains the same as it was at *aa*.

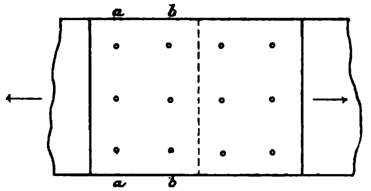


FIG. 160.

The metal is therefore not so highly stressed at *bb*.

In fact, in this example it is only half stressed, having given up half its stress to the cover plate through the rivets at *aa*. It is therefore capable of being more highly stressed than it is, *i.e.*, of being utilised to do some other work.

This principle may be briefly stated thus:

If a plate of constant effective section gives up or transfers a stress in one direction through *n* rivets, it is then capable of being loaded with another additional stress in the same direction, transferred to it by *n* other similar rivets; or

If it gives up stress through 4 rivets at one section, it has 4 rivets' worth of strength made available for taking other stresses.

This principle is utilised in many ways, but the following is a common application.

JOINT OF FOUR PLATES BY DOUBLE COVER PLATE.

Let Fig. 161 be a longitudinal section through four similar plates, *l*, *m*, *n*, *o*, that are all to be jointed under two covers *uv* and *wx*. Number the rows of rivets across the plates, 1, 2, 3, to 9.

Assume for illustration that three rows of rivets are enough to transmit the stress from one plate to the cover plate.

Thus the rows 1, 2, and 3 take up and transmit the stress in plate *l* to the cover above it, and 4, 5, and 6, transmit it back to the other portion of the plate *l*.

Now between rivets 3 and 4 the cover plate is fully strained, and between 4 and 5 it has given back $\frac{1}{3}$ of the stress to plate *l*, and is only $\frac{2}{3}$ loaded.

It is therefore able to take another $\frac{1}{3}$ of its full stress between 4 and 5.

Similarly it can again take a further $\frac{1}{3}$ between 5 and 6.

Note also that between rivets 4 and 5 plate *l* has only received $\frac{1}{3}$ of its proper stress from the cover plate, and can therefore take $\frac{2}{3}$ more between 4 and 5 if required. So that between rivets 4 and 5 the cover plate and plate *l* can both assist in acting as a cover plate to a joint in plate *m*, which is made between rivets 4 and 5.

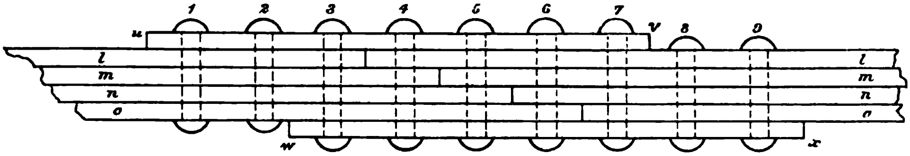


FIG. 161.

Assume these two between them to take $\frac{1}{3}$ of the stress in *m*; this will leave $\frac{2}{3}$ of the stress in *m* to be taken by the lower cover plate *wx*, which must therefore begin at rivet 3, *i.e.*, two rivet holes to the left of the joint in plate *m*.

Again, between rivet holes 5 and 6 plate *l* has still only received $\frac{2}{3}$ of its proper stress from the upper cover plate, and having given up at rivet 5 the stress that it had due to plate *m*, is capable of taking up $\frac{1}{3}$ of the stress in plate *n*, which has its joints between rivets 5 and 6; also plate *m* can take $\frac{1}{3}$ of the stress from plate *n*.

Therefore, the lower cover plate, which already has taken $\frac{2}{3}$ of the stress in plate *m*, can take the remaining $\frac{1}{3}$ from plate *n*.

And so on for any number of plates.

The top cover plate is no longer required to the right of rivet 7.

This and similar joints are most useful in the top and bottom flanges of long-span girders.

The rivets must only be considered as in single shear.

COMPRESSION JOINTS.

The calculations for riveted joints in tension all apply to compression joints.

Many engineers, however, allow for the fact that really well-made rivets completely fill the holes into which they are pressed, and can therefore take compression.

If this be so, there is no need then to make any reduction at all in the effective section of compression members due to rivet holes.

On this point every designer must be his own judge.

It is necessary in the first place to differentiate between work done in shops or with first class machinery, and work executed in foreign countries under difficulties, and often with indifferent appliances.

When joints are made away from the shops or without proper up-to-date riveting machinery, all members must be designed full strength as if the rivets did not really fill the holes, and took no compression.

Again, if rivets have more than four thicknesses of plate to traverse or a total thickness of more than three times their own diameter of shank to go through, there is some considerable risk of their not completely filling the rivet holes however carefully they may be put in, since the rivet may cool before it is thoroughly squashed home.

There must, therefore, remain some little risk in not deducting rivet holes from compression members. Such deduction, however, is frequently omitted unless there are other conditions and circumstances involved.

For example, the upper or compression boom, or flange, of a light plate girder is usually made of exactly the same size as the lower or tension flange, purely on account of simplicity of construction, plating, etc. The saving in weight in not deducting the rivet holes in the compression member does not compensate for the alteration of sizes.

In the opinion of the author, no allowance should be made for this advantage, and compression members should be designed in the same way as tension members, as far as rivet holes are concerned, in all girders such as are treated in this work.

Examples of riveted joints will be found throughout the book in the different designs there worked out.

FRICTION IN RIVETED JOINTS.

No allowance has been made here for the friction of plates one upon another when they are closely joined together by rivets.

When two or more plates are pressed closely together by rivets the friction between them tending to resist their movement by sliding one upon another is so great that it vastly increases the strength of the rivet joint.

It has been proved by experiment that a double-riveted tension joint, in which the rivets are much weaker than the tension bars or plates, will fail by the bars or plates breaking across before the rivets will fail by shearing or bearing.

This is entirely due to the extra strength given by the friction between the plates.

No allowance is usually made for this in design, and rivets are calculated of the full strength required in the joint.

It will be found that in old plate girders which have stood the test of time the rivets near the ends of the girders which join the web to the flange are quite inadequate in bearing or shearing or both, and it must be largely due to the friction that they have stood as they have.

It will be shown in the chapter on plate girders that these portions of plate girders are not easy to arrange as regards rivet stresses, and it will be consoling to know that there is this force to be relied upon giving additional strength.

Not enough is as yet known of this matter to enable us to calculate upon it as a sure means of obtaining strength, and this is of course the only reason for ignoring it in designing.

CHAPTER X

ROLLING LOADS AND THEIR BENDING MOMENTS ON BEAMS

Suppose a load rolls or moves across a beam from one end to the other; then, at any given point in that beam, the bending moment produced by that load will vary during the passage of that load; and when it is a maximum, it is called the *Max.* M_B *at that given point.*

Any load or loads moving across a beam or span may produce different maximum bending moments at all points on the span; and that point at which the bending moment produced is the greatest is called the *point of Max.* M_B; and the value of this maximum bending moment is called the *Max.* M_B *on the beam or span.*

This latter is what the beam must be designed to resist.

Referring to pp. 15—18, it will be seen that a load W placed anywhere on a beam causes a bending moment or M_B at every portion of the beam, but this M_B is greatest immediately under the load and is called the *Max.* M_B or maximum bending moment in the beam due to the load.

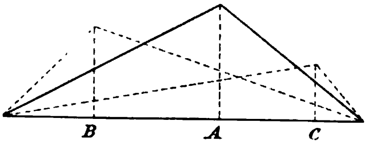


Fig. 162.

If now the load moves along the beam, the position of the *Max.* M_B moves too, always remaining under the load.

Further, at any point B, Fig. 162, the bending moment due to the load will be greatest when the load rolls over the point B; *i.e.*, the *Max.* M_B at B will be produced when the load is at B.

Similarly the *Max.* M_B at point C will be produced when the load is over C.

It will, therefore, be seen that there is a maximum value of M_B (caused by the load in its passage over the beam) for every point in the beam, and this *Max.* M_B will be produced when the load is at that point.

The greatest of these values of maximum bending moment in the beam, is produced at the middle point of the beam (when only loaded with one load), at the moment when the load is at the middle point, and is called the *Max.* M_B in the beam.

MAX. M_B DUE TO A SINGLE ROLLING LOAD (ANALYTICAL SOLUTION).

Consider a beam as in Fig. 163, and a load W rolling over it, distant x feet from A. Then the bending moment under the weight $= R_A \times x$.

Now $$R_A = \frac{W(l-x)}{l}.$$

Therefore $$Max.\ M_B \text{ at the load} = \frac{W(l-x)}{l}x.$$

Erect a perpendicular de, to represent this on any scale and join Ae and Be.

Then AeB is the M_B diagram for this particular position of W.

Now let the load roll on to some position d_1 distant x_1 feet from A.

Then as before the greatest M_B in the beam is under the load,

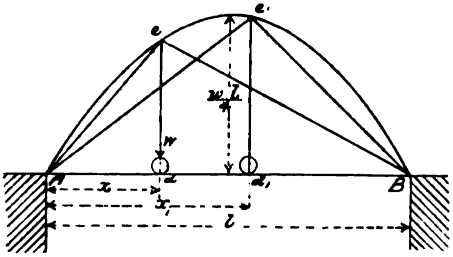


FIG. 163.

and $$= R_A \times x_1 = \frac{W(l-x_1)}{l}x_1.$$

Draw diagram Ae_1B as before.

Similarly for all positions of the load on the span.

It is thus obvious—

(1) That the maximum M_B occurs at each point as the load passes over that point.

(2) That we can draw an infinite series of triangles for different positions of the load, the height of each being the value of the maximum M_B at the load.

Now if we find some expression that will give the value of M_B at any point when the load stops there; or find the locus of the point e, as the load rolls over the bridge, we shall obtain a curve, giving the maximum values of the M_B at each point which the beam or bridge will be called upon to bear during the passage of the load.

Now the *Max.* M_B at any point

$$= \frac{W(l-x)}{l}x$$

$$= \frac{W(lx-x^2)}{l}$$

This is of the form of the equation to a parabola.

The locus of the point e will, therefore, be represented by a parabola passing through the points A, B, having a central ordinate

$$= \frac{Wl}{4}, \text{ since at the centre of span where } x = \frac{l}{2},$$

Max. M_B on the span $= \frac{Wl}{4}$.

A parabola so drawn will, therefore, be the diagram for maximum bending moments at any point. Inversely, if it be required to ascertain the M_B at all points of the beam, a parabola may be drawn to scale having its central ordinate equal to the *Max.* M_B on the span, or $\frac{Wl}{4}$, plotted to scale.

Then the vertical ordinate at any point will give the *Max.* M_B at that point.

MAX. M_B DUE TO TWO ROLLING LOADS (ANALYTICAL SOLUTION).

Two loads rolling over a span are assumed to be kept at a fixed distance apart from one another, similar to the wheels of a road engine.

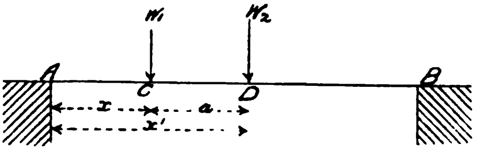


FIG. 164.

Let the span be loaded as in Fig. 164.

Let C and D be points of application of the weights W_1 and W_2.

Let the weights be a fixed distance apart $= a$.

Then $R_A = W_1 \frac{l - x}{l} + W_2 \frac{l - x - a}{l}$

and $R_B = W_1 \frac{x}{l} + W_2 \frac{x + a}{l}$

Then M_B at $C = R_A \times x$

$$= (W_1 + W_2) \frac{l - x}{l} x - \frac{W_2 a x}{l} \text{ . . . . . . } (a)$$

and M_B at D at a distance x' from A.

$$= R_A \times x' - W_1 a$$

$$= (W_1 + W_2) \frac{(l - x)}{l} x' + \frac{W_2 x' a}{l} - W_1 a$$

$$= \frac{x'}{l} \Big[(W_1 + W_2)(l - x) - W_2 a \Big] - W_1 a$$

Now for x substitute $x' - a$,

$$M_B = \left[W_1 \frac{l - x' + a}{l} + W_2 \frac{l - x'}{l} \right] x' - W_1 a$$

$$= \frac{x'}{l} (W_1 + W_2)(l - x') + W_1 \frac{a x'}{l} - W_1 a$$

$$= \frac{x'}{l} (W_1 + W_2)(l - x') - \frac{W_1 a}{l} (l - x') \text{ . . . . } (\beta)$$

Now to find the value of x that will give the maximum value of M_B, due to load W_1,

Differentiate (a) for a maximum value of x.

$$\frac{d M_B}{d x} = \frac{(W_1 + W_2)(l - 2x)}{l} - \frac{W_2 a}{l} = 0$$

$$\therefore x = \frac{l}{2} - \frac{W_2 a}{2(W_1 + W_2)}$$

Similarly, $x' = \frac{l}{2} + \frac{W_1 a}{2(W_1 + W_2)}$

Now $\frac{W_2 a}{W_1 + W_2} =$ distance of W_1 from c of g of W_1 and W_2.

Similarly, $\frac{W_1 a}{W_1 + W_2} =$ distance of W_2 from c of g of W_1 and W_2.

Hence the two positions of any two loads that will give maximum bending moments are such that the centre of the span bisects the distance between W_1 and the *c.g.* of W_1 and W_2, or W_2 and the *c.g.* of W_1 and W_2.

And this *Max.* M_B will be underneath W_1 or W_2.

If the weights are not equal, the *Max.* M_B on the span will be produced when the heavier weight is nearer to the centre of the span than the other.

This rule is applicable to all cases of two loads on a span; and since x and a are known the actual value of *Max.* M_B can be found by substitution in (α) or (β) above. If the spacing a of the loads exceeds ·59 of the span, the heavier load will produce a bigger M_B by itself when placed at the middle of the span than any combination of the two loads.

Example.

Assume a road engine with axle loads of 14 tons and $2\frac{1}{2}$ tons on back and front axles respectively, and a wheel base of 11 feet

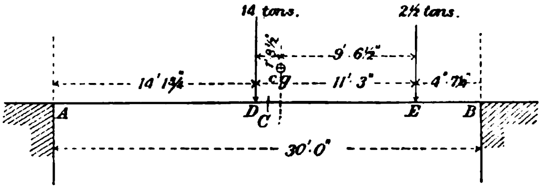


FIG. 165.

3 inches between axles, to be crossing a 30-foot span bridge (Fig. 165).

To find value and position of *Max.* M_B on the span.

Position.

The spacing of the axles is 11 feet 3 inches and is less than ·59 of the span; hence the rule given above for two rolling loads defines the position of *Max.* M_B.

First therefore find the *c.g.* of the loads. Let it be distant x feet from the heavier axle. Then:—

$$14 \text{ tons} \times x = 2\tfrac{1}{2} \text{ tons} \times (11 \text{ feet } 3 \text{ inches} - x)$$

$$\therefore 16\tfrac{1}{2}x = \frac{225}{8}$$

$$x = 1 \text{ foot } 8\tfrac{1}{2} \text{ inches nearly.}$$

The *Max.* M_B will therefore occur under the 14-ton axle, when the centre of the span is half-way between the 14-ton axle and the *c.g.* of the two axles.

i.e., the 14-ton axle is $\dfrac{1'\ 8\tfrac{1}{2}''}{2}$.

or $10\frac{1}{4}$ inches from the centre of the span.

The other dimensions follow, Fig. 165.

Value of Max. M_B on the span.

$$R_A = 14 \times \frac{DB}{AB} + 2\tfrac{1}{2} \times \frac{EB}{AB}$$
$$= 14 \times \frac{15'\ 10\tfrac{1}{4}''}{30} + 2\tfrac{1}{2} \times \frac{4'\ 7\tfrac{1}{4}''}{30}$$
$$\therefore \text{Max. } M_B \text{ at } D = R_A \times 14 \text{ feet } 1\tfrac{3}{4} \text{ inches.}$$
$$= 94{\cdot}23 \text{ foot-tons.}$$

This is in foot-tons since units taken have been feet and tons. The M_B at E for this position of the load is.

$$M_B = R_B \times BE$$
$$= \left(2\tfrac{1}{2} \times \frac{25'\ 4\tfrac{3}{4}''}{30} + 14 \times \frac{14'\ 1\tfrac{3}{4}''}{30}\right) 4'\ 7\tfrac{1}{4}''$$
$$= 16{\cdot}03 \text{ foot-tons.}$$

Max. M_B at other points in the beam.

The value and position of *Max.* M_B on the beam have now been found, *i.e.*, the greatest M_B that can occur anywhere in the span during the passage of any two loads.

All that remains now is to find the value of *Max.* M_B or greatest bending moment that can occur at all other points of the span due to the loads in their passage across the beam. These can be found analytically for any or every point from equations (α) and (β) on p. 127, by simple substitution.

This is easy but laborious; and once the value and position of *Max.* M_B on the span have been found, the graphic method given on p. 131 is the best, giving at once the values of *Max.* M_B at all other points along the beam.

The graphic method is more especially useful when the beam itself is not merely supported only, but is fixed, or half fixed, or hinged, or cantilevered, etc., containing points of contraflexure. This method is given lower down.

GENERAL REMARKS

on rule for position of *Max.* M_B on the span due to two loads.

From the solution on p. 127 it will be seen that there are two positions of the two loads, in both of which the vertical through the centre of the span bisects the distance between one of the loads and their common c. of g. Either of these positions gives a theoretical maximum M_B under the load nearest to the centre of the span.

Fig. 166 shows ordinate at D erected to scale to represent the value of *Max.* M_B found in equation (a), the centre line bisecting the distance between the load W_1 and their common c. of g.

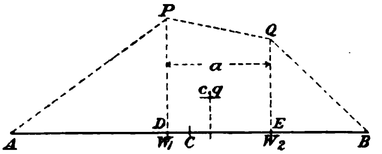


FIG. 166.

The ordinate at E represents the M_B at E under this loading.

The diagram is composed of straight lines, the loads being momentarily considered at rest at the position of *Max.* M_B.

Fig. 167 shows the ordinate at E erected to scale to represent the other value of *Max.* M_B found in equation (β), the centre line bisecting the distance between W_2 and the c.g.; and the ordinate at D represents to scale the M_B at D under this loading.

This is a second maximum value for the *Max.* M_B under these two loads.

The *Max.* M_B in this case occurs at E since W_2 is nearest to the centre of the span.

Now, if W_1 and W_2 be equal, their c.g. will be half-way between them, and the M_B under each will be the same.

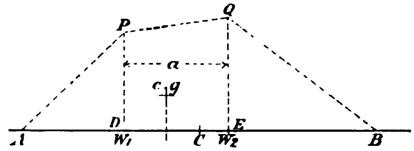


FIG. 167.

Therefore there will clearly be two points of *Max.* M_B, one at each side of the centre, the *Max.* M_B being as stated the same in either case.

If, however, one load is greater than the other, then the two values of *Max.* M_B will not be the same, but the greatest will occur under the heavier load when it is nearest to the centre, according to rule.

The lesser maximum value, when the lesser load is nearest to the centre, is a maximum in theory, but being less than the other

M_B is not required as far as the design of beams and bridges is concerned, and may be neglected. Referring to Figs. 166 and 167 the two maxima are explained thus:—

Suppose that the loads roll on to the span from right to left.

As W_1 approaches and passes the centre, it causes a wave of M_B, reaching a maximum at D (Fig. 166), and dying away again as it passes D.

But as W_2 approaches the centre, it causes a second though lesser wave of M_B, which reaches a maximum value at E (Fig. 167), dying away again to zero.

The first or larger wave, or *Max.* M_B, is what concerns the design of the bridge or beam over the span.

MAX. M_B DUE TO TWO ROLLING LOADS (GRAPHIC SOLUTION).

The graphic method for two loads is very simple and has the advantage of showing the *Max.* M_B at every point of the span.

The construction is as follows:—

(1) Draw the diagram of *Max.* M_B for each load separately as it passes over the span, *vide* p. 125.

(2) Combine these two diagrams having regard to the constant spacing or distance apart of the two loads.

The final diagram gives the *Max.* M_B on the span and also the *Max.* M_B or greatest moment that can occur due to the combined loads at all points of the beam.

EXAMPLE.

Suppose that there are two loads W_1 and W_2 at a distance a apart, rolling over the span AB from left to right (Fig. 168).

The *Max.* M_B due to each of these loads, considered separately, during their passage over the span, are $\frac{W_1 l}{4}$ and $\frac{W_2 l}{4}$.

Construct two parabolæ on base AB with heights $\frac{W_1 l}{4}$ and $\frac{W_2 l}{4}$. (Figs. 168 and 169).

These two are combined in Fig. 170.

For when load W_1, the leading load, reaches a position P (Fig. 168), W_2 is not yet on the span, and the M_B due to W_1 at P is measured by the ordinate PP_1.

Plot this in Fig. 170.

Then suppose the load W_1 to move on to some point Q, so that W_2 comes on to the span to the point q (Fig. 169). Then the M_B at Q due to W_1 is represented by QQ_1; plot this in Fig. 170 at Q.

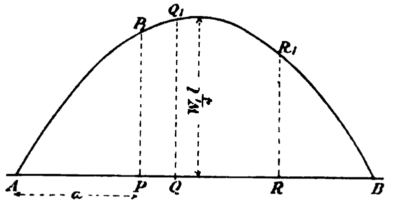


FIG. 168.

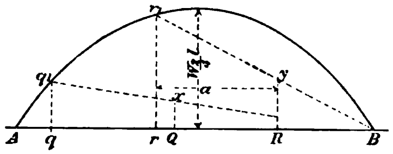


FIG. 169.

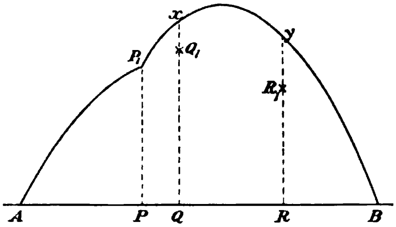


FIG. 170.

But since W_2 is on the span there is also an M_B due to W_2.

Erect a vertical qq_1 in Fig. 169; this represents the M_B due to W_2 at the point q; join $q_1 B$.

The intercept xQ (Fig. 169) is the M_B due to W_2 at the point Q; add this to the ordinate QQ_1 in Fig. 170.

Then Qx is the total M_B at Q (Fig. 170) due to the two loads.

Similarly let W_1 roll on to some point R (Fig. 168.)

Plot RR_1 in Fig. 170 (equal to RR_1 in Fig. 168).

At the new position of W_2, viz., r (Fig. 169), draw vertical (rr_1) and join r_1 B.

The intercept yR at R (Fig. 169) must be added to RR_1 in Fig. 170, giving Ry as the ordinate of total M_B at R. This fixes the point y on the combined M_B curve.

In this manner by considering the two loads in successive positions on the span (say 2 feet apart), a series of points on the combined curve of *Max.* M_B can be found, and the greatest ordinate can be selected by inspection and can be measured, giving the value and position of *Max.* M_B on the span.

If the loads are equal, the combined diagram will be symmetrical. If they are not equal the diagram will be unsymmetrical; and in this case it must be borne in mind that the loads may roll across the bridge in either direction, or either load leading, and that the greater half of the diagram or the half having the greatest ordinates must be taken for all measurements.

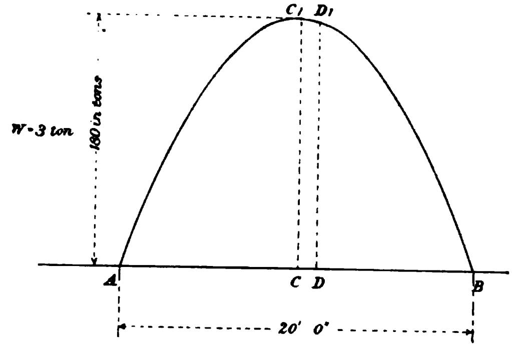


FIG. 171.

EXAMPLE.

Two loads of 3 tons and 2 tons are rolling across a span of 20 feet, from left to right.

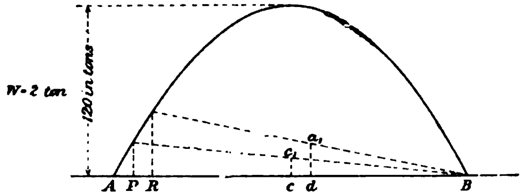


FIG. 172.

They are 9 feet apart.

First draw the *Max.* M_B diagrams for these two loads separately.

These diagrams are parabolæ with vertical heights $=$ *Max.* $M_B = \frac{Wl}{4}$.

The *Max.* M_B on the span due to these loads separately are 180 and 120 inch-tons respectively.

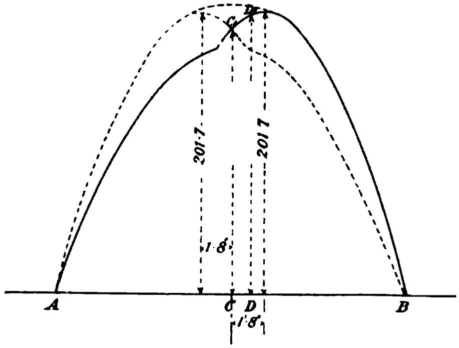


FIG. 173.

Figs. 171 and 172 are these diagrams, having vertical heights of 180 and 120 inch-tons respectively.

They must then be combined into one diagram (Fig. 173).

As the 3-ton load advances on to the span from left to right, the M_B caused will be that due to itself alone, until it gets 9 feet on to the span, and the 2-ton load is just about to come on to the span as well. Up to this point, therefore, the ordinates at Fig. 173 are the same as those of Fig. 171. Now consider the 3-ton load at C; its M_B is represented by CC_1 (Fig. 171).

Then the 2-ton load is at P (Fig. 172), and its M_B on the beam at C is represented by cc_1.

Add CC_1, and cc_1, and plot them in Fig. 173 at C.

Then let the loads advance 1 foot further to D and R respectively.

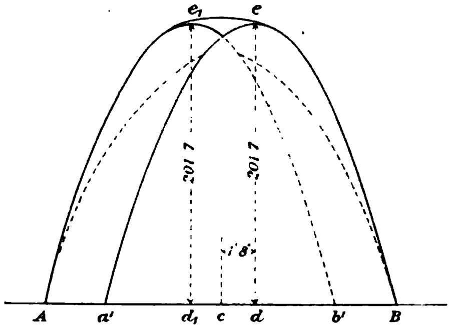


FIG. 174.

Add the ordinates DD_1 (Fig. 171), and dd_1 (Fig. 172), and plot their sum in Fig. 173 at D.

And so on.

The curve shown in full lines (Fig. 173) is thus obtained. It is unsymmetrical.

The highest ordinate is found to scale 201·7 inch-tons, and is situated 1·8 feet from the centre of the span.

It is shown dotted in Fig. 173.

Since the loads may also roll on to the span from right to left, a corresponding diagram for this latter case is shown dotted in Fig. 173.

The two curves will be seen to intersect one another, forming a re-entrant angle at the top.

This should be disregarded; in fact it is usual to join the two outer portions of the two M_B curves together at the top, as shown in Fig. 174.

The value and position of *Max.* M_B on the span may be verified analytically.

ANOTHER GRAPHIC METHOD.

It is possible to draw an approximate diagram straight off, without having to draw separately the M_B diagrams for the two loads.

In the above example, where the loads are moving from left to right, the *Max.* M_B on the span can be found analytically (by rule), to be 201·7 inch-tons, under the 3-ton load, when it has passed 1·8 feet beyond the centre of the span, *i.e.*, to the right of it in this case.

Plot AB as a base line to represent the span (Fig. 174).

Then plot 201·7 as an ordinate at d, where d is 1·8 feet to the right of C, the centre.

Describe a parabola Bea', with vertical axis de, and height 201·7 inch-tons.

(a' is 1·8 × 2 feet from A, *i.e.*, the distance of the leading load from the *c.g.* of the two loads. As the loads are advancing from left to right, the leading load would have reached the point a_1 just as the *c.g.* of the two loads arrived at A and was about to come on to the span.)

Now since the loads may cross the span in the opposite direction, a similar parabola $b'e_1A$ is also drawn, and the two parabolæ thus obtained are united into one whole curve at the top, as in Fig. 174, at ee_1.

This figure gives the value of the *Max.* M_B at all points of the beam without the trouble of combining two diagrams into one. But it is of no practical use for more loads than two.

NOTE.—Before going further, attention is again called to the meaning of the parabolic and similar diagrams of *Max.* M_B at any point, and *Max.* M_B on the span.

The greatest ordinate of such a diagram drawn for either one or two, or more loads, is the *Max.* M_B on the span or greatest bending moment that can be caused by the load or combination of loads in their passage over the span, and it occurs at that point of the span indicated by the diagram, and called the point of *Max.* M_B.

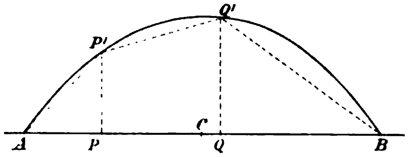


FIG. 175.

The remaining ordinates denote the *Max.* M_B that can occur at all other points due to the passage of the load or loads over the span. But at any particular moment during the passage of the load or loads over the span or at any particular position, *e.g.*, P and Q (Fig. 175), the maximum values of M_B are produced at P and Q under the loads, but those produced in the remainder of the span are indicated by the ordinates of a curve, consisting of straight lines, formed by joining AP', $P'Q'$, $Q'B$ (Fig. 175). And as the loads move onwards, P' and Q' will move along the parabolic curve, whereas the intermediate values of M_B will be bounded by the straight lines AP', $P'Q'$, $Q'B$.

SEVERAL ROLLING LOADS ON A SPAN.

General.—The *Max.* M_B due to several rolling loads such as the wheels of an engine or train crossing a bridge is what now remains to be found.

The methods employed in the case of two loads can be extended and applied to this case also.

Thus, the *Max.* M_B on the span can be found analytically by trial and error, or more directly by the use of differential calculus, or by graphic solution.

The graphic solution of such problems is by far the easiest, and has the advantage of giving the maximum stresses at every point in the span, which in the analytical solution would have to be calculated separately.

Further, the graphic solution gives the points of contraflexure in the beam, if such exist, as well as the sign of the shear stresses, and the extent and magnitude of the reversal of stress.

The graphic solution will therefore be treated first.

M_B DUE TO SEVERAL LOADS (GRAPHIC SOLUTION).

The principle of solving these cases by graphics is as follows:—

(1) Draw separate *Max.* M_B diagrams for each load, supposing it to roll across the span by itself.

(2) Combine these diagrams into one, having regard to the fixed spacing or distance apart of the loads.

This will be most clearly exemplified by a numerical example from practice.

EXAMPLE.

Assume an engine with axle loads as shown in Fig. 176 to be crossing a 60-feet span.

The first step is to plot parabolæ of *Max.* M_B for each axle load considered separately as it crosses the span.

This is done (refer example, p. 125) by plotting central vertical ordinates representing the *Max.* M_B on the span or $\frac{Wl}{4}$ for each load, and drawing parabolæ through the points thus found.

The maximum central M_B for

axle No. 1 = 6·8 × $\frac{60}{4}$ = 102·00 foot-tons
No. 2 = 7·0 × „ = 10·500 „
No. 3 = 16·5 × „ = 247·50 „
No. 4 = 14·5 × „ = 217·50 „
No. 5 = 9·0 × „ = 135·0 „
No. 6 = 9·0 × „ = 135·0 „
No. 7 = 8·12 × „ = 121·80 „

Plot seven parabolæ with central vertical ordinates as above. These parabolæ have been numbered for reference in Fig. 176 with the same numbers as the engine wheels.

It is now required to combine these parabolæ into one curve which will give the position and value of *Max.* M_B on the span, and the values of *Max.* M_B at all points of the span.

Suppose that the engine roll on to the span from left to right.

As wheels Nos. 1 and 2 advance on to the span they produce bending moments, but these may be neglected as they obviously are not so great as those that will be produced when wheels 3 and 4 come on to the span.

In fact a little practice will make it possible to judge very nearly the position of the engine on the span which will produce the greatest M_B.

Consider the case when wheel No. 3 has advanced 8 feet on to the span, and is for the moment stationary at P_3, on parabola No. 3 (Fig. 176).

The corresponding positions of axles 1 and 2 can now be plotted on their own parabolæ, viz., 1 and 2, at P_1 and P_2.

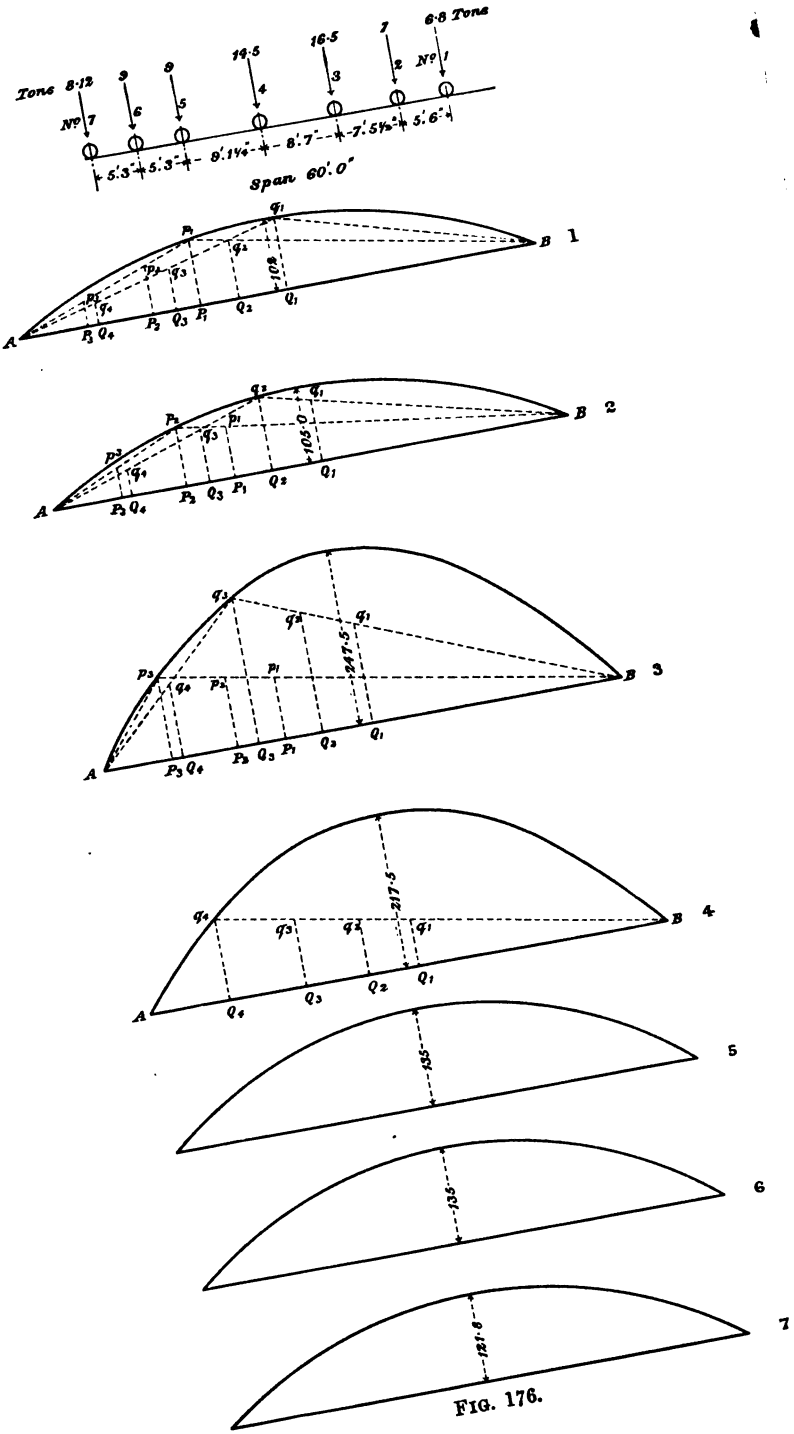

Tons 8·12
9
9
14·5
16·5
7
6·8 Tons
Nº 7
6
5
4
3
2
Nº 1
5′.3″
5′.3″
9′.1¼″
8′.7″
7′.5½″
5′.6″
Span 60′.0″
A
B
1
p_1 q_1 q_2 p_2 q_3 p_3 q_4
P_3 Q_4 P_2 Q_3 P_1 Q_2 Q_1
108
2
q_2 q_1 p_2 p_1 q_3 p_3 q_4
P_3 Q_4 P_2 Q_3 P_1 Q_2 Q_1
105·0
3
q_3 q_2 q_1 p_3 p_1 q_4 p_2
P_3 Q_4 P_2 Q_3 P_1 Q_2 Q_1
247·5
4
q_4 q_3 q_2 q_1
Q_4 Q_3 Q_2 Q_1
217·5
5
135
6
135
7
121·8

FIG. 176.

Erect a perpendicular at P_1, viz., P_1p_1, representing the M_B at P_1 due to axle No. 1, and join p_1A, p_1B.

Erect similar perpendiculars at P_2 and P_3 and join to A and B.

Axle No. 4 will not yet be on the span.

Then draw the base line AB in Fig. 177 and mark the points P_1, P_2, P_3.

To combine the separate parabolæ, add the ordinates together at each point where a wheel is resting.

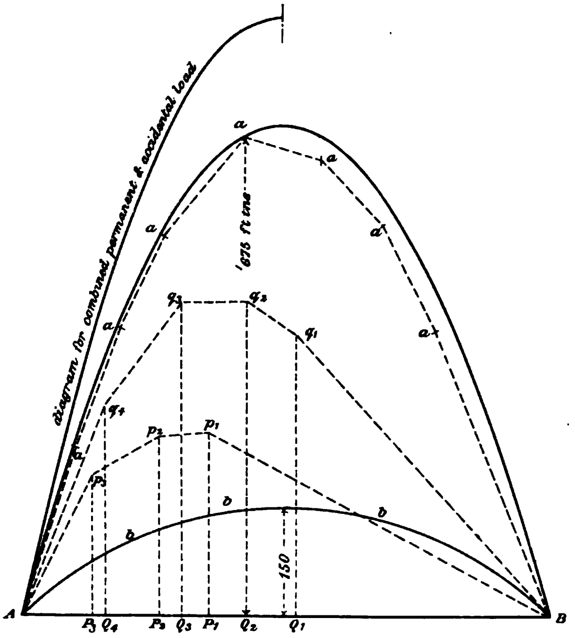


FIG. 177.

The ordinate at P_1 is made up thus (Fig. 177) :—

First there is the whole M_B due to No. 1, or P_1p_1 in Fig. 176.

Then there is also the portion P_1p_1 on parabola No. 2, or the M_B at P_1 due to axle No. 2.

This axle is momentarily stationary at the point P_2, and its M_B on the beam or span is given by the ordinates of the triangle Ap_2B.

Hence the M_B at P_1 due to load No. 2 is the ordinate P_1p_1 in parabola No. 2.

Similarly there is also the effect at P_1 of the axle No. 3 resting at P_3, *i.e.*, the ordinate P_1p_1 in parabola No. 3.

These three amounts are added together, and are plotted as the ordinate of M_B at P_1 in Fig. 177.

Similarly ordinates are plotted for the total M_B for all three axles at the points P_2 and P_3.

The ordinate P_2p_2 in Fig. 177 is the sum of the ordinates P_2p_2 in parabolæ Nos. 1, 2, and 3 (Fig. 176).

And the ordinate P_3p_3 is the sum of the ordinates P_3p_3 in the same three parabolæ.

Therefore A, p_3, p_2, p_1, B, are points on the M_B diagram in Fig. 177 and should be joined up.

This diagram gives the effect of the engine on the span when at the position assumed.

Now suppose the engine to advance on to the span until axle No. 3 is 18 feet from A.

Axle No. 4 will have now come on to the span.

The positions of the leading four axles are plotted on their respective parabolæ at Q_1, Q_2, Q_3, Q_4 (Fig. 176), and also on the base line of Fig. 177.

Again ordinates of M_B must be erected in each parabola at the positions where the axle loads are applied.

The M_B at Q_1 (Fig. 177) is the sum of the M_B's at Q_1 due to all four axles that are on the span, *i.e.*, the ordinate Q_1q_1 in Fig. 177 is the sum of ordinates Q_1q_1 in parabolæ Nos. 1, 2, 3, 4 in Fig. 176.

Similarly for the other points Q_2, Q_3, Q_4.

The diagram $Aq_4q_3q_2q_1B$ in Fig. 177 represents the combined M_B due to the engine when wheel No. 3 is 18 feet from A.

Now the engine must be supposed to advance still further on to the span, and the M_B must be plotted for different positions of the axle loads, advancing (say) 5 feet at a time.

As the engine advances the polygon or curve of total M_B will increase to a maximum, and a position of the engine will eventually be obtained beyond which any further advance will lead to a reduced M_B on the span.

In order to avoid confusing the figures, the construction of these subsequent positions has not been shown, but the *Max.* M_B diagram is as shown in Fig. 177, $A\ a\ a\ a\ a\ a\ a\ B$, the *Max.* M_B

measuring 675 foot-tons, and occurring at a point 4 feet from the centre under wheel No. 4.

As the diagram is a polygon made up of straight lines, it is advisable to enclose or circumscribe it with a continuous curve, which approximates in shape to the polygon, and stands symmetrically on the base line of the span.

This will allow for the engine crossing the bridge in the opposite direction.

It is desirable that it should be a parabola, *i.e.*, that parabola which will most nearly enclose the polygon.

The reason of this will appear later.

If it is desired to use this curve of *Max.* M_B for the design of the girders of a bridge, it will be necessary also to include the M_B due to the dead weight of the bridge itself. This dead weight will be a uniformly distributed load on the span, and its M_B diagram will be a parabola.

This parabola can be drawn on the same base as the *Max.* M_B diagram and can be combined with it if desired; or it may be left separate.

Such a parabola is shown in Fig. 177, viz., *A b b b B* having a vertical ordinate of 150 foot-tons, derived from the formula $M_B = \frac{Wl}{8}$ for uniform load, and allowing for a weight of $\frac{1}{3}$ tons per foot of span.

The total *Max.* M_B at any point is measured by the sum of the ordinates at that point, of both the outer M_B curve and the inner dead load curve.

It should be borne in mind that the outer *Max.* M_B curve represents live loads, and that the curve for the dead load represents dead loads only, due to the weight of the bridge.

Corrections for the live load would usually be made before calculating or plotting the vertical heights of the *Max.* M_B parabolæ of the separate axle loads.

In plotting curves of this nature it is necessary to exercise great care in plotting the individual parabolæ. As large a number of points on the parabola as possible should be found and the horizontal scale should be a large one. If the horizontal scale is small the sides of the parabola near the abutments will be very nearly vertical, and a very small error in the position of the loads on the span will cause a very much larger error in taking off the bending moments.

In the above example, if the final curve is not symmetrical it must be made so, the ordinates for any two points symmetrically situated on opposite sides of the centre of the span being made equal to each other, and also equal to the maximum value found for either of them by the polygon of *Max.* M_B.

This will make the curve symmetrical about the centre line of span and will allow for the engine crossing the span in either direction.

Plate II. shows such a *Max.* M_B diagram worked out.

When there are many wheels the figures are apt to become confusing; different colours have therefore been used in order to prevent this as far as possible, and each colour represents a position of the engine on the span.

Suppose wheel 3 to be at the position shown in red in parabola No. 3, then the red lines in all the other parabolæ represent the *Max.* M_B due to all the other wheels at that moment, and all the red ordinates vertically above each wheel must be added and plotted to obtain the red curve of total *Max.* M_B due to this position of the engine.

Suppose the engine rolls on so that wheel 3 is in the position shown in blue lines in parabola No. 3.

Then the blue lines in all the other parabolæ represent the *Max.* M_B due to the other wheels at that moment, and all the blue ordinates over each wheel must be added and plotted to obtain the blue polygon of total *Max.* M_B.

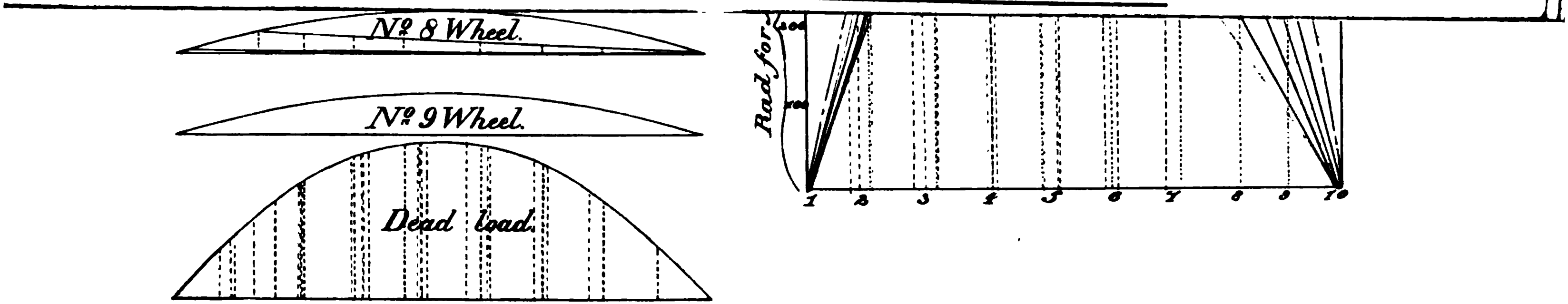
NOTE.—It will not be necessary to plot the total curve of *Max.* M_B for every position of the engine.

Examination of the axle loads, and experience, will enable a fair guess to be made at the position of the engine that will give the greatest M_B, and a few trials to one side or the other will decide this position definitely, and the value of *Max.* M_B on the span, quickly and accurately.

This is the method which is recommended for the solution of problems in all solid beams loaded with three or more rolling loads, and certainly for all plate girders.

PLATE II.

CURVES OF MAXIMUM BENDING MOMENTS.



CHAPTER XI

ROLLING LOADS AND THEIR SHEAR STRESS ON A BEAM

SHEAR DUE TO ONE ROLLING LOAD.

It has been shown in Chapter V. that the vertical shear at an abutment or support is equal to the reaction at that abutment or support.

In Fig. 178 the reaction at A due to W at $C = W\frac{CB}{AB}$ and that at $B = W\frac{AC}{AB}$.

Therefore the shear at $A = W\frac{CB}{AB}$ and at $B = W\frac{AC}{AB}$.

If now the load move to C', then the shear at A increases to $W\frac{C'B}{AB}$ and that at B decreases to $W\frac{C'A}{AB}$.

The shears therefore depend on the magnitude of CB and CA or on functions of the first degree, and as the load moves the shear increases or decreases uniformly, and the ordinates denoting the shear on either side of the load at any point, when the load is at that point, will be bounded by straight lines.

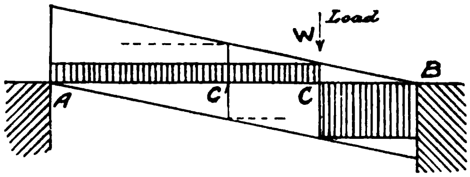


Fig. 178.

When the load approaches either abutment, in the limit when it reaches it, the shearing force at that abutment = load = W, and the shear at the other abutment = zero.

When the load is at the other abutment the same occurs, the sign of the shear being changed.

To plot the diagram therefore erect ordinates at A and B

equal to W plotted to scale, one positive and the other negative, and join as in Fig. 179.

This gives the maximum possible values of shear at all points due to that load. Fig. 179 shows also the shearing stress that occurs when the load is at any particular point of its passage.

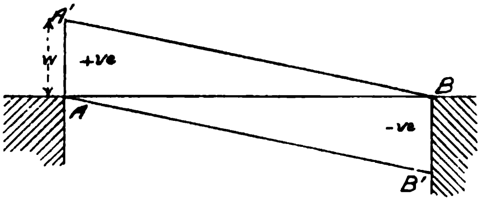


FIG. 179.

The above only treats of the vertical shear.

It must not in any way be supposed that this is the only shearing stress that has to be met by the beam.

But this solution gives the vertical shearing stress and this must be equal to the horizontal shearing stress, so that this latter can easily be found.

The real diagonal shearing stress can also be found if required by compounding the vertical and horizontal shearing stresses.

VERTICAL SHEAR DUE TO TWO ROLLING LOADS (ANALYTICAL SOLUTION).

Suppose W_1 and W_2 to be two loads, l' apart and rolling across a span, l feet long, from right to left (Fig. 180).

When W_1 is on the span and W_2 is just about to come on to it, the shear on the beam on either side of W_1 is the algebraic sum of the vertical forces acting on the beam on that side of W_1. Thus on the portion AD of the beam, the forces are:—

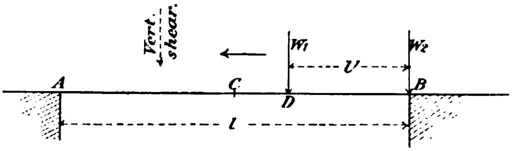


FIG. 180.

R_A upwards.

Shear downwards.

$$\therefore \text{ Shear} = R_A$$

$$= W_1 \frac{l'}{l} \text{ and is positive by convention,}$$

and shear in DB

$$= R_B$$

$$= W_1 \frac{l - l'}{l} \text{ and is negative by convention.}$$

Now suppose the loads roll on further as in Fig. 181.

The shear at any point in the beam will be found by taking the alegebraic sum of the forces on one or other side of the point, acting upon the beam vertically, *i.e.* :—

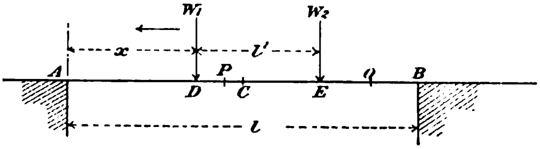


FIG. 181.

The shear at any point in $AD = R_A$

$$= W_1 \frac{l - x}{l} + W_2 \frac{l - x - l'}{l}$$

This is positive.

Again the shear at any point in DE such as P, is found in the same way.

The forces acting on the beam on the left of P are :—

R_A upwards.

W_1 downwards,

Shear at P.

Equating these we have,

Shear at $P = R_A - W_1$

$$= W_1 \frac{l - x}{l} + W_2 \frac{l - x - l'}{l} - W_1$$

This may be positive or negative according to the magnitude of W_1 and W_2.

The shear at any point Q is found in the same way.

Consider the forces to the left of Q.

Shear $= R_A - W_1 - W_2$

$$= W_1 \frac{l - x}{l} + W_2 \frac{l - x - l'}{l} - W_1 - W_2$$

$$= - \left[W_1 \frac{x}{l} + W_2 \frac{x + l'}{l} \right]$$

This is a negative shear.

Or the shear at Q may be found from the reaction at B.

Consider the forces to the right of Q.

$$\text{Then shear} = R_B = W_1\frac{x}{l} + W_2\frac{x+l'}{l},$$

and is negative by convention.

The maximum positive shear occurs as one or other of the two loads approaches the abutment A, viz., when $x = o$ or $x = l'$, depending on the relative values of W_1 and W_2.

Similarly for negative shear at B.

VERTICAL SHEAR FOR TWO ROLLING LOADS ON A SPAN (GRAPHIC METHOD).

Assume that two loads W_1 and W_2 fixed at a distance l_1 apart are rolling on to a span, l in length, from right to left.

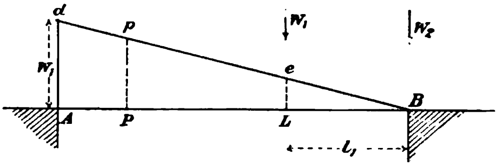


FIG. 182.

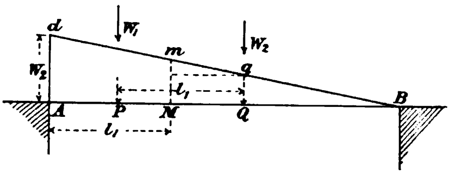


FIG. 183.

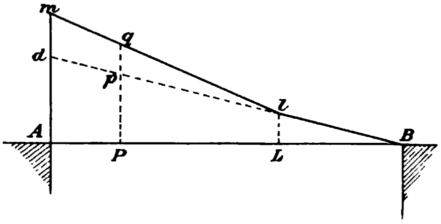


FIG. 184.

Draw separately the positive maximum vertical shearing stress diagram for each load as it crosses the span.

Thus in Figs. 182, 183, erect vertical ordinates at $A = W_1$ and W_2 respectively, and join Bd in each.

These represent the maximum values of vertical positive shear that the loads can each cause in their passage over the span.

Then combine the two in Fig. 184 thus :—

When W_1 reaches a point L distant l_1 from B, the whole shear on the beam is represented by the ordinate Le (Fig. 182).

Plot this in Fig. 184. So far W_2 has not got on to the span.

Now suppose W_1 to be at P.

The shear on the left of P due to W_1 is represented by Pp (Fig. 182).

Plot this in Fig. 184.

The shear at P due to W_2 which is at Q distant l_1 from P is Qq (Fig. 183).

Add this to the ordinate Pp in Fig. 184, making a whole ordinate Pq (Fig. 184), which represents the combined shear at P.

Similarly for any number of different points such as P, selected (say) 2 feet apart along the span AB.

When the leading load W_1 arrives at A, W_2 will have arrived at M (Fig. 183).

Plot Ad at A on Fig. 184 from Fig. 182, and add to it the ordinate Mm from Fig. 183.

Then Am represents the whole shear at A when W_1 is close up to it.

This method is similar in principle to that employed for *Max.* M_B, *i.e.*, adding the ordinates of parabolæ together.

This completes the diagram of maximum shear.

The ordinate measured at any point will give the maximum shear that can occur at that point.

[NOTE.—After the point l has been found (Fig. 184) it is sufficient to erect the ordinate Am and to join ml.]

ANOTHER GRAPHIC METHOD.

The following is a simpler construction arriving at the same result:—

Consider W_1 as rolling on to the span from B to A as in Fig. 185.

Erect a perpendicular Ad at A to represent W_1. Join Bd.

Then let l_1 be the distance between W_1 and W_2.

Suppose W_1 to have rolled l_1 on to the span, so that W_2 is just coming on at B.

Then the shear under $W_1 = c\,k +$ the incipient shear due to W_2 which is still just $= 0$.

Draw $A\,a_1 = l_1$.

Erect a perpendicular $a_1 d_1$ to represent W_2; join $d_1 c$, cutting $A d$ at m.

Then as the load W_1 comes to any point on the beam such as p, the shear at that point = the sum of the ordinates at that point measured from the two diagrams AdB and $a_1 d_1 c$, viz., pt and ps. This sum can be obtained graphically by laying off $d d_2 = Am$, and joining $d_2 k$. Then $pu = ps + pt$.

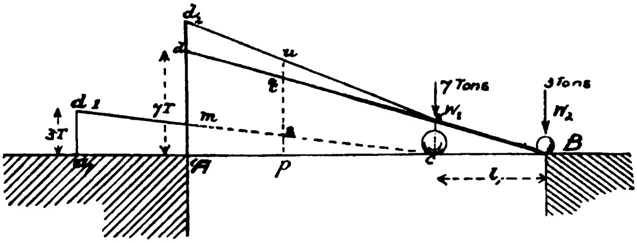


FIG. 185.

The diagram for the two loads is then $a_1 d_1 m d_2 k B$.

When W_1 reaches A, shear at $A = Ad_2$.

When W_1 passes A, shear drops to Am due to W_2 only, and increases again up to $a_1 d_1$ as W_2 approaches A.

NOTE.—Shear must always be measured under W_1.

This therefore completes the diagram for positive vertical shearing stress.

This process is applicable for any number of loads.

GENERAL.

It must not be forgotten that the intensity of horizontal shearing stress at any point is equal to the intensity of the vertical shearing stress at the same point; and that both the vertical and horizontal shearing stresses are only the two components of the real shearing stress which in a solid homogeneous beam acts upwards towards the centre of the span at an angle of 45° to the vertical, and is equal to the "*vertical component of shear* $\times \sqrt{2}$" at any point. This is general. It does not apply to all beams, but is true for solid homogeneous beams at their neutral axes.

NEGATIVE SHEAR.

Hitherto the shear has only been considered at one end of the span.

Referring to Fig. 185, it is clear that the moment the leading load (7 tons) rolls on to the span at *B*, it causes a vertical shear on the span at *B* equal to itself, *i.e.*, 7 tons.

This decreases as the load rolls on towards *A*.

Or were the loads to roll on to the span from the other end towards *B*, the same effect would be produced when the load reached *B*.

Hence the shear diagram must be duplicated as shown in Fig. 186, in which the negative shears are shown below the line. The diagram below the line is of course identical with that above it, although of opposite sign.

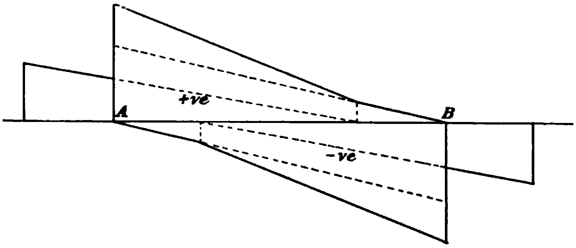


FIG. 186.

From this it will be apparent that any portion of the beam may be subjected to either positive or negative shear, causing reversal of stress in the beam.

It should be noted that in the case of moving loads, where no notice is taken of the weight of the beam itself, reversal of stress may occur at all points of the beam, although towards the supports the reversal is very small.

EXAMPLE.

Consider the loads given in the example on p. 133, viz., 3 tons and 2 tons, 9 feet apart, rolling across a span of 20 feet.

It is required to draw the shearing stress diagram and to find the maximum shear at either abutment.

Suppose the loads to roll on to the span from right to left, the 3-ton load leading (Fig. 187).

Then at A plot a vertical $A\,d$, equal to 3 tons on any convenient scale.

Join $d\,B$.

Then $A\,d\,B$ is the diagram of shear stresses due to the leading load, 3 tons.

Lay off $B\,P$ along $B\,A$ equal to 9 feet on the same scale as $A\ B$.

Erect a vertical ordinate $P\ P_1$. This ordinate will represent the shear at P when the leading load reaches P.

Then lay off $A\ a_1$ to the left of A equal to 9 feet.

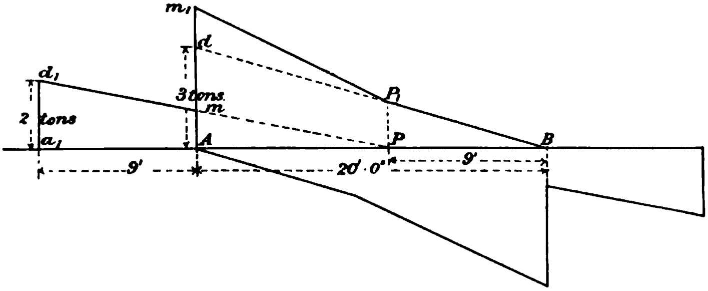


FIG. 187.

At a_1 erect $a_1 d_1$ equal to 2 tons, to scale. Join $d_1 P$ cutting $A\,d$ in m.

Measure $A\,m$ and lay it off at $d\,m_1$ above d. Join $m_1 P_1$.

Then $A\ m_1$ represents the whole maximum shear at A due to both loads when the leading (or 3-ton load) has just reached A.

Also $m_1 P_1 B$ is the diagram showing the maximum positive shear at all points between A and B due to the two loads.

The maximum shear at A is found to be $A\,m_1 = 4{\cdot}1$ tons by measurement.

The diagram for negative vertical shear is equal and opposite, and is drawn below the base line $A\ B$.

SHEAR DUE TO SEVERAL ROLLING LOADS (GRAPHIC METHOD).

The method of plotting the maximum shearing stress diagram for several rolling loads is merely an elaboration of the preceding for two rolling loads, and consists of combining the shearing stress diagrams for each separate axle into one.

The same data as those taken for the example of *Max.* M^B diagram (p. 136) are here taken into consideration, and the engine (Fig. 176) is supposed to be running backwards on to the span from right to left with wheel No. 7 leading.

The process is as follows:

(1) Draw separate diagrams of maximum shear for each axle load (Fig. 188).

(2) Combine them into one having regard to the spacing of the engine wheels (Fig. 189).

Now we know that the maximum positive vertical shear that a given load can produce at any point of the span during its passage is given by the ordinate of its maximum shearing stress diagram, and that this diagram is drawn with an ordinate equal to the load at one end and equal to 0 at the other.

Therefore for each axle load erect a perpendicular at *A* to represent the axle load to scale on a base line *A B* representing 60 feet span, and join the points so found to *B* (Fig. 188). These are maximum shear stress diagrams for the different loads (No. 7 diagram is drawn first as No. 7 wheel is leading over the span).

Now we know that the position of maximum shear is close to one or other abutment when the loads have rolled across the span and are close up to that abutment.

Let us therefore assume that axle No. 7 is just about to reach the abutment *A*, and to roll off the span to the left.

Then in Fig. 189, on a base line *A B* erect a perpendicular at *A* to represent the shear at *A* due to axle No. 7.

This will be *ab* from diagram No. 7 (Fig. 188), since axle No. 7 is at *A*.

Now there is also the shear at *A* due to axle No. 6, etc.; and axle No. 6. will be 5 feet 3 inches from *A* at this instant, and the ordinate at this point in diagram No. 6, viz., *Pc*, represents the shear at *A* due to axle No. 6, and must be added to *Ab* in Fig. 189, making a total ordinate *Ac*.

Similarly in diagram No. 5, *P* indicates the position of axle No. 5 on the span for this position of the engine, viz., 5 feet

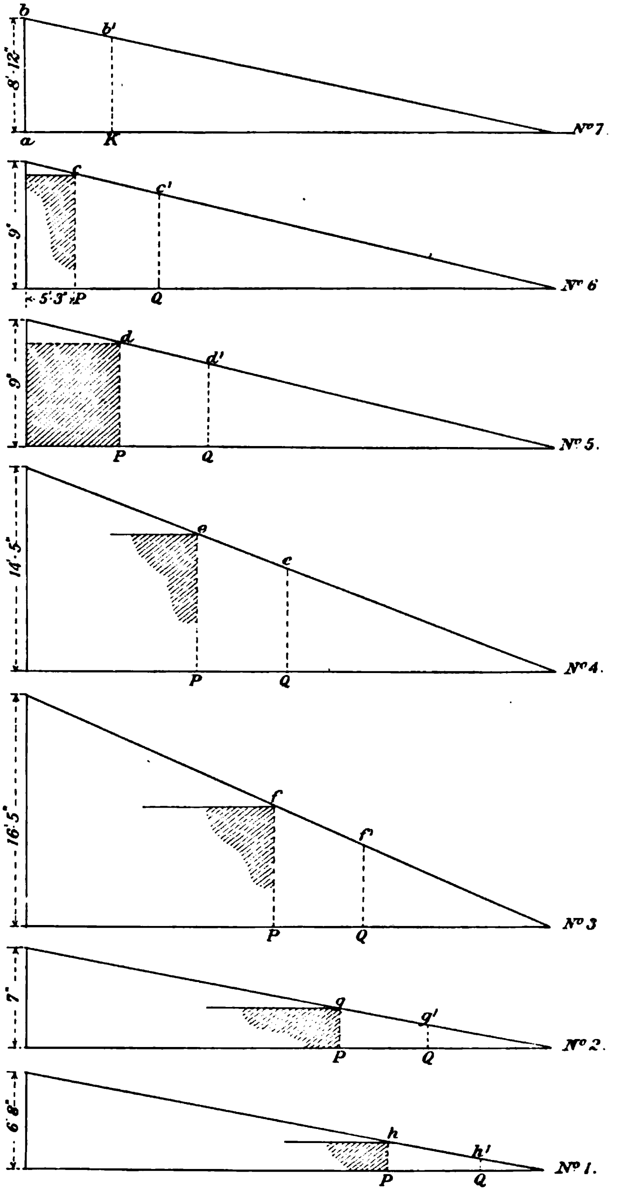

b
b'
8'·12"
a
K
Nº 7.
c
c'
9'
5'3"
P
Q
Nº 6
d
d'
9'
P
Q
Nº 5.
14'·5"
e
e'
P
Q
Nº 4.
16'·5"
f
f'
P
Q
Nº 3
7'
g
g'
P
Q
Nº 2.
6'8"
h
h'
P
Q
Nº 1.

FIG. 188.

3 inches to the right of axle No. 6, and *Pd* represents the shear at *A* due to axle No. 5 and must be added to *Ac* in Fig. 189, making a total ordinate *Ad*. Similarly for all the axle loads:—

The ordinates above *P* in the axle diagrams (Fig. 188) are added together to form the ordinate *Ah* in Fig. 189.

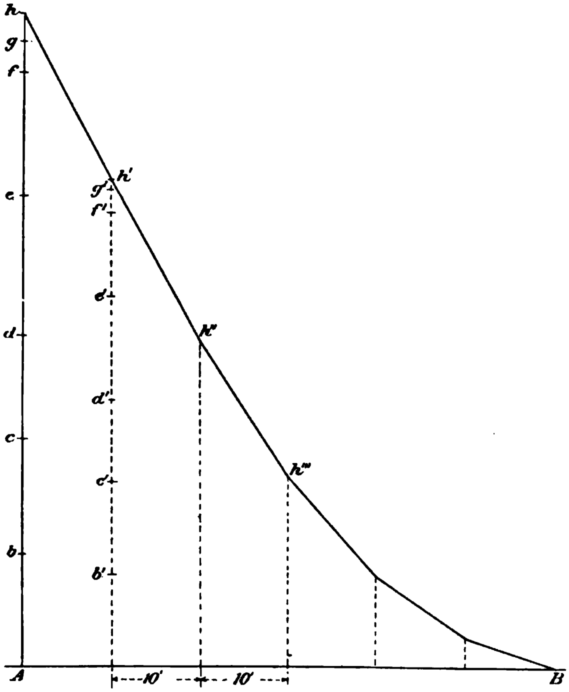


FIG. 189.

This establishes the maximum shear at *A* or the maximum reaction at the abutment.

Now suppose that the engine is a little further back, viz., axle No. 7 is 10 feet back from *A* to the right, at some point *K*. Plot the corresponding point *K* in Fig. 189 and erect an ordinate above it as follows:

The shear at *K* due to axle No. 7 is *Kb'* (Fig. 188), and is plotted in Fig. 189.

The shear at *K* due to axle No. 6 being 5 feet 3 inches to the right of axle No. 7, or 15 feet 3 inches from *A*, is *Qc′* and is plotted at *K* in Fig. 189.

Similarly the shear due to axle No. 5 is given in diagram No. 5 and is *Q d′*, axle No. 5 being 10 feet 6 inches to the right of *K*, or 20 feet 6 inches from *A*, and is plotted at *K* in Fig. 189.

Similarly for the other axle loads.

This gives a total ordinate *Kh′* in Fig. 189, representing the shear at *K* due to all the axle loads when wheel No. 7 is at *K*.

The ordinate *Kh′* (Fig. 189) is made up of the ordinates above *K* and *Q* in the diagrams (Fig. 188).

This process must be repeated for different positions moving the engine or loads 5 feet or 10 feet at a time along the span. The points so obtained, viz., *h′ h″ h‴ h⁗*, etc., are joined, and the graph *h′ h″ h‴ h . . . B* so obtained is the diagram of maximum shearing stress, positive, vertical, at any point in the span.

NOTE.—The maximum shear at *A* scales 46·75 tons in this example.

ANOTHER METHOD.

A shorter method of plotting the complete shearing stress diagram is that shown in Fig. 190.

It is similar to that shown for two rolling loads in Fig. 185. The method consists merely in plotting the different diagrams of maximum shearing stress on the same base line *AB*, or *AB* produced, in such positions or relative spacing, depending on the engines under consideration, that the ordinates of each shear diagram intercepted above *A*, by the vertical line at *A*, represent the shear at *A* due to each axle load respectively, and may be plotted together vertically above one another at *A* to give the maximum shear at *A*.

Assume the same engine, as in the foregoing case (Fig. 176), to be crossing a 60-foot span *BA*, from right to left, with axle No. 7 (8·12 tons) leading.

Suppose this axle to be just approaching the support at *A* though not actually resting on it.

Then the shear at *A* due to axle No. 7 is 8·12 tons.

Plot this to scale vertically above *A*, viz., *A* 7. Join *B* 7.

Now axle No. 6 is 5 feet 3 inches from No. 7; therefore mark off *c* and *c'* along *BA*, 5 feet 3 inches to the left of *B* and *A* respectively.

At *c'* erect a perpendicular and plot the point 6 so that *c'* 6 represents 9 tons (axle No. 6).

Join 6*c*. Then the triangle *c'*6*c* is the maximum shear stress diagram for axle No. 6 and is in its right place.

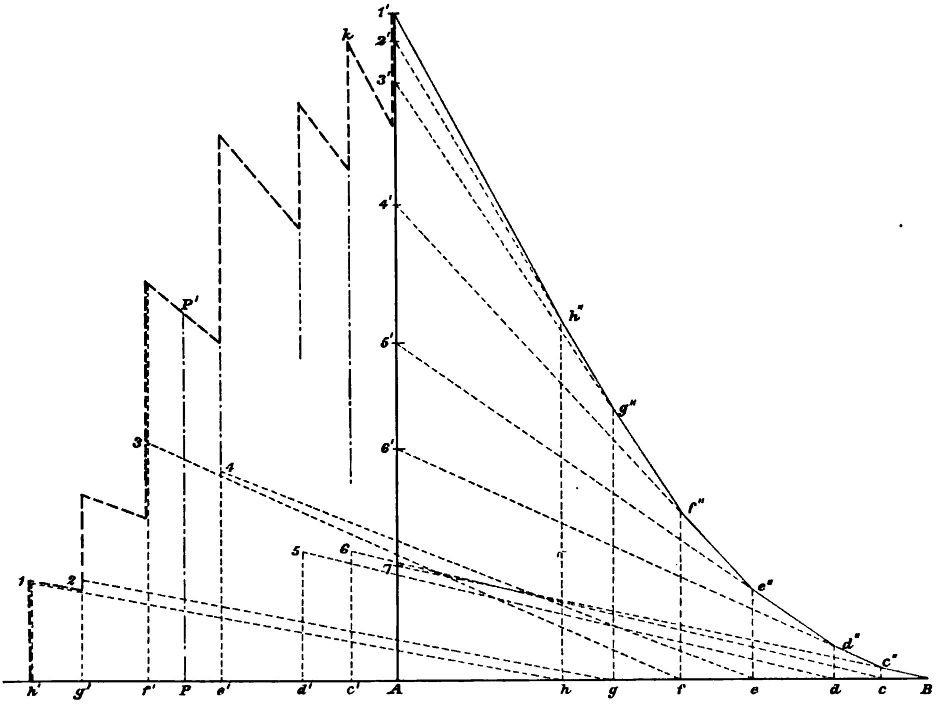


FIG. 190.

Add this to the triangle *A*7*B*.

On the vertical at *A* measure the intercept between *A* and the line 6*c*. This represents the shear at *A* due to axle No. 6.

Plot this above the point 7, viz., 7 6'.

Also at *e* erect a perpendicular *c c''* cutting *B* 7 at *c''*. Join *c''* 6'.

Then for axle No. 5 lay off points *d* and *d′*, 5 feet 3 inches further to the left of *c* and *c′*.

Erect perpendicular of 9 tons for axle No. 5, viz., *d′*5. Join 5*d*.

Measure intercept above *A* and lay it off above point 6′, viz., 6′ 5′.

Erect perpendicular at *d* meeting *c″* 6′ at *d″*. Join 5′ *d″*.

Then measure points *e′ e*, 9 feet 1½ inches further to the left.

Erect *e′*4, to represent 14·5 tons, join 4*e*.

Measure intercept on the vertical through *A* and lay it off as at 5′ 4′, and so on.

This completed diagram gives the maximum shear at any point for the loads in question, measured vertically above that point on the span, and occurs when the leading axle (No. 7 in this case) reaches that point in its passage over the span.

The maximum shear at *A* scales 46·75 tons in this example.

The closer together the wheel loads are, or the more nearly they approximate to a uniform distributed load, the more nearly does the diagram approach to a regular curve, namely, a parabola.

The portion of the diagram to the left of *A* may be completed as shown in Fig. 190 in thick dotted lines, so as to give the decreasing values of the shear at *A* as the engine rolls off the span to the left.

It is obvious that as each axle reaches *A*, and passes off the span, the shear at *A* must decrease with a sudden drop, equal to that axle load; it will then increase again gradually as the remaining axles approach it, until the next one reaches *A*, in its turn, and passes off the span, when the shear will again drop by that amount.

Or, conversely, if the engine were to return across the span from left to right, with axle No. 1 leading, the increasing shear at *A* would be measured by the ordinates of the thick dotted curve over axle No. 7, and would go on increasing until axle No. 7 reached *A*, when it would reach the maximum (*Ah* in Fig. 189 and *A*1′ in Fig. 190) and then decrease again according to the diagram, as the axle No. 7 moved away from *A* across the span.

It is also worthy of note that if axle No. 7 is small compared with axle No. 6, it is quite possible for the shear at *A* to be a maximum when axle No. 7 has passed off the span, and axle No. 6 is close up to A. This is equivalent to saying that the point k in Fig. 190 may be higher than point $1'$.

To complete the diagram add the ordinates at c', d', e', f', g', and join the points so obtained, making allowance for the sudden decrease due to one axle at each point.

Then after axle No. 7 has passed off the span to the left of A the shear at A can be ascertained for any position of the engine by measuring the ordinate *above axle No.* 7.

Viz., if axle No. 7 is at P, then $P\,P'$ represents the value of *the shear at A* at that moment.

It should be noted that if the engine were to cross the span with axle No. 1 leading, instead of axle No. 7, the maximum stress diagram would be different, giving either larger or smaller shear stresses. Which of these two positions gives the larger shear stresses must of course be determined in every case, and that giving the smaller may be neglected. In this case the engine running backwards, *i.e.*, axle No. 7 leading, has been found to give the larger shear stresses, and has accordingly been so considered for plotting the maximum shear stress diagram, Figs. 189 and 190.

NEGATIVE SHEAR.

The shear diagrams in Figs. 189 and 190 give the maximum value of positive vertical shear that can occur at any point of the span due to the loads in question traversing the span from right to left, with axle No. 7 leading.

When the leading axle reaches any point on the span the vertical ordinate at that point gives the value of the shear immediately in front of the load, the shear behind it being either considerably less, or of opposite nature and negative in sign.

If the engine were to cross the span in the opposite direction, viz., from left to right, with the same wheel leading, viz., No. 7, a shear stress diagram could be drawn precisely similar to the above in every detail, but having the maximum shear stress ordinate at B instead of A, and the zero at A instead of B. In this latter case the shear stresses would be of an opposite nature to those caused when the engine .crosses from right to left, and are called negative for convention, since they would tend to neutralise those called "positive."

This diagram is plotted below the datum line *AB*, for convenience, *vide* Fig. 191, and the complete shear stress diagram for both directions of crossing is given in Fig. 191.

The reasons for calling one shear positive and the other negative have been given on p. 62.

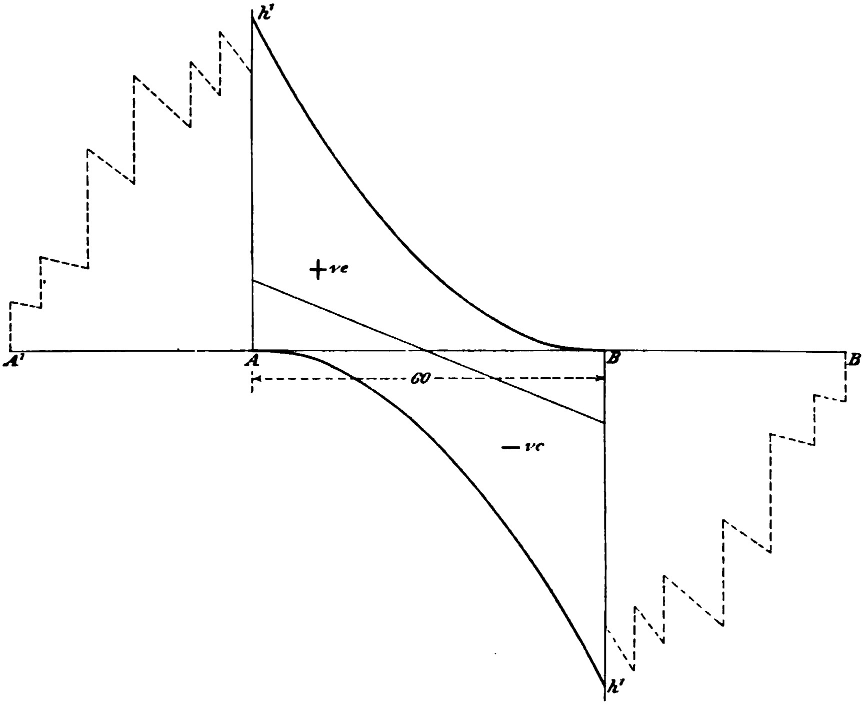


FIG. 191.

It is essential that the difference between positive and negative shear should be thoroughly understood.

To make this matter clear, or to call it under review again, consider Fig. 192.

The leading wheel *P* is on the span moving from right to left, and the small rectangles *a* and *b*, drawn on the side of the beam or girder (greatly exaggerated in shape and size), tend to deform or elongate to a diamond shape in opposite directions, the deforming stress on "*a*" being considered positive, and that on "*b*" being considered negative.

The positive stress in front of the load *P* would be ascertained from the positive or upper diagram in Fig. 191, while that behind *P* is not obtainable from the diagram since it is not nearly equal to the maximum negative shear for that point.

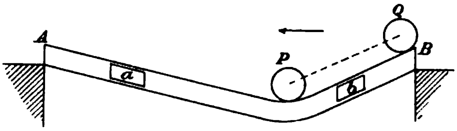


FIG. 192.

In Fig. 193 the loads have advanced until both are on the span. Then the shear to the left of *P* is positive and that to the right of *Q* is negative; that between *P* and *Q* may be either positive or negative, depending on the relative weight of the two loads and their positions on the span.

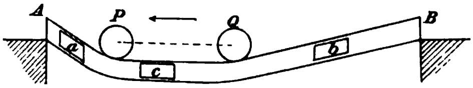


FIG. 193.

These two opposite kinds of shear stress must be most carefully kept separate, as they stress the beam, or girder, in opposite ways, and neutralise each other when they occur on the same portion of the span.

DEAD LOAD.

So far in the investigation of shear stresses the weight of the girder has been ignored, and only the accidental or super-imposed loads have been considered.

In the example of *Max.* M_B, p. 141, the weight of the girder was taken as 20 tons, or $\frac{1}{3}$ ton per foot run.

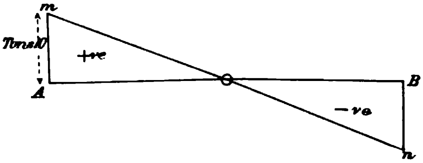


FIG. 194.

This is a permanent distributed load, entirely independent of the accidental load, and its diagram of shear stress on the span is given in Fig. 194.

This is plotted to the same scale as the diagrams representing the shear due to the axle loads.

For convenience or compactness this is transferred to the base line *AB* in Fig. 195, and is lettered *mn*.

The shear stresses due to the dead load must of course be added to those due to the accidental load, in order to ascertain the total maximum shear stress at any point, due account being taken of the sign of the shear.

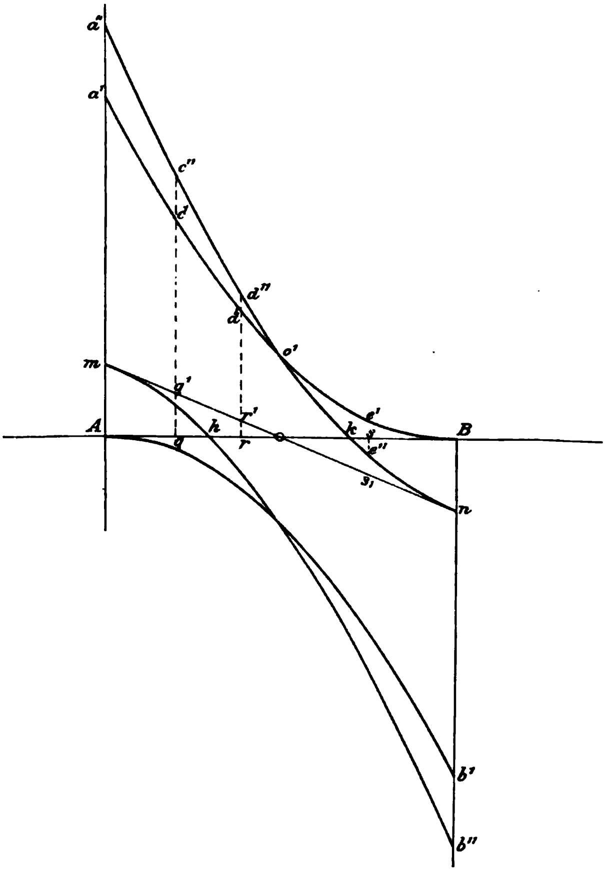


FIG. 195.

Suppose, for example, that the engine is crossing backwards as before, from right to left, causing positive shear stresses on the span as shown by the diagram *Aa'c'd'e' B* (Fig. 195), and that the shear stresses due to the weight of the girder itself are shown by the diagram *mon* (Fig. 195), then in order to combine these two diagrams, the ordinates must be added, thus :—

Lay off $ma'' = Aa'$; then Aa'' represents the total positive shear at *A*.

At any points along the span, say 5 feet apart, add the ordinates, viz., the total ordinate qc'' is the sum of qq' and qc' laid off above *q*.

rd'' is the sum of rr' and rd' laid off above *r*.

At *o* there is only one ordinate which remains unchanged.

se'' is negative and is the algebraic sum of ss' (negative) and se' (positive) laid off downwards from *s*.

At *B* there is no positive ordinate and *Bn* is the sum of the shears at *B*, being negative.

The total combined diagram is therefore $Aa''c''$ . . . . $kc''nB$.

AB remains the datum line from which ordinates are measured, and it will be noted that the net shear due to both the accidental and the permanent loads is positive to the left of the point *k*, and negative to the right of it.

The same superposition applies to the negative sheer stress diagrams below the line *AB*, the final curve being mhb''.

This complete diagram shows that to the left of the point *h*, *i.e.*, between *A* and *h*, the shear is *always* positive, though it varies greatly in magnitude, while to the right of *k* or between *B* and *k* the shear is *always* negative.

Between *h* and *k* the shear stress may be either positive or negative.

REVERSAL OF STRESS.

A girder or beam over the span *AB* in Fig. 191 is liable to either positive or negative shearing stress throughout its length. Or, in other words, all portions of the beam are liable to be stressed, either in tension or in compression, by the shearing stresses.

When this state of affairs exists, the girder is said to be liable to reversal of stress throughout its length.

Again, the girder over the span in Fig. 195, in which diagram the permanent weight of the girder itself has been included, is liable to positive or negative shear stress between the points

h and k only, or may be said to be liable to reversal of stress between the " limiting points " h and k.

The greater the weight of the girder or other permanent load compared to the accidental load, the closer together h and k will be.

Between h and k special means must be taken in designing a girder in order to meet the reversal of stress shown in the diagram.

In Fig. 194, in which the girder is loaded with a purely uniform distributed load, there is no reversal of stress.

MAXIMUM SHEARING DIAGRAM. UNIFORM DISTRIBUTED LOAD.

This is a case often occurring in bridge design, in which the engine and train crossing a span are not considered as a collection of separate wheel loads, but the uniform equivalent distributed load is used instead.

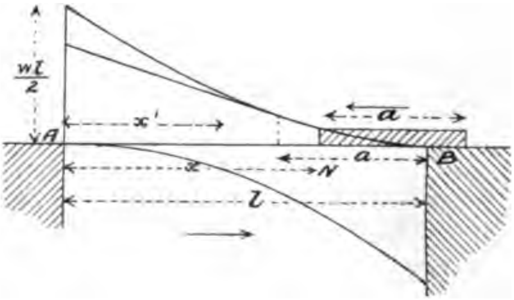


FIG. 196.

Assume that a train of uniform weight w per foot run is running on to a span (Fig. 196) from right to left.

Let l be the length of the span and a that of the train (less than the span).

Then the total weight of the train $= wa$.

Then suppose the train to have reached some point N; then the maximum shear possible at N occurs when all possible loads are added on the right of N, and when there are no loads to the left of N, *i.e.*, when the entire span between N and B is covered by the train, and the head of the train has not passed N.

For adding any load to the right of N increases the positive shear at N, while adding a load to the left of N (by allowing the train to pass to the left of N) causes a small negative shear at N, making a reduction in the total shear at N.

Hence the maximum shear stress at any point occurs when the head of the load has advanced just up to that point.

Let N be distant x from A.

Now the value of the positive shear at $N = R_A$.

$$= w\frac{l-x}{l}\cdot\frac{l-x}{2}$$

$$= \frac{w}{2l}(l-x)^2.$$

When the whole load has come on to the span then $(l-x) = a$.

Therefore shear at the head of load $= \frac{w}{2l}a^2$.

Now the equation $S = \frac{wa^2}{2l}$ is of the form of the equation to a parabola.

Hence, the maximum values of the shear stress at any points along the span from B to the head of the load are given by the ordinates of a curve, which is a parabola, with equation $S = \frac{wa^2}{2l}$ having its vertex at B and its maximum ordinate $\frac{wa^2}{2l}$ at a distance a from the vertex.

If the load move on beyond the point N to some point distant x' from A,

the shear at $A = R_A$ $= \frac{wa}{l}\left(l - x' - \frac{a}{2}\right)$

$$= wa - \frac{wax'}{l} - \frac{wa^2}{2l},$$

and the curve of maximum shear stresses, as the load continues to advance, becomes a straight line, tangential to the parabola. Hence a diagram can be drawn for the above load.

Now if the train or load completely covers the bridge (*i.e.*, $a = l$, or is greater than l), which is the usual case, the equation becomes:

$$S = \frac{wl^2}{2l}$$

$$= \frac{wl}{2}.$$

M 2

Hence to draw the diagram of maximum shearing stresses for a long uniform load advancing on to a span, lay off at A and B ordinates $= \frac{wl}{2}$, the one positive and upwards, and the other negative and downwards, and draw two half parabolæ, having vertices at B and A, and passing through the two points found above respectively.

Then the ordinate at any point will give the shear at that point when the advancing load reaches that point, or, in other words, will give the maximum shear that the moving load can produce at that point during its passage across the span.

CHAPTER XII

LOADS

BRIDGE LOADS.

All loads on beams or bridges may be divided into two main classes.

(1) Dead loads which are gradually and steadily applied and remain constant.

(2) Live loads which are either suddenly applied or vary in amount or point of application, or produce impact and vibration.

DEAD LOADS.

Dead loads are always known in magnitude and direction, and the stresses which they produce are easily calculable, and are constant.

The principal dead load on a bridge is the weight of the bridge itself with the roadway, of whatever kind that may be.

In designing a bridge of known width for any given span, to carry any given load, the approximate weight per foot run of the bridge can generally be obtained from some tables, or from data of a similar nature, with sufficient accuracy for inclusion in the calculations for the strength of the girders.

This weight must of course be actually checked after the girder has been designed in the rough, and the design must be altered if there be any considerable discrepancy between the actual and the estimated weight.

Tables of weights of girders are compiled for different engine loads, and for different types of girders and flooring, and are to be found in various pocket-books.

American tables, such as are given in American pocket-books, are of little value for English practice, and are quite useless for single lines of development in new countries of various gauges.

All engineering offices of railway companies and steel manufacturing works, however, have tables of the weights per foot of their usual types of girders and spans, which will prove very accurate when used for the rolling stock in use by that particular company, etc. These tables sometimes separate the weight of the girders themselves over the clear span from that due to the portion over the bearings and that due to the floor or decking. But as a rule the decking or floor should be estimated separately, as this can be done with considerable accuracy at the outset.

Table XIII. gives the weight of plate girders for usual English practice.

So far, therefore, as railway bridges are concerned, the dead load presents no great difficulties beyond the accurate computation of the exact weight of the whole girder after the design is complete, as a check or correction of the original assumptions either from tables or experience.

For road bridges the matter is not so easy. Tables do not exist of a reliable nature, owing to the fact that the loads that road bridges have to carry vary so tremendously.

The weight of the floor of the bridge and the superimposed roadway must first be found in detail as accurately as possible from any ordinary tables of weights of brickwork, etc., similar to Table XVIII. in this book, and must be added to the live load for which the girders are to be designed.

The actual weight of the girders may then be obtainable roughly from makers' tables, but the accurate assessment of the weight of road bridges can only be obtained in practice by trial and error, which though a tedious process is not difficult, and must be gone through.

LIVE LOADS.

Live loads produce greater stresses than those directly due to their own weight or magnitude. The actual amount of extra stress is indeterminate, and depends on a variety of things. In the first place the effect of live loads on main girders of a bridge is quite different to those on the floor members which are more directly in contact with the moving load.

Also the position of the main girders is in the main parallel to the direction of motion of the loads, while the floor members are generally at right angles to the direction of motion. This matter of position is an important one, affecting greatly the effect of the load on the structure.

The effect of a live load is also, generally speaking, reduced or lessened by transmission, so that if a load imparts impact to one member through another member, the effect is reduced thereby.

This is not always true of vibration, which must depend largely on the rigidity, tuning, fixing, and other qualities of the members.

The same applies not only to bridge floors, but also to all structures; direct impact is always greatest, and transmitted impact is always lessened.

We must, therefore, separate the consideration of main girders in bridges from that of cross girders and other members of a bridge floor, and we shall deal with the former first.

There are, broadly speaking, two ways of allowing for the extra effect, impact, and blows caused by live loads, and those for main girders in bridges will be considered first.

LIVE LOADS ON MAIN GIRDERS.

The two methods of correction are—

(a) To allow a very reduced working stress in the material of which the beam or girder is made, and to add no percentage to the actual load.

(b) To add a percentage to the actual load, obtaining the equivalent dead load, and to allow the full safe working stress of the material.

(a) This is the American practice. The live load is treated as dead load, and the following reduced values are allowed for "*f*," or stress per square inch:—

For Steel.

4½ tons per square inch	for spans ...	up to 20 feet.
5 tons per square inch	,,	from 20 feet to 30 feet.
5½ tons ,, ,,	,,	from 30 feet to 80 feet.

For Iron.

Stresses are 20 per cent. less.

It will be noticed that the span is an important factor.

(b) This is the English practice.

The stresses allowed are:—

7 tons per square inch in steel,
5 „ „ „ iron,

for all spans, and a percentage is added to the live load to obtain the load which is used for calculation.

This load after the percentage has been added to it is called the "equivalent dead load," and it can be used for purposes of calculation as if it were a dead load.

The percentage for live load must of necessity vary for different kinds of loads and bridges, and also for other different structures.

Deflection.—In calculating deflection by the formula $\Delta = \frac{n'fl^2}{Ey}$, the full value of f, *i.e.*, 7 tons per square inch, must be given. The correction of live load by using a reduced value of f, viz., 4½ tons, implies that vibration and impact may actually cause the intensity of stress in the member to rise as high as 7 tons per square inch, and it is the maximum deflection under such loading that is required to be found.

This present book deals mainly with two kinds of structures, namely, " Road Bridges " and " Railway Bridges."

Road bridges have to carry slowly moving loads subject possibly to much oscillation, and causing great impact due to the road surface.

Railway bridges, on the other hand, have high speed loads to contend with but not so much impact, due to the even nature of the permanent way.

LIVE LOADS ON RAILWAY BRIDGES.

Railway bridges are subject mainly to violent loads, but in dealing with them we have an advantage over other types of bridges, since the load is always of the same nature, running on a well-laid permanent way, and fixed to a definite position on the bridge, generally about half-way between the two girders of the bridge. The well-laid permanent way is the main factor, enabling the live load percentage or the resulting equivalent dead load to be kept low.

It has been found by experiment that trains crossing a bridge can produce 20 per cent. more deflection in the girders than the same dead stationary load does. This is not an exact measure, but a fair indication, of the increased effect of live loads.

The effect of speed is considerable since the energy of moving loads varies as the square of the velocity. Badly-laid track causes blows and impacts of great violence on the bridge, floor, and cross girders, which are transferred of course to the main girders. Also the slight deflection of the rails or rail-bearers has the same effect.

The rhythmical oscillation of badly-balanced machinery in engines is also an important factor.

All these forces must be allowed for in some way when calculating any beam or girder.

The accurate analysis of the stresses produced by live loads is practically impossible.

Although the effect due to live loads is very great on small spans, on large spans this effect decreases, and on spans of 250 feet and over it is almost negligible, the load being treated as dead without any percentage addition.

The reason for this is not definitely known, but is probably as follows :—

Any girder or beam under a live load, pulsating at regular intervals, is put into a state of vibration or sway.

The natural period of its vibration, and of that of the moving load, may synchronise, in which case the amplitude may increase to a very dangerous degree; but like a rod or string the girder, if fairly long, may sub-divide itself into nodes and loops; and the greater the span the more subsidiary vibrations are set up all tending to neutralise each other.

In conclusion, to allow for the effect of live loads a percentage is added to the actual load in order to obtain the equivalent dead load. These percentages for different spans are given in Table XIV. At the same time every possible precaution should be taken to ensure good track on all bridges.

SELECTION OF LOADS ON RAILWAY BRIDGES.

The load usually consists of two or more of the heaviest engines running on the line coupled together.

In practice, not more than two such may be expected, but the custom is to allow for more, if the bridge span be long enough to take them.

It is not necessarily the heaviest engine and tender that will produce the greatest M_B in the cross girders and flooring, but it is nearly always the heaviest that will have to be considered in regard to the main bridge girders or "beams," unless the span be very short. In this latter case, the heaviest axle may be the governing factor.

When a bridge is designed for a double line, a train of engines must be allowed for on both tracks at one time.

The tendency of modern locomotive design is to increase their weight.

For this reason the engineer is rather at a disadvantage in trying to design a bridge which will suit a future generation. 120 tons is not excessive for the weight of engines to-day, and 140- and 150-ton engines are being tried. Large express engines have long tenders; it is, therefore, possible that a string of tank engines, not quite so heavy in gross weight, may produce a greater M_B on a bridge span than a row of heavier engines with tenders. The engine or row of engines producing greatest M_B on any given span must, therefore, be selected for calculation for each railway line or system.

EQUIVALENT UNIFORM DISTRIBUTED LOAD.

Sometimes, for sake of convenience, an equivalent distributed load of so many tons per foot run of span is used for calculating the M_B on bridges.

This is such a load per foot run, that if it were applied all along the span, it would produce stresses in the girders equal to those actually produced by the heaviest engines on the line.

It varies for different spans, and is given in Table XVI. for all heavy modern engines, on a 4 foot 8½-inch gauge.

It is not accurate for spans under 30 feet, but is very nearly so for all spans of 30 feet and over.

It is of course only used for the design of main girders, since the flooring of the bridge, running transversely to the bridge, i.e., at right angles to the main girders, is dependent on the maximum "axle" loads of engines.

This equivalent distributed load is obtained from the actual *Max.* M_B produced by the heaviest engines, coupled together, which is found by the method of actual wheel loads.

Thus in Fig. 177, the diagram $A\,a\,a\,a\,a\,a\,a\,B$ is the *Max.* M_B diagram for a given engine, and a parabola has been circumscribed round it.

The maximum ordinate can be measured, and is found to be 690 foot-tons.

Now let w be the load per foot run, which would produce such a *Max.* M_B if uniformly spread over the span AB (60 feet).

$$\text{Then} \qquad Max.\ M_B = \frac{wl^2}{8} \text{ foot-tons} = 690 \text{ foot-tons.}$$

$$\text{therefore} \qquad w = \frac{690 \times 8}{60 \times 60}$$

$$= 1{\cdot}54 \text{ tons per foot run.}$$

This would be the equivalent uniform distributed load for this span.

In this particular example the engine is not the heaviest type, nor does it quite cover the span, so that the result is not the maximum for the span.

But if the heaviest engines were selected and were coupled close together in the most unfavourable position on the span, then the value of w obtained would be the equivalent uniform distributed load for that span, corresponding with the figure in Table XVI.

This equivalent uniform load will not give quite the same diagram of maximum shear stresses as the actual wheel loads would, but the divergence is not great. Table XVII. gives the uniform equivalent loads for metre gauge and 5 feet 6 inches.

LIVE LOADS ON ROAD BRIDGES.

Live loads may be of all degrees of violence on road bridges, and vary from quiet loads to very violent loads.

It is by no means so easy to arrange a suitable percentage for correcting these as is the case for railway bridges.

PERCENTAGE ADDITION FOR MAIN GIRDERS.

It is not possible to lay down any very definite rules for this percentage correction, since each case is usually so different from all others; the matter must therefore depend largely on the individual judgment of the engineer.

It is necessary to separate the correction for live load for main girders and for floors here, as in the case of railway bridges; main girders will be considered first.

As in the case of railway bridges, the span is a very important factor and the percentage decreases as the span increases.

Table XV. shows the percentages advocated as a general guide for addition to loads in calculating main girders.

These apply to a good road surface such as good macadam, asphalte, wood block, or other smooth hard surface. They do not apply to setts or other laid stone surface.

The material of the road surface itself absorbs a great deal of the shock and does not assist the transmission of vibration at all well.

It will also be found that in road bridges the dead load due to the weight of the bridge itself and the roadway, etc., is much larger than in the case of railway bridges; and the greater the inert dead load in comparison to the live load, the less correction is required to compensate for shock, etc.

Further, comparing road loads with railway loads, there is no regular pulse or beat about the blows which the loads strike the road, as in the case of railway loads. Engines have a regular period to their vibration, and all the rolling stock strike the rail ends at a regular interval; but in the present case there is no rhythm or time period about the blows, and the vibration induced thereby is consequently much less.

On the other hand, there are no smooth rails to carry the traffic; engineers may do what they will, but bad places in roads are bound to occur, and these cause impact. The observed effect on road bridges is just as great as on railway bridges.

The percentages given in Table XV. are not by any means intended as anything more than a general guide. They are all on the safe side and there will be many cases where they might with advantage be reduced.

NATURE OF LOADS ON ROAD BRIDGES.

Pathway Loads. People.—These weigh 60 to 80 lbs. per square foot.

These need not be taken into consideration on the actual roadway of the bridge.

Table XIX. gives the corrected loads for a few cases of floor loads.

Road Loads. Vehicles.—These consist of carts, drays, heavy wagons, timber long wagons, guns, etc., etc.

They do not weigh as much as steam or motor vehicles and so may be neglected as a rule.

Steam Traction Engines, Lorries, etc.—The Lion type tractor is about the heaviest type to be considered; it outweighs all other forms of traction, *e.g.*, lorries, commercial vans, etc. (*vide* Table XX.).

Tractors may weigh as much as 19 tons in all when loaded, but 14 tons on the back axle is quite common, with 2½ tons on the front axle, making 16½ tons in all.

The back axle of a traction engine is generally the ruling load for all road bridge floors.

Bridges are not usually designed for very heavy ordnance, and all guns mounted on travelling carriages weigh less than the back axle of a traction engine.

Tram Cars.—If the bridge under consideration is anywhere near a town where developments may be reasonably expected, tram cars should be allowed for. The weights of these are tabulated in Table XX.

DISTRIBUTION OF LOAD.

When the bridge is wide enough for two streams of traffic two streams must be allowed for, and if it be wide enough for three or four these must be allowed for.

As a rule, the floor of a road bridge transmits the load to the main girders at many points, since the floor rests upon the main girders at many points.

Thus the load when transmitted is nearly a uniformly distributed load.

For main girders, therefore, it will suffice to arrive at a load which may be considered as being distributed along the span. For this the bridge must be supposed to be covered with traffic on all the traffic streams that are possible, and a load per foot run must be worked out.

It is not possible to lay down exactly what vehicles must be supposed to be on the bridge at one time. This is a matter of local circumstance. Town bridges and country bridges are not the same thing. A string of traction engines on both sides of a road would be too heavy.

A string of commercial lorries or a train of one traction engine and two loaded trucks would be suitable to distribute over the bridge. The total load should then be divided up by the number of feet span and a load per foot run be obtained.

It must be remembered that road vehicles do not travel fast, and that consequently their effect on the floor is not so great. At the same time the road surface is a very important thing. Everything possible should be done to keep this in first class order at all times.

A bad road surface causes most terrible blows and impact, and its effect cannot be exaggerated.

LOADS ON FLOORS OF BRIDGES.

So far the effect of live load has only been considered with regard to main girders, or beams, or similar members. Now the effect of live loads on floors is a different thing, being much greater than on the main girders, and at the same time much more difficult to determine.

RAILWAY BRIDGE FLOORS.

In the case of railway bridges, rails deflect between sleepers under the advancing load, and as the load reaches the sleeper, where no deflection is possible, it strikes a decided blow at that point, upon the floor.

In the same way rail bearers or road bearers running longitudinally with the bridge, deflect slightly, and when the load above them comes over a point of support under the bearer, where no deflection is possible, it strikes a decided blow on the support, whatever it may be. This support is generally a cross girder, which spans from one main girder to the other. The cross girder, therefore, receives a violent blow as the load reaches it.

Hence it is usual to allow a percentage of 100 per cent. on all live loads passing over members at right angles to the direction of motion, in the floors of a railway bridge.

The methods in which this percentage is applied to floor members are two.

Either the percentage may be added to the load and the usual value of "f" may be taken in calculating, or a reduced value of "f" may be introduced and no addition be made to the load.

It is the customary practice to take f as being 4½ tons per square inch for floor members. This is the same as an addition of 100 per cent., since this covers the dead load as well as the live load.

Tables XIV. and XV. of percentage additions to live loads do not, therefore, apply to floor loads.

When this reduced value of "f" is taken, as is the usual practice, reduced values of "f" must also be taken for the other constants, viz., rivets, etc.

The constants adopted are:—

Shearing 4·8 tons per square inch.

Bearing 6·4 tons per square inch.

These same constants apply also to the floor members themselves.

ROAD BRIDGE FLOORS.

Live loads.—This matter is more difficult than the floors of railway bridges.

In considering the addition to live load for floor stresses, several factors have to be taken into consideration.

In the first place it is unusual for a long span bridge to have only two main girders; a number of girders or ribs of comparatively small depth are frequently used, and hence the floor joists, transverse to the bridge, will have a very small span, and will be firmly fixed at such frequent intervals as to be rigid and incapable of great vibration.

Distribution of load.—The material of which the road surface is made is such as to greatly distribute the weight.

The weight may be taken as distributed through a cone of the material under it, having its apex at the point where the load is resting and an apex angle of 90°.

If the road metal is thick, then this cone will cover a comparatively large area of the rigid floor, or other member below it.

This cone of dispersion should always be considered. It is the standard method in considering distribution in reinforced concrete floors, etc.

From this it will be seen that if the cone of dispersion only covers one trough of a trough floor (Fig. 250), the load must be assumed to be borne entirely by this one trough.

Again, the material of the road itself absorbs a great deal of the shock, in some cases, and does not assist in the transmission of the vibration.

Percentage addition.—It is clear that there is no need for such a large addition as 100 per cent., as is given to cross girder loads in railway bridges.

The impact due to the slow moving load is not at all the same.

It is difficult to lay down any definite rules; but this may be said with certainty, that where a proper road is laid upon the bridge, such as a concrete foundation, and macadam on top, or asphalte or wood block paving, the live load corrections for the floor members need not be much more than for the main girders.

An extra 15 per cent. added to the percentages given in Table XV. for main girders should always suffice.

The following points should be borne in mind in designing:—

The better the road surface the less the impact will be.

Cobbles and setts, frequently laid beside tram lines, are to be avoided.

Multiple ribs under a bridge minimise the vibration in cross girders.

The deeper the road formation and metal over the steel floor members, the greater the distribution of load and impact.

It will be found in road bridges that the dead load is very large compared to the live load, and this is more so the greater the span of the bridge.

In this case it would not be fair to use a reduced value of f for calculations, as this would be correcting the dead load as well as the live load.

This is not a matter of any serious importance in railway bridges, where the floor is light, and all the extra dead load due to long spans is in the main girders.

LOADS FOR FLOORS.

These have been enumerated in the earlier part of this chapter.

Floors are governed by the heaviest axle loads, as in the case of railway floors.

It must be remembered that the load is not confined to one part of the floor, as it is in the case of railway bridges by the rails, but may occur anywhere.

A pair of heavy wheels will not produce the greatest M_B when placed at the middle of a cross girder, but when slightly to one side of the middle, so that the centre point of the cross girder bisects the distance between one wheel and the c. g. of the two.

The cross girders in a road bridge will generally be spaced so close together as to preclude any weight being brought on to one cross girder by other axles than the one immediately above it.

CHAPTER XIII

PLATE GIRDERS

A PLATE girder is the simplest and most common form of girder in use for bridges, between spans of 20 and 60 feet. It is similar in general shape to an I section beam, but being too large to be rolled whole in the mills, is built up in different parts (Fig. 197).

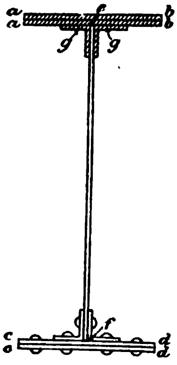


FIG. 197.

The plates *ab* and *cd*, either single or multiple, compose the top and bottom flanges, taking the compressive and the tensile stresses in the girder.

They are joined together by a web plate *ef*, which may be very thin, and only serves to resist shearing stress; *i.e.*, it unites the top and bottom flanges and prevents them moving with regard to one another.

The top and bottom plates are joined to the web by angle steels, *g*, *g*, which are riveted to both the flanges and the web, and which are included in the cross sectional area of the top and bottom flanges, which are utilised for resisting compression and tension.

SIMPLE PRINCIPLES OF DESIGN.

In designing a plate girder for any particular case, it is necessary to select the plates composing the top and bottom flanges, and to fix upon the depth of the beam, so as to give exactly the moment of resistance required. After due consideration of the loads to be borne, including the dead weight of the bridge, and percentage for live load, a *Max.* M_B diagram should be drawn for these conditions.

The M_R at each point must then be made equal to the M_B at each point as shown on this diagram.

The shearing stress diagram must also be drawn and the depth of the girder and thickness of the web must be made sufficient to resist this shearing.

The M_B and M_R will be dealt with first.

The formula usually employed for plate girders on account of its simplicity is $M_R = fad = M_B$.

"f" may be taken as 7 tons per square inch for steel, either in compression or tension.

"d" is the effective depth of the girder, and may be altered to suit each case at will.

It is usually measured from the top of the upper angle steel to the bottom of the lower angle steel, or in other words is the same as the depth of the web plate.

It is generally kept constant throughout the girder.

If it is varied it produces a girder with either a curved upper flange or a curved lower flange.

It is generally confined within narrow limits by the circumstances of the particular case.

"a" is the flange area or effective section of the flange. This is another variable and is that which is most useful to the designer in increasing or decreasing the M_R of the girder as required.

This factor "a," as has been shown, is really the equivalent area of the portion of the girder above or below the neutral axis; and since the bulk of the cross-section of the flanges is concentrated as near the top and bottom edges of the girder and as far from the neutral axis as possible, their equivalent area is practically equal to their actual effective area; *i.e.*, they will to all practical purposes be stressed all over to the maximum intensity of f, all fibres being considered as "extreme fibres."

In order to be on the safe side in the above assumption any tensile or compressive stress exerted by the web is ignored, the web being considered as resisting shear stresses only.

In Fig. 197 the shaded portion represents "a," which is taken as the actual flange area, on the assumption given above.

Rivet holes must of course be deducted.

GENERAL DIMENSIONS AND PRACTICE.

Parallel Girders.—A parallel girder is one in which the top and bottom flanges are straight, and parallel to one another.

This is the commonest type of construction.

Effective Depth.—The depth "*d*" is usually $\frac{1}{10}$ to $\frac{1}{16}$ of the span in plate girders.

Flanges.—The flanges are generally made $\frac{1}{30}$ to $\frac{1}{40}$ of the span in width. They vary between 12 inches and 21 inches, rarely more, with a single web. The width is constant throughout the girder. They are built up of one or more plates, and angle steels or tees, riveted together, the latter serving as connections between the flange plates and web plate.

Standards.—In designing any flange, standard sizes must be adhered to. These are laid down by the British Standards Committee. Other sizes can of course be obtained, but are not generally kept in stock by makers. Any variations lead to expense.

It is also desirable for the sake of economy to keep to one size of rivet throughout the work, one pitch of rivets, one section of L and T steels, or as few such sizes as possible, and also to keep the girders symmetrical end for end.

Where the girders are placed on a skew, it may be desirable to make one end of one main girder different to the other, but the other main girder should be similar to it throughout end for end.

Dimensions.—All dimensions of any girder should be decided with reference to the pitch of the rivets, so that joints, etc., will fall in with the pitch of rivets without having to alter the latter.

Board of Trade Regulations.—The Board of Trade Regulations lay down the amount of clearance required all round railway rolling stock, so that this and the gauge decide the width of the bridge (Chap. XVI.).

Camber.—$\frac{1}{2}$ inch in 20 feet is usually given to all main girders. This does not affect the design, except in so far as that the camber is stated in the specification by the designer, but it is not shown in the drawings, and its execution is left with the makers, who arrange to give the required camber during the operation of plating.

Bearings.—All girders are made longer than the clear span which they have to bridge, so as to obtain a good bearing at the ends on the abutments.

This bearing should hardly ever be less than 2 feet 6 inches, along the centre line of the girder, and is more usually 3 feet for spans of 20 feet to 75 feet.

Effective Span.—The span for calculation is always taken as the distance from centre to centre of bearings, *i.e*, if a girder over a clear span of 75 feet has a bearing of 3 feet at each abutment, the theoretical span is 78 feet.

This span is called the effective span.

EXAMPLE.

To design a pair of plate girders for a railway bridge to carry a single line of railway with all ordinary rolling stock, over a clear span of 32 feet.

A typical engine is shown in Fig. 198. Gauge, 4 feet 8½ inches.

This is a small span, and the girders will be made parallel, *i.e.*, having the top and bottom flanges parallel to one another, and thus "*d*" will remain constant throughout their length.

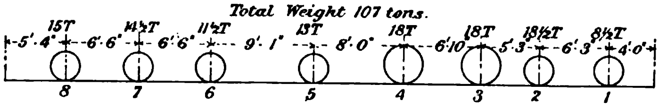


FIG. 198.

The completed design is shown in Plate III.

A bearing of 2 feet 6½ inches will be given at the ends of the girders, making the total length of the girders 32 feet + 5 feet 1 inch or 37 feet 1 inch, and the theoretical span = 34 feet 6½ inches, or, say, 34 feet 6 inches.

(NOTE.—The ½ inch odd amount in the length of the bearing is introduced by the thickness of the end plate of the girder.)

The two girders will be identical with one another in all respects.

Depth.—The depth "*d*" will be made about $\frac{1}{12}$ of the span, viz., 3 feet.

This is the actual depth of the web plate.

Width of Floor.—The next consideration is the total width of floor between the two girders.

This is governed by the gauge and also by Board of Trade regulations. These regulations lay down that no standing work projecting more than 2 feet above the level of the rails is to be nearer than 2 feet to the side of a railway carriage, making the total width between girders 13 feet 2 inches; *vide* p. 264.

In the present case the girders will be less than 2 feet above rail level, and they consequently may be allowed as close as 4 feet 7 inches to the centre of the track. This would give a total width of 9 feet 2 inches between girders, or 9 feet 2 inches + 1 foot 4 inches between centres of girders.

This is rather narrow, however, for a single track bridge, and would allow no room for fixing and bracing handrails on top of the girders.

If we allow 12 feet 4 inches between centres of girders, this will give 13 feet 8 inches between handrails and plenty of room (Fig. 199).

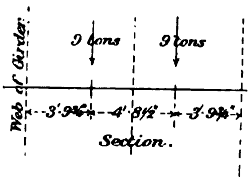


FIG. 199.

Live Load.—The load for this span might be worked out for the engine shown in Fig. 198, by the graphic method for separate wheel loads; but for ordinary rolling stock on the 4 feet 8½ inch gauge, Table XVI. of Uniform Equivalent Loads is very accurate and will here be used.

This Table gives an equivalent load of 3 tons per foot run for a span of 34 feet 6 inches approx., or:—

$$W = 103{\cdot}5 \text{ tons on the span,}$$
$$= 51{\cdot}75 \text{ tons on each girder.}$$

This is live load, and must be corrected by the addition of a percentage.

Table XIV. gives 15 per cent. addition for this span, hence the equivalent dead load for this span = 59·51 tons.

Dead Load.—This consists of

(a) The weight of the flooring.

(b) The weight of the permanent way.

(c) The weight of the girders themselves.

In this case the flooring is to be composed of steel troughing placed transversely to the bridge; the ends of the troughs rest on the top of the inner portion of the lower flange of the girders.

The troughs are riveted together side by side. No ballast is used.

On top of the troughs are placed longitudinal sleepers of 8-inch × 12-inch timber, on their flat, and on these rest the rails.

The calculations and details of this floor are worked out in example on p. 171.

(a) The troughing selected for the floor is 2 feet wide × $9\frac{3}{8}$ inches deep, and weighs 35 lbs. per foot super.

Hence the weight of the troughing on one girder is :—

$$= \frac{12 \text{ feet } 4 \text{ inches}}{2} \times 34 \text{ feet } 6 \text{ inches} \times 35 \text{ lbs.} = 3{\cdot}3 \text{ tons.}$$

(b) The weight of the permanent way and longitudinal sleepers may be taken as 1·5 cwt. per foot run of track.

Hence the weight on one girder

$$= \frac{34 \text{ feet } 6 \text{ inches}}{2} \times 1{\cdot}5 \text{ cwts.}$$

$$= 1{\cdot}3 \text{ tons.}$$

(c) It is necessary to add the actual weight of the girders themselves; this may be estimated at 4 tons for each girder.

Hence W, or total load on one girder

$$= 68{\cdot}11 \text{ tons.}$$

$$\text{Now the } Max.\ M_B = \frac{Wl}{8}$$

$$= \frac{68{\cdot}11}{8} \times 34 \text{ feet } 6 \text{ inches} \times 12 \text{ inch-tons}$$

$$= 3524{\cdot}7 \text{ inch-tons.}$$

Therefore we plot a parabola on a base of 34 feet 6 inches having a central vertical ordinate 3524·7 inch-tons (Fig. 200), and the ordinates of this parabola will represent the *Max.* M_B at every point of the girder.

Shearing Stress.—The diagram of maximum shear stresses is also drawn at the same time. In Fig. 201 the ordinates *Am* and *Bn* are drawn, each equal to half the dead load, *i.e.*,

$$= \tfrac{1}{2} \times 8{\cdot}6 \text{ tons}$$

$$= 4{\cdot}3 \text{ tons.}$$

Join *mn*.

This is the diagram for the dead load.

The diagram for the live load is a parabola, since the load is

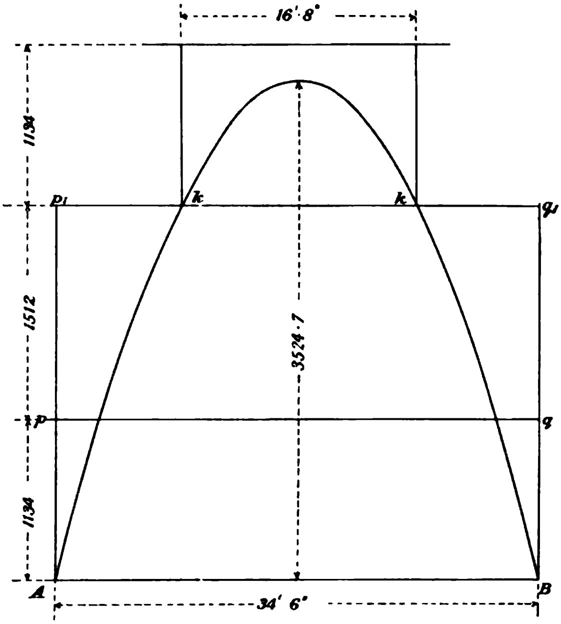


FIG. 200.

considered as a uniformly distributed load, *i.e.*, 3 tons per foot run.

Therefore make the ordinate $Ab = \frac{W}{2}$ after the addition of 15 per cent.

$$= \frac{59 \cdot 51}{2} \text{ tons}$$

$$= 29 \cdot 76 \text{ tons,}$$

and draw a parabola with vertex at B passing through b.

Combine this with $A\ m\ o\ n\ B$.

Similarly draw the negative shear diagram.

NOTE.—The shearing stress diagram for the moving load represents the maximum shears produced on the span by the load advancing across it. These are greater than what would be produced by the load merely standing still upon the span.

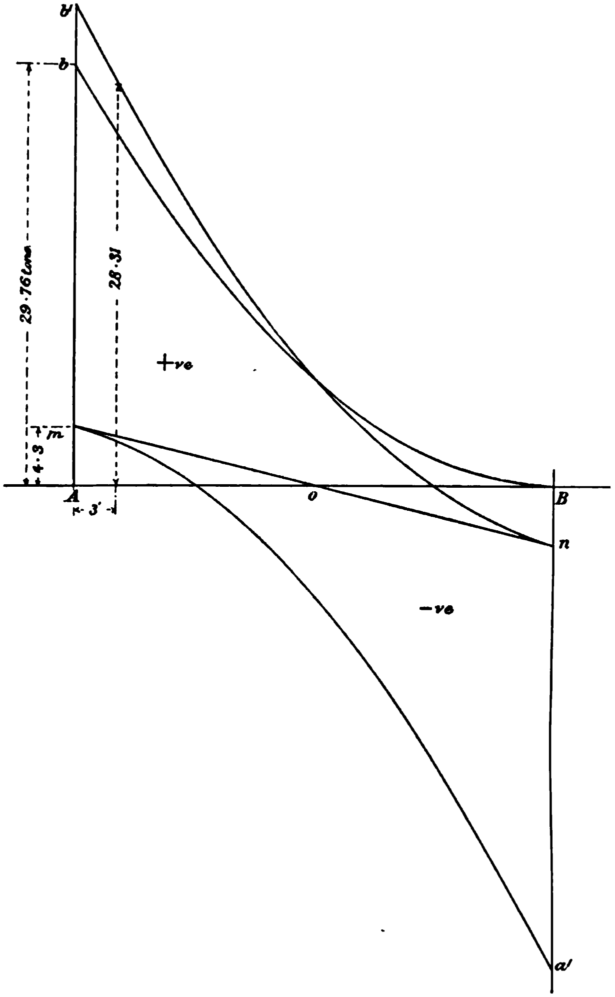


FIG. 201.

M_R.

Now let us assume that the top and bottom flanges are each to consist of one plate 16 inches wide × ½ inch thick, connected to the web by two angle steels $3\frac{1}{2}$ inches × $3\frac{1}{2}$ inches × $\frac{1}{2}$ inch.

Refer to Plate III., in which the section of the girder is shown.

The effective area or "*a*" for the two angles only, deducting two rivet holes, 1 inch diameter each, is:—

$$\begin{aligned} &= 2 \times (3\tfrac{1}{2} + 3 - 2) \times \tfrac{1}{2} \\ &= \tfrac{9}{2} \\ &= 4\tfrac{1}{2} \text{ square inches.} \end{aligned}$$

Therefore the M_R that these two angles will supply is

$$\begin{aligned} M_R &= fad \\ &= 7 \times \frac{9}{2} \times 36 \\ &= 1{,}134 \text{ inch-tons.} \end{aligned}$$

Now this M_R due to the two angle steels can be plotted on the *Max.* M_B diagram in Fig. 200.

Erect perpendiculars Ap and Bq each equal to 1,134 inch-tons on the same scale as that to which the M_B is drawn.

Join pq.

Then any ordinate of the diagram $ApqB$ represents the M_R in the girder due to these angle steels.

Similarly the area or "*a*" for the $\frac{1}{2}$-inch plate deducting rivet holes $= fad$

$$\begin{aligned} &= 7 \times (16 - 4) \times \tfrac{1}{2} \times 36 \\ &= 7 \times 12 \times \tfrac{1}{2} \times 36 \\ &= 1{,}512 \text{ inch-tons.} \end{aligned}$$

Plot the M_R due to this plate on top of that due to the angle steels, viz., erect ordinates pp_1 and qq_1 in Fig. 200, each equal to 1,512 inch-tons.

Join p_1q_1.

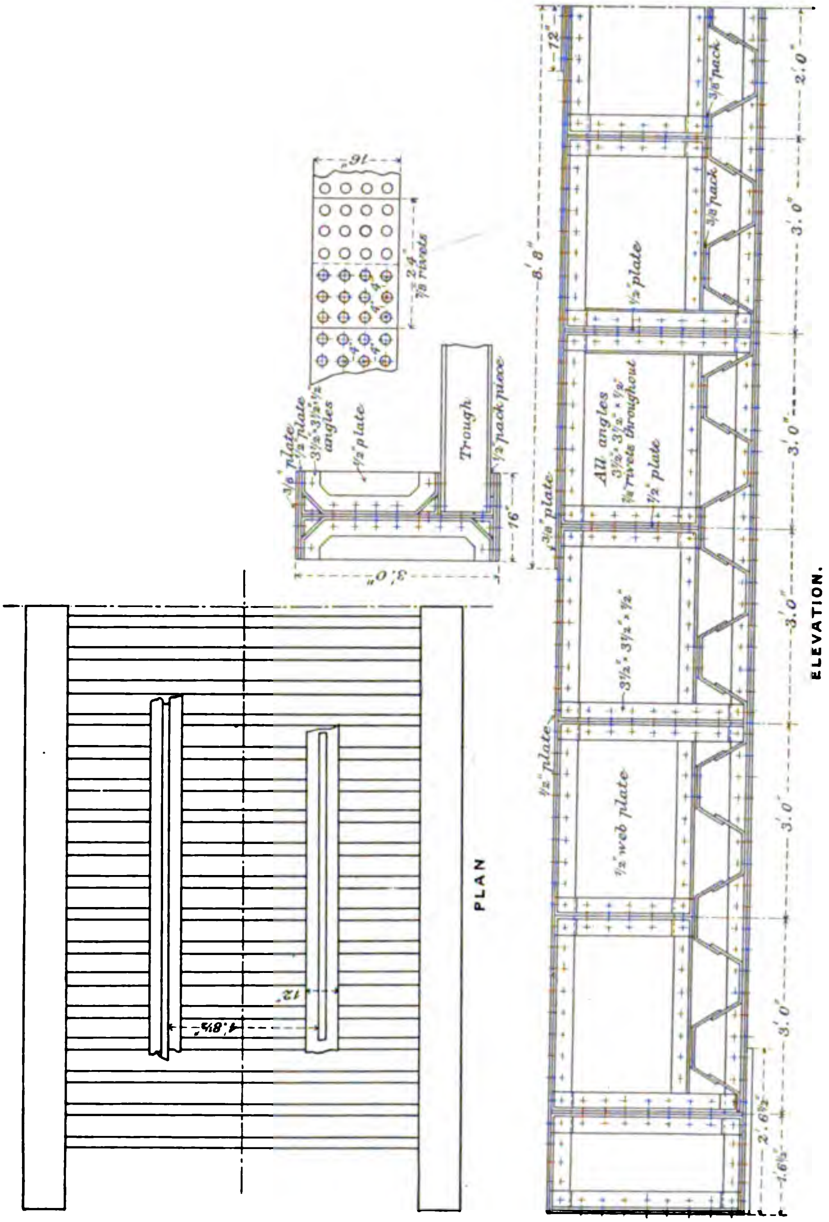
These with the angle steels give a total M_R of 2,646 inch-tons throughout the girder.

The diagram shows that this is enough for the ends of the girder, but is insufficient for the central portion.

The points where the M_R diagram, or the line p_1q_1, cuts the M_B parabola are 8 feet 4 inches on either side of the centre line of the span.

Hence for 8 feet 4 inches on each side of the centre the M_R is

PLATE III.


16"
2'4"
⅞" rivets
4"
⅜" plate
½" plate
3½"×3½"×½" angles
½" plate
Trough
½" pack piece
16"
3'0"
12"
8'8"
⅜" pack
⅜" pack
½" plate
All angles 3½"×3½"×½"
⅞" rivets throughout
½" plate
⅜" plate
3½"×3½"×½"
½" plate
½" web plate
2'0"
3'0"
3'0"
3'0"
3'0"
3'0"
2'6½"
1'6½"
ELEVATION.
PLAN
12"
1'8¾"

insufficient, and an extra plate 16 inches × $\frac{3}{8}$ inch must be riveted on to the top and bottom flanges.

This will increase "*a*."

The effective area "*a*" of this plate will be

$$= (16 \text{ inches} - 4 \text{ inches}) \times \frac{3}{8}$$
$$= 4{\cdot}5 \text{ square inches,}$$

and its $M_R = fad$

$$= 1{,}134 \text{ inch-tons.}$$

This gives a total M_R of 3,780 inch-tons at the centre which is well in excess of the *Max.* M_B on the span.

This is plotted in Fig. 200.

THE WEB.

The web is to be 36 inches deep and will be made of $\frac{1}{2}$-inch plate.

In practice $\frac{3}{8}$ inch is the minimum thickness permissible in this size of girder, as such plates and thinner are liable to buckle, and require too much stiffening.

They also prove deficient in bearing area for riveting.

The most usual practice is to use $\frac{1}{2}$-inch plate.

Rivets.—Assume that, in conformity with general practice, the pitch of rivets is to be 4 inches from centre to centre, or three rivets per foot run, and that they are $\frac{7}{8}$ inch in diameter near the ends of the girder.

These will require a hole $\frac{15}{16}$ inch in diameter, which may be considered as 1 inch diameter for calculation, since the drilling processes tend to damage the metal round the edges of the hole.

Now it is necessary to have four rows of rivets along the top and bottom flange plates, where there are two plates together, for although two rows of rivets are sufficient to connect these plates to the angle steels, two more rows outside these are necessary to keep the two plates together in intimate contact with one another, so as to prevent the ingress of water between them, etc., and consequent rust, and in fact to make them act as one plate. They are also strengthened against buckling, when in compression, by being firmly fixed together.

It is now necessary to test the $\frac{7}{8}$-inch rivets, spaced 4 inches apart, in order to see whether they are strong enough to connect the flange plates to the angles and the angles to the web plate, *i.e.*, to resist horizontal shearing stress.

We know that the intensity of the horizontal shear is equal to the intensity of the vertical shear at any point.

The values of maximum horizontal shear that can occur in the girder at every point can therefore be obtained from the shearing stress diagram.

These horizontal stresses are considered positive or negative in a similar manner to the vertical stresses, according to whether they are acting from left to right or from right to left in the girder.

It should be remembered that the vertical and horizontal shears are both components of the real diagonal shear which is acting upwards at an angle of 45° to the horizontal, and is considered positive or negative according to whether it acts upwards from left to right or upwards from right to left, *i.e.*, whether it is in the first and third, or second and fourth quadrants, considered from horizontal and vertical axis with an origin at the point at which it is acting.

Refer to the shear stress diagram from which the maximum ordinate of shear is found to be 34·06 tons.

This is not really acting at the end of the girder, since we have taken a theoretical span of 34 feet 6 inches for calculation instead of 37 feet 1 inch, but is 1 foot 3 inches from the end, viz., at a point immediately above the centre of the bearing of the girder, a point at which the reaction of the abutment may be taken as acting.

This value, 34·06 tons, is the vertical component of the whole shear acting upwards at 45° towards the centre, and the horizontal component must be equal to the vertical component in intensity per foot run, or per inch run.

Now the web plate is 36 inches deep, hence the 34·06 tons of vertical shear are spread over 3 feet of vertical web plate, or in other words there are 34·06 ÷ 3 tons, or 11·35 tons of vertical shear per foot run of web plate, on a vertical section through the web.

This may also be expressed by saying that the intensity of vertical shear at the point is = 11·35 tons per foot run. The horizontal shear at this same point must therefore also be 11·35 tons per foot run.

Now consider the shear at another point 3 feet further along the girder towards the centre.

Here the shear stress is 28·31 tons or 9·44 tons per foot run.

Therefore the average intensity of shear between these two points is 10·39 tons per foot run.

Let n be the number of rivets required *per foot run* to resist shearing due to this stress.

Then $n \times \pi \left(\frac{7}{16}\right)^2 \times 6{\cdot}5 = 10{\cdot}39$

$\therefore n = 3$ rivets.

Again, let n be the number required to resist bearing stress. The weakest point in bearing is where the $\frac{7}{8}$-inch rivets bear against the $\frac{1}{2}$-inch web plate.

Hence $n \times \frac{7}{8}$-inch $\times \frac{1}{2}$-inch $\times$ 8 tons $= 10{\cdot}39$

$\therefore n = 3$ rivets.

Therefore the 4-inch pitch, giving three rivets per foot run, will suffice, and what is sufficient near the abutment is of course sufficient near the centre, where the shear stress is less.

It would not be worth while on account of reduced shearing stress to make any alteration of the pitch of rivets near the centre.

The vertical shear is unimportant here.

However, the web plate must of course be able to resist it, *i.e.*, it must have a sufficiently large vertical cross-section to prevent being sheared across by the vertical shearing stress.

The vertical cross-section of the web plate allowing for rivets 4 inches apart is

(36 inches − 9 inches) × $\frac{1}{2}$ square inch which will take a shearing stress

$$= (36 - 9) \times \tfrac{1}{2} \times 6{\cdot}5 \text{ tons}$$

$$= 88 \text{ tons, which is ample.}$$

The web plate might be made thinner if it were not for the bearing area required for the rivets.

The arrangement of rivets near the ends of plate girders is a very common trouble; it frequently occurs that either the pitch has to be reduced to less than 4 inches, or that the diameter of the rivets has to be increased for a certain distance from both girder ends.

If this is done both ends of the girder should be made alike, and the pitch or diameter of rivets should change at a stiffener.

JOINTS.

The question of joints in plates must now be considered.

The flange plates selected are 16 inches wide and 36 feet 1 inch long by $\frac{1}{2}$ inch thick.

Table XXIII. gives the maximum sizes to which plates can be rolled in the usual course at the foundry.

It will be found that the flange plates can be obtained in one piece. The same applies to the $\frac{3}{8}$-inch extra plate on the central portion of the girder, and also to the web plate. There need therefore be no joints.

But in order to illustrate the principles involved in making joints in plates, one joint will be made in the centre of the $\frac{1}{2}$-inch plate in the upper flange, and two joints in the web plate.

Joint in the Centre of Upper Flange.—The actual joint of the plates is a plain butt joint, and must be made to occur halfway between two rivets, *i.e.*, at the exact centre of the span.

On no account should the pitch of rivets be disturbed by the joint, but the joint must be made to fit in with the rivets.

This joint will necessitate a plain single cover plate, riveted on the top of the upper flange, above the extra $\frac{3}{8}$-inch plate which is added to the flange over its middle.

The $\frac{1}{2}$-inch plate of the top flange will be in compression, and its fibres will be fully stressed at 7 tons per square inch.

The net cross-section of the upper plate has been found to be (16 inches — 4 inches) × $\frac{1}{2}$ inch.

$$= 6 \text{ square inches.}$$

Hence the total stress in the top plate is

$$= fa$$
$$= 7 \times 6 \text{ tons}$$
$$= 42 \text{ tons.}$$

The cover plate must be able to carry this and transfer it from one side of the joint to the other.

The rivets also must be able to do the same.

A piece of plate 16 inches by $\frac{1}{2}$ inch with four rivet holes will have the same net section as the flange plate and will suffice.

Now let n be the number of rivets required on each side of the joint to connect the cover to the flange plate.

The rivets are to be $\frac{7}{8}$-inch rivets.

The safe resistance to shear for rivets is 6·5 tons per square inch.

Therefore $n \times \pi (\frac{7}{16})^2 \times 6{\cdot}5 = 42$

$$\therefore n = 10{\cdot}74$$

$$n = 11 \text{ rivets.}$$

As there are four rows of rivets across the flange, *i.e.*, four rivets in a line at right angles to the flange, a multiple of 4 must be taken and 12 rivets will therefore be used.

Now test these for bearing:

The safe resistance to bearing is 8 tons per square inch.

Let n equal number of rivets required.

Then the resistance to bearing of the rivets in the $\frac{1}{2}$-inch plate is:

$$= n \times \tfrac{7}{8} \times \tfrac{1}{2} \times 8$$

$$\therefore n \times \tfrac{7}{8} \times \tfrac{1}{2} \times 8 = 42 \text{ tons}$$

$$\therefore n = 12 \text{ rivets (nearly).}$$

Therefore 12 rivets will suffice.

This determines the length of the cover plates (Plate III.), in which all the joints are shown.

The end of the cover plate must fall half-way between two rows of rivets, and is made square.

Therefore, half the length of the cover plate measuring from the joint to the extreme edge of the plate,

= 2 inches + (three rows of rivets) + 2 inches
= 2 inches + 8 inches + 2 inches
= 12 inches.

Therefore the cover plate is 24 inches long.

The rivets on the top flange are of course carried right through from the angle steels underneath to the cover plate on the top of the flange.

JOINT IN THE WEB PLATE.

A joint will be made 5 feet on either side of the centre of the span, where a stiffener of two angle steels occurs. This joint will be made of two cover plates one on either side of the web, of $\frac{1}{2}$-inch plate.

These cover plates will have to resist vertical shear and transmit it from one side of the joint to the other. The maximum vertical shear at a point 5 feet from the centre is: = 13·8 tons, measured from the shear stress diagram. This is spread over the whole vertical section of the web. It should be noted that the two $\frac{1}{2}$-inch cover plates are together stronger than the web plate; two $\frac{1}{4}$-inch plates would be sufficient in theory, but $\frac{1}{2}$-inch cover plates are used for convenience so as to bring their outer surface flush with that of the angles of the flange.

We have now to find the number of rivets required for the joint.

These rivets, being in double shear, will take $1\frac{3}{4}$ times the stress that they will in single shear, and it is obvious that they will be stronger in shearing than in bearing.

To test for bearing the $\frac{1}{2}$-inch web plate must be considered.

The bearing surface is: $n \times \frac{7}{8} \times \frac{1}{2}$ square inches, and allowing 8 tons per square inch for resistance to bearing

$$n \times \tfrac{7}{8} \text{ inch} \times \tfrac{1}{2} \text{ inch} \times 8 = 13{\cdot}8 \text{ tons.}$$

$$\therefore n = 4 \text{ rivets.}$$

Now there are 12 rivets in the angle stiffeners on each side of the joint (*vide* drawings), and these will be sufficient for the web plate joint without any extra being added.

The stiffeners are fixed outside the $\frac{1}{2}$-inch covers, the latter being in contact with the web plate.

Extra Flange Plate.—Referring to Fig. 200 it will be seen that the extra $\frac{3}{8}$-inch flange plate is required to resist M_B between the points k,k, which are 8 feet 4 inches from the centre of the span. Now this centre plate must end half-way between two rivets, and the rivets are spaced 4 inches apart, starting 2 inches on either side of the centre.

Therefore the position of any rivet is:—

2 inches + $n4$ inches from the centre, and the end of the plate must therefore be:—

2 inches + $n4$ inches + 2 inches on either side of the centre with a minimum of 8 feet 4 inches.

Now 8 feet 4 inches would appear to suit these conditions, but the first rivet in the plate would only be 8 feet 2 inches from the centre, and this would be the effective length of the plate, since it could not be said to be doing any work outside the leading rivet.

This is therefore cutting the length too fine, and it should be made 8 feet 4 inches + 4 inches or 8 feet 8 inches from the centre, or 17 feet 4 inches over all.

Stiffeners.—Stiffeners consist in practice of T steels, or angles singly or in pairs, placed vertically on either side of the web plate at suitable intervals along the girder, and riveted to the web plate and the flanges. Stiffeners are sometimes made of T steels alone, having a very broad table compared to the stem of the T, and sometimes of two angles with a plate riveted in between them. The strength of the stiffener required depends on the depth and span of the girder, and is a matter for judgment.

The sizes of angles or tees adopted are nearly always those which are in use in other parts of the bridge as well.

This avoids having to use too many different sizes.

Stiffeners fit close up against the web of the girder through which they are riveted.

A plain standard stiffener is shown on extreme left of Plate IV., and consists of a T steel, butting on the inner surfaces of the flanges.

The kink in it is formed hydraulically when the T is hot. This is useful for light girders with a narrow flange.

A better stiffener is that shown in Plate III., in which the T or angles are bent twice so as to pass diagonally clear of the flange angles and lie flat underneath the flange plates, to which they are riveted.

In this example two different types have been used, viz., two angles, 3½ inches × 3½ inches × ½ inch, back to back, riveted together without any plate between them, and also two similar angles with a ½-inch plate between them. This plate is carried out so as to have its edge flush with the edges of the top and bottom flanges of the girder.

This latter arrangement is a " plate stiffener," and is of course stronger than the plain stiffener without the plate, giving greater support to the outside edge of the top or compression flange where it might be likely to buckle.

Spacing of Stiffeners.—The best spacing of stiffeners is practically indeterminate; in small girders up to 50-foot span, the spacing is always decided upon without any calculations, being based on practice.

The end of the girder forms one stiffener.

Another, generally a plate stiffener, is always placed above the bearing plate of the girder, standing over the abutment, and forming in conjunction with the end of the girder a sort of braced column or upright end to the girder.

In this example the bearing is 2 feet 6½ inches, and the plate stiffener has been placed 1 foot 6½ inches from the end.

This distance is governed to a certain extent by the pitch of the rivets and by the pitch of the floor troughing used.

The flooring as a rule begins at this point, and the elevation in Plate III. shows one of the two angles of this stiffener inside the girder shortened by ·625 inches and finished on the lower flat of the troughing.

If the bearing is longer (3 feet), and the girder is deep (4 feet), it may be desirable to have two stiffeners over the bearing plate as well as the end plate of the girder, in order to strengthen the vertical action of the girder end.

The spacing of the remaining stiffeners is governed solely by practical experience.

As a general rule:—

(a) Stiffeners are required closer together near the abutments where the shear is greatest than near the centre where the shear is less.

(b) Stiffeners should hardly ever be further apart than the depth of the girder, and above the abutments should not be further apart than half the depth.

(c) The actual spacing depends partly on the spacing of the flooring, whether cross girders or troughing.

(d) Deep girders require more stiffening than shallow.

In this example the stiffeners are 3 feet apart all through, with the exception of the centre portion, where there is one interval of 4 feet.

The bottom ends of the stiffeners are turned over on to the troughing of the floor.

Some of the stiffeners are made plate stiffeners in order to give rigidity to the girder and prevent it snaking or buckling, and to support the top flange which is in compression.

GIRDER END.

Ends of girders are of many kinds and shapes.

Plain girders such as the one under consideration have only square or rounded ends. The latter are peculiar to small spans or the centre girder of a three girder bridge. They look neater than square ends, and are not so inconvenient to platelayers or pedestrians.

There is little difference in the matter of cost, the latter being perhaps a little more expensive.

Square ends are the simplest for handling.

The flange plates are generally made the full length of the girder going from end to end.

The flange angles and the web plate are usually slightly shorter, namely ½ inch in this example; and a ½-inch end plate is fixed to the end of the web plate, fitting in between the top and bottom flange plates, and being secured by angle steels forming a half stiffener. The thickness of this plate is added on to the length of the girder, making the total length 37 feet 1 inch. If the girder were only 37 feet long, the vertical row of rivets nearest to the end would be very cramped for room.

Pitch of Rivets.—The horizontal pitch of rivets has been altered from 4 inches to 3½ inches for the end 1 foot 6 inches of the girder, *i.e.*, between the end and the first stiffener.

This could not be otherwise with a total length of girder of 37 feet 1 inch and 2-foot troughing.

It would not be convenient to have a rivet at the centre of the girder, since the fixing of the troughing necessitates a rivet 2 inches each side of the centre line of the girder. This arrangement of rivets at the centre, carried along outwards, at a 4-inch pitch to the ends of the girder, would bring a rivet exactly at the girder ends, and the next 4 inches from the end. This would of course not do.

We therefore alter the pitch for a distance of 1 foot 6 inches from each end, and place the end four rivets 3½ inches apart instead of 4 inches, thus saving 2 inches in all, and bringing the last rivet into the right place.

BEARINGS.

Two feet six and a-half inches has been allowed for bearing, including the ½-inch end plate. Underneath this portion of the girder a bearing plate is fixed, viz., a plate 1 inch thick, and 16 inches wide by 2 feet 6½ inches long.

It is riveted to the lower flange with countersunk rivets, so that their heads shall not project underneath, but be smooth and flush.

This plate is called a "bearing plate" and rests direct on the bedstone of the abutment. It is free to slide so as to allow for expansion and contraction of the girder.

One end of the girder is, however, usually fixed, with two or four lewis bolts, passing through the lower flange and bearing plate and concreted into the bedstone.

The troughing ends in a hollow or valley, over each abutment, so that its edge is close to the masonry of the abutment, although not touching. It is free to slide, although it keeps ballast from falling through from the track.

DRAINAGE.

In this example the troughs are open, having no ballast upon them, and water will drain away quickly.

This is arranged for by giving the troughs a very slight fall to one end, *i.e.*, bedding one girder a little higher than the other, and providing a small escape hole in the bottom of each trough.

The water dripping through is collected by a common rain-water pipe, underneath the bridge floor, and is carried away to the abutments and down.

DEFLECTION.

The girder must be tested for deflection. When the girder is fully loaded with the load for which it has been designed, then the flanges are fully loaded up to the safe working values of "*f*" or resistance to tension or compression, and the formula may be applied :—

$$\Delta = \frac{n^1 f l^2}{E y}$$

Here $n^1 = \frac{5}{48}$ for a supported beam with a uniform load.

$E = 13{,}400$ tons.

$y = \frac{d}{2} = 18$ inches.

$f = 7$ tons.

$l =$ length in inches $= 414$ inches.

$$\therefore \Delta = \frac{5}{48} \times \frac{7}{13400} \times \frac{(414)^2}{18}$$

$$= \cdot 518 \text{ inches.}$$

This is $\frac{1}{800}$ of the span, which is well within the limit of permissible deflection, viz., $\frac{1}{480}$ of the span.

CAMBER.

An initial camber or upward deflection of 1 inch should be specified. Thus when the girder is fully loaded it will still have a slight upward curve. This camber is merely specified, and need not be shown in drawings. It is given to the girder during manufacture in the shops, by cutting the web plate with the camber in it, and pulling the other members up to it in the plating up. The camber is composed of one and a-half times the deflection plus a fraction to spare, or ½ inch per 20 feet of span, the result being about the same in either case.

WEIGHT.

The actual weight of the girder should now be taken out carefully as a check on the estimated value, and the calculations depending thereon.

This amounts to 4·41 tons, which is slightly larger than the assumed weights. A sufficient margin of safety has been left, however, for this in the design, and there is no need to recalculate.

CHAPTER XIV

BRIDGE FLOORS—RAILWAYS

BRIDGE FLOORS—GENERAL.

In this chapter on bridge floors it is not proposed to enter into the comparison of different kinds of floors with their relative advantages, but rather to explain them and to exemplify the calculations for each, giving details of construction, etc.

Their respective merits are included in the general discussion of bridge design (Chap. XVI.) since the general nature and type of floor to be adopted in any specific case are interdependent with the description of girder selected, and the general conditions of the case.

Bridge floors are here divided into two general classes—

(a) Railway bridge floors.

(b) Road bridge floors.

In the calculation of floors, the loads used are not quite the same as those used for main girders.

The heaviest axle or wheel load of a vehicle is generally the ruling load.

For example, referring to Fig. 198, any member of the floor of a bridge, such as an I steel joist, placed transversely to the bridge from one girder to the other, will be most heavily loaded when the heaviest axle, viz., No. 3 or 4 of 18 tons weight, is immediately above it. The wheels on either side of the heaviest axle may have some influence on the M_B, but those beyond will have none.

The spacing of the members of the floor is of importance in this consideration.

Dead Loads.—These are the same as described in the chapter on loads, and are best exemplified by such examples as follow.

Live Loads.—The question of impact and vibration is important in floors; it is greater than in the case of main girders, and it has been dealt with in Chap. XII.

To minimise the impact due to the deflection of rails in the case of railway floors, the sleepers are laid closer together than on the ordinary track; this gives a smaller span to the rails and distributes the load better on the floor below.

Steel Troughing.—The use of steel troughing for floors is nearly universal, and is exceedingly convenient.

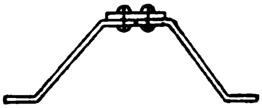


FIG. 202.

Troughing can be used either transversely to the bridge, resting on the lower flange, or on top of the upper flange of the girders; or it may be used longitudinally, parallel to the girders, resting on cross girders.

It is light and readily fixed with a minimum of riveting. It can be used for railway bridges with or without ballast, and also for road bridges.

It is made at all steel works and is standardised by the British Standards Committee.

Troughs are made in various sizes, given in B.S. tables or in makers' catalogues, where their properties and section moduli are given.

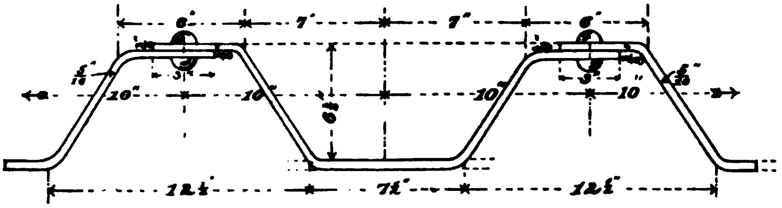


FIG. 203.

Figs. 202 onwards show various kinds of troughs and how they are joined together when placed in a floor. The sections shown in Figs. 202 to 205 are pressed into shape when hot.

The form of trough shown in Figs. 202 and 203 is to be preferred to that in Figs. 204 and 205.

These latter have a tendency to close up, *i.e.*, the upper on to the lower, bringing great shearing stresses on to the rivets at the junction of the upper and lower portions. In some cases the rivets have sheared, causing failure, but this should not be so if enough rivets are given.

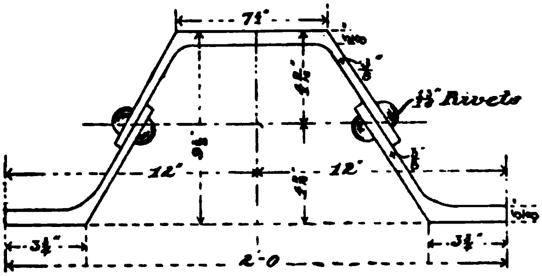


FIG. 204.

All these trough sections can be obtained from makers up to 40 feet in length even in the heaviest sections.

With proper tackle these heavy troughs are quite as easy to handle as the lighter sections; they must, however, be provided with tie-plates.

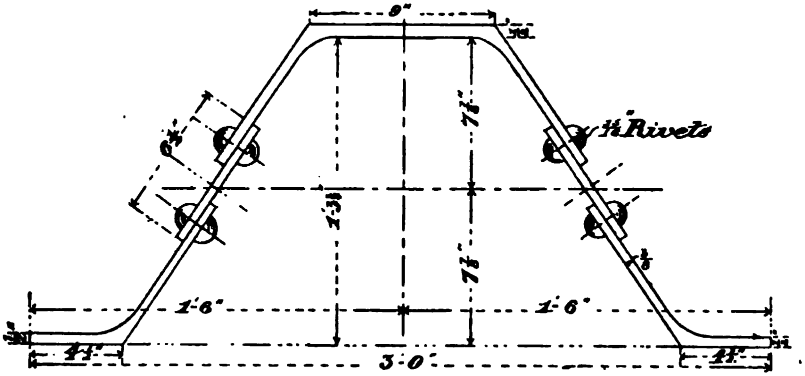


FIG. 205.

These prevent spreading of the troughs during handling and erection, which must otherwise occur, making riveting up well nigh impossible.

Those troughs shown in Figs. 206 and 207 are built up of plates and angles riveted together, and would rarely be used, certainly never if stamped troughing were available. Troughing can be strengthened by riveting an extra plate on to the top or on to the bottom flange, or on to both. This gives a stronger trough without much increasing the depth.

The pitch of troughing, or distance from the centre of the table of one trough to the centre of the table of the next trough when laid, is always arranged to be some multiple of 4 inches, *i.e.* the most usual pitch of rivets.

Troughing can be had with a pitch of 1 foot, 1 foot 4 inches, 1 foot 8 inches, 2 feet, 2 feet 8 inches, 3 feet, the last being a composite trough and uncommon.

Troughing is usually despatched from the mills riveted together in sections of three troughs. These sections are themselves riveted together when in position in the bridge.

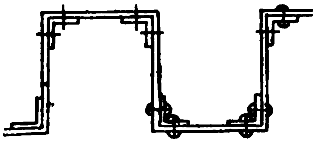


FIG. 206.

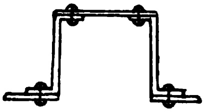


FIG. 207.

They generally rest on the angle of the lower flange of the girders and fit underneath an angle steel which is fixed to the inside of the web of the girder at the right height. This angle, and also the stiffeners on the inside of the girder, which are intended to be riveted to the top of the troughs, is kept up about $\frac{3}{8}$ inch above the exact height of the top of the troughing so as to allow room for fitting the troughing into place after the girders have been put into position, and this space is filled up with a packing piece of steel plate when riveting up (Plate III.).

Distribution of Load.—It may be assumed that the loads coming on to the trough floors are very much distributed by the rails, sleepers, and ballast, so much so that only one half the actual load comes on to the trough immediately beneath it, the remainder being borne by adjacent troughs.

This has been proved by experiment, and applies both to troughing placed transversely to the bridge and also to troughing placed longitudinally with the bridge. This distribution is the same for both railway and road bridges, with very few exceptions.

Such exceptions are rare and are always obvious.

Each of the adjacent troughs takes a quarter of the load.

Some engineers allow an even greater reduction of load in the case of transverse troughs under longitudinal sleepers, the load on the trough being taken as one-third the concentrated superload.

RAILWAY BRIDGE FLOORS.

These may be tabulated thus:

(a) Transverse trough floors.
(b) Cross girders and longitudinal trough floors.
(c) Cross girders and rail bearers with plate floor.
(d) Cross girders and rail bearers with buckle plate floor.
(e) Cross girders and timber floor.
(f) Cross girders and buckle plates.
(g) Cross girders and reinforced concrete.

(a) *Tranverse Trough Floors.*—This type of floor is in common use for spans of 20 to 50 feet, where three main girders are used, or for single-line bridges.

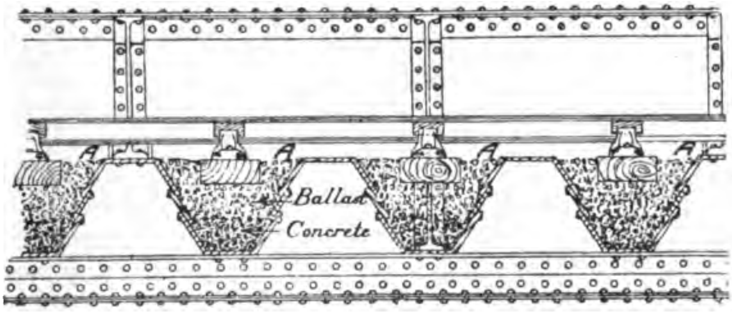


FIG. 208.

The troughs have thus only a short distance to span, *i.e.*, that under a single line, say, 13 feet 6 inches to 15 feet, and can be made of light section; they are simple to erect, cheap, occupy little headroom, and distribute the load well on to the main girders.

They rest on the bottom or the top flanges of the main girder as required, or on an angle steel fixed some way up the girder web.

They are very convenient for use with skew girder bridges.

They can be covered with ballast or not at discretion.

If ballasted, they must be covered with at least 3 inches of ballast at their highest part. Sleepers may then be transverse or longitudinal, the former being the general custom in order to preserve uniformity of track, though the latter gives better distribution of loads on to the troughs.

If headroom is scarce, the transverse sleepers can be sunk in the troughs, having at least 3 inches of ballast underneath them (Figs. 208 and 209).

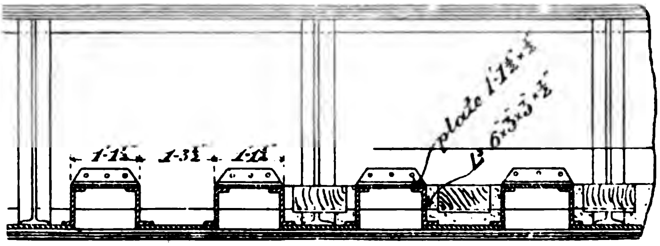


FIG. 209.

By this means 8 or 9 inches of headroom can be saved.

This dropping of sleepers is undesirable, as the distribution of the load is bad, and the extraction of sleepers for repair by platelayers is very inconvenient, involving lifting the rails.

If the floor be not ballasted, longitudinal sleepers must be used. This arrangement admits of easy inspection of the floor and easy painting and repair. Also rainwater runs away quickly, and the weight of ballast is saved, dead load being very greatly reduced thereby.

Four to four and a-half tons per square inch is the safe value of "f" for these floors.

EXAMPLE.

A Transverse Trough Floor.—To design a floor for a single line, 4 feet 8½ inches gauge carried between two plate girders, viz., those shown in the example of plate girder design (Chap. XIII.).

The flooring is to be of troughing placed transversely to the bridge, carrying longitudinal sleepers under the rails, and no ballast (Plate III.).

There is plenty of headroom available, so that there is no need for a very shallow trough.

Width.—The width of the floor is chiefly determined by Board of Trade Regulations, and, of course, by considerations of economy. It was decided on p. 182 to make this floor 12 feet 4 inches wide.

The clearance gauge of railways laid down by the Board of Trade is given on p. 264, and shows that above the height of 2 feet above rail level, there must be at least 6 feet 7 inches from the centre of the track to the nearest standing work, which in this case would be the edge of the upper flange of the girders; while below this height of 2 feet only 4 feet 7 inches is required.

In this case the troughs are to be 9⅜ inches deep, and the longitudinal sleepers 8 inches, and these with the rails and chairs will together make up about 1 foot 11 inches, so that the top of the girder will only be about (3 feet − 1 foot 11 inches) or 1 foot 1 inch above rail level.

Nine feet two inches is therefore the greatest width that need be given.

At the same time, for general reasons given in the design of the girders, p. 182, the width is not curtailed so much, and 12 feet 4 inches is the width allowed, and is the length of troughing to be used.

The load for calculation is shown in Fig. 198, where the heaviest axle load is 18 tons.

The live load will be compensated for by using 4½ tons as the value for "*f*."

Consider the two wheels of the 18 tons axle to be resting on the rails immediately above one trough (Fig. 199).

Then the *Max.* M_B on the trough occurs under each wheel, and *Max.* M_B = Reaction × distance from one rail to end of trough.

= Reaction × (3 feet 9¾ inches)
= 9 × 45¾ inch-tons
= 411·75 inch-tons.

Now, only half the load is carried by any one trough, the remainder being borne by adjacent troughs.

Hence the M_B to be met is:

$$= 411{\cdot}75 \div 2$$
$$= 205{\cdot}88 \text{ inch-tons.}$$
$$\therefore M_R = 205{\cdot}88.$$
$$\therefore f\frac{I}{y} = 205{\cdot}88.$$

Hence if f be taken as $4\frac{1}{2}$ tons per square inch, then the $\frac{I}{y}$, or section modulus, for the trough required will be

$$\frac{I}{y} = 205{\cdot}88 \div 4\tfrac{1}{2}$$
$$= 45{\cdot}75.$$

On referring to tables, a suitable section can now be selected, viz.: a trough having

2 feet pitch,
$9\frac{5}{8}$ inches depth,
·625 inch flange thickness,
35·02 lbs. per foot-run weight,

and having $\frac{I}{y} = 51{\cdot}45$ (Fig. 204).

This leaves a small margin of safety.

There is no other section of convenient pitch having its section modulus equal to 45·75.

It should be noted that it is very important to make the pitch suit both the span of the girders and the spacing of the stiffeners.

In the above calculations the weight of the trough itself has been taken as negligible and all other dead load as well.

If the floor were a ballasted floor it would be necessary to include the dead load.

It would then be easiest to find the *Max.* M_B by graphic methods.

Rivet holes: Deductions.—Rivet holes are necessary at the ends of the trough to secure it to the girders, but there is no need to make any deduction from M_R for these, as the M_B near the ends of the trough is very small.

The rivets also which are used for joining adjacent troughs to one another are all located along the neutral axis exactly halfway between the top and bottom flanges of the troughs, and cause no deductions. The longitudinal sleepers will be secured to the top flange of the troughs by angle brackets, forming a chair, and these will have to be riveted to the troughs, causing two holes in the top flange of the trough close to the point of *Max.* M_B.

Now, the metal section represented by these rivet holes has a M_R of 19 inch-tons. The total M_R of the trough selected is 231·52 inch-tons. Therefore deducting that lost through rivet holes, the net M_R is 212·52 inch-tons, which is still greater than the *Max.* M_B.

Transverse Troughing in Three-Girder Bridge ; Double Line.—This is another case in which transverse troughing is generally used.

For a double line of railway, for spans up to about 50 feet, three girders are generally employed, and the centre one is kept so low that it does not project more than 2 feet above the level of the rails. This permits the inner rails of the two tracks to be closed in until there is only 6 feet between them ; *vide* Board of Trade Regulations.

This makes a great saving of weight in the bridge floor, and owing to the presence of the extra girder enables very light troughing of comparatively short span to be used, placed transversely to the bridge between the girders.

The calculations for the troughs are exactly the same as those in the previous example, for a single line, with this difference only, that the loads or rails are not centrally placed on the troughs, being nearer to the centre girder than the outside ones. In this case the M_B under the outer rail, *i.e.*, that furthest from support, will be the greater of the two, and if it is desired to combine this with the dead weight of ballast, etc., the graphic method of doing so is the best.

EXAMPLE.

Let Fig. 210 represent a section through a double line three-girder bridge, on which the floor is to be of transverse troughing, with ballast 3 inches deep over the top of the troughing, and with the ordinary track, laid with sleepers 2 feet apart, centre to centre.

The span of the main girders is 50 feet, and the heaviest axle load to be carried is 20 tons.

There is plenty of headroom.

Now it will be possible to keep the top of the girders so low that they do not project up above the 2-feet limit above the rail level, and in this case the inner lines of rails of the two tracks may be 6 feet apart in the clear.

Fig. 210 shows a cross-section of the bridge, giving the spacing of the girders and span of the transverse troughs which compose the floor.

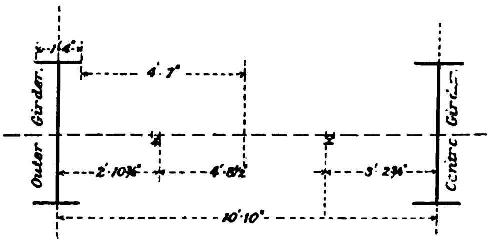


FIG. 210.

The minimum distance from the centre of the track to the nearest standing work, *i.e.*, the edge of the flange of the outer girders, is 4 feet 7 inches, and the flange of the girder is assumed to be 16 inches wide.

The troughs will rest upon the lower flange of the girders, and will butt up against the web.

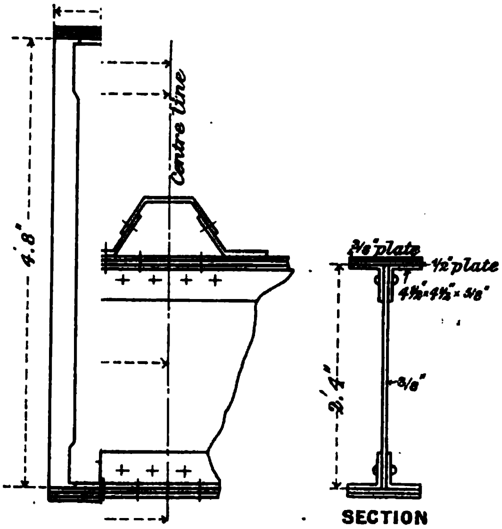
They may therefore be assumed to project 8 inches beyond the edges of the flange of the girder, so that the outer end of a trough will be 4 feet 7 inches + 8 inches, or 5 feet 3 inches from the centre of the track.

The dimensions are in accordance with the Board of Trade Regulations.

The total length of trough from centre of outer girder to centre of centre girder is therefore 10 feet 10 inches (Fig. 210).

Now the M_B at each rail must be found.

If the dead load were negligible, and the live load were the only consideration, then the *Max.* M_B occurring at the inner rail would be all that was required. But the floor is to be


4'8"
Centre line
3/8" plate
1/2" plate
4½"×4½"×3/8"
3/8"
2'4"
SECTION

ballasted, and this will mean a dead load that is not negligible.

Hence the live and dead loads must both be considered, and the easiest way to do this is by graphics.

Dealing with the live load first:—

The M_B at the outer rail is the reaction at the outer girder multiplied by 2 feet 10¾ inches.

$$= R \times 2 \text{ feet } 10\tfrac{3}{4} \text{ inches}$$

$$= 10 \text{ tons} \left(\frac{7'\,11\tfrac{1}{4}'' + 3'\,2\tfrac{3}{4}''}{10'\,10''}\right) \times 2'\,10\tfrac{3}{4}'' \times \tfrac{1}{2} \times 12 \text{ inch-tons}$$

$$= 178{\cdot}1 \text{ inch-tons.}$$

The factor ½ is introduced because each trough only takes ½ of the axle load above it.

Similarly the M_B at the inner rail;

$$= 10 \times \left(\frac{2'\,10\tfrac{3}{4}'' + 7'\,7\tfrac{1}{4}''}{10'\,10''}\right) \times 3'\,2\tfrac{3}{4}'' \times \tfrac{1}{2} \times 12 \text{ inch-tons}$$

$$= 187{\cdot}8 \text{ inch-tons.}$$

The base line 10 feet 10 inches must be drawn out to scale, and these two amounts of M_B must be plotted as ordinates to some suitable scale, as in Fig. 211.

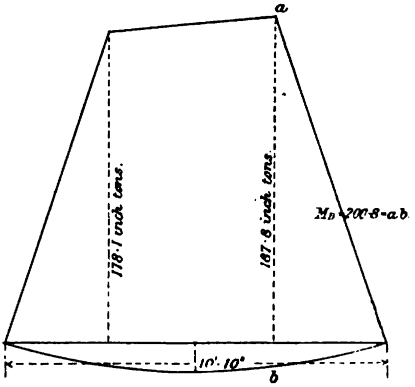


FIG. 211.

Now the dead load must be considered.

The troughing to be used will have 2 feet pitch, and probably will be 9 inches deep, weighing 35 lbs. per foot run.

If the ballast be filled up to 3 inches above the top of the trough, the amount of ballast on each trough at 110 lbs. per cubic foot = ·665 tons.

Similarly the weight of each trough

= 35 lbs. × 10 feet 10 inches

= ·171 tons.

Also the weight of rails, fish plates, sleepers, and chairs, etc., may be taken at 100 lbs. per foot run of track, or 200 lbs. per trough.

$= \cdot 09$ tons per trough.

(NOTE.—This may be taken as distributed load as it is spread by sleepers and ballast.)

This gives a total dead load of ·926 tons, which is distributed over each trough.

The *Max.* M_B due to this

$$= \cdot 926 \times \frac{10 \text{ feet } 10 \text{ inches}}{8} \times 12 \text{ inch-tons}$$

$$= 14 \cdot 95 \text{ inch-tons.}$$

Plot this below the base line of Fig. 211.

Then the maximum can be measured, and is 200·8 inch-tons at a point 3 feet 2¾ inches from the centre girder, *i.e.*, under the inner rail.

Hence the M_R must be made

$$= 200 \cdot 8 \text{ inch-tons} = f\frac{I}{y},$$

and allowing "f" to be 4½ tons per square inch for this floor, then the section modulus $\frac{I}{y}$ must be 44·6 or greater.

Referring to tables, the most suitable trough is that shown in Fig. 204, and the sizes and weights assumed for calculating dead load are correct.

The troughs should have 1 inch cut off at each end so as to allow them to fit in between the web plates of the girders, and the total length of each trough will therefore be 10 feet 8 inches.

Height of Floor.—The height of the rail level above the underneath edge of the web of the girder is 2 feet 2 inches.

This is made up thus:—

	Ft.	Ins.
Depth of trough	0	9⅜
Depth of ballast over trough.	0	3
Thickness of sleeper	0	7
Depth of rail and chair.	0	6
Thickness of angle steel of lower flange of girder	0	0⅝
Total	2	2

If the girder be 3 feet 8 inches deep, the top flange of the girder will then be 1 foot 6 inches above the top of rail level which is less than the 2 feet limit allowed.

Fixing the Floor.—An angle steel should be fixed on the inside of the web of the main girders at a convenient height to allow for riveting it to the top of the troughs after a ⅜-inch packing piece has been inserted. The exact size of this angle does not matter, and it would be made to agree with the general sizes adopted in the design of the main girder (Figs. 212 and 213).

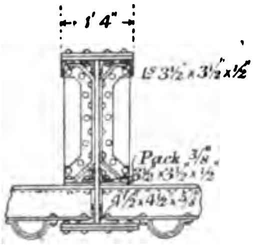


FIG. 212.

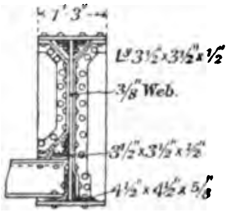


FIG. 213.

CROSS GIRDERS.

Practically all other types of bridge floor are supported on cross girders.

These are I section steel joists or light girders placed transversely to the bridge, and upon these rests the floor, whatever it may be.

Troughing is sometimes used placed across the cross girders, *i.e.*, longitudinally, or lengthwise with the bridge, with ballast on top of it.

Rail bearers may be used underneath the position of the rails spanning from one cross girder to another.

These rail bearers, like the cross girders, consist of I beams, or light built-up girders, and they serve to carry the weight of the track.

When they are used, a floor covering or decking is required as well, consisting of flat plates or buckled plates, for carrying the ballast, people, platelayers, etc., walking across the bridge.

These types of floor are equally suitable for two-girder and three-girder bridges.

Spacing of Cross Girders.—The rolling, or accidental load is the principal load on the cross girders.

It is obvious that nothing is gained by very close spacing of these girders, as in any case each cross girder must bear the whole weight of the heaviest pair of driving wheels when they are above it.

Therefore it is economy to space cross girders so that the weight on them due to the intermediate railbearers cannot exceed the greatest axle load of engine.

This will generally happen when the spacing is a little more than the distance apart of the two heaviest pairs of wheels.

Other governing factors are the symmetry of the spacing over the span; and the desirability of having no cross girder actually at the centre of the span, in order to lessen the $Max.\ M_B$ on the main girders.

Where there is a skew this latter may form an important factor.

In braced girders having panels, cross girders are nearly always symmetrical with panel points. Both main girders should be kept alike.

Supports of the Ends of Cross Girders.—The ends of cross girders are usually similar to those of a plate girder. If plain rolled section I steel joists are used, they are merely cut off square at the ends.

In through bridges these ends usually rest on the lower flange of the main girders, butting close up against the web; or they may be supported at some height up the web of a plate girder by an angle bracket in the same way as troughing.

This angle must, of course, be strong, and the web must have a stiffener at this point, as otherwise it would tend to buckle (Fig. 254).

In under bridges the cross girders rest upon the top of the main girders, and are fixed in position by rivets through their lower flanges.

The weight of the ends of cross girders is taken partly by the flange or bracket, upon which they rest, and partly by vertical angle brackets.

These, consisting of short pieces of angle steel, are riveted to both sides of the web of the cross girder and also to the web of the main girder.

Fig. 254 shows these angle brackets.

When the cross girder is not a rolled steel joist, but is built up of angles and plates, its end is finished in a similar manner to a plate girder, and these angles form its end.

Where the surfaces upon which a cross girder rests, or against which it butts, are uneven, they should be made flush or even by introducing packing pieces; Plate III., where packing pieces are placed underneath the trough ends to make the bearing flat; also Plate IV., where the same is shown for a cross girder.

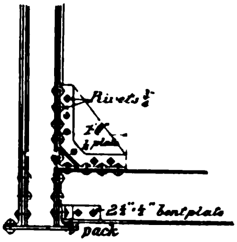


FIG. 214.

With braced girders the connections are not so easy.

Sometimes the cross girder can rest on the lower flange, but as a rule such bearing is insufficient.

Fig. 214 shows such a joint; a bent plate or angle bracket is riveted to the web of the cross girder so as to keep it in position, a packing piece is inserted underneath it to give a flat bearing, and a triangular bracket is fixed above it, between the cross girder and the vertical member of the main girder, to give general stiffness.

Sometimes the flange does not give sufficient bearing surface, or would be subjected to torsion if loaded, in which case the cross girder might be fastened to the vertical member of a braced girder, or even slung underneath the flange.

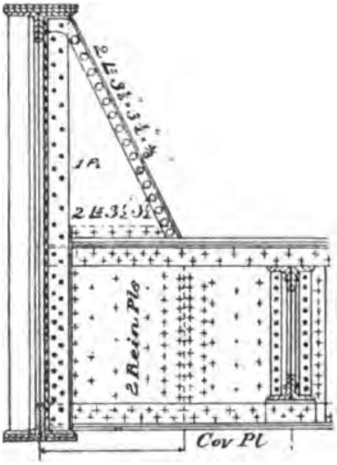


FIG. 215.

Fig. 215 shows the web plate of the cross girder carried in to butt up against the web of the main girder, between the two angles of stiffener of the main girder.

These two stiffener angles transfer the weight of the cross girder to the main girder, forming at the same time the end of the cross girder.

The top and bottom plates and angles of the flange of the cross girder are of course stopped short so as to allow the angles of the stiffener to lie flat against the web of the cross girder.

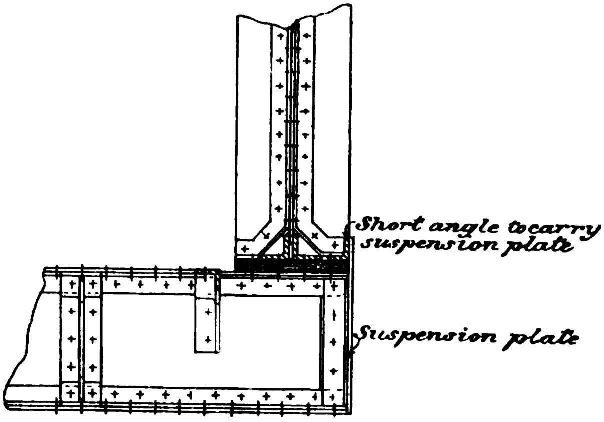


FIG. 216.

This is a very strong form of attachment.

Figs. 216 and 217 show a cross girder suspended underneath the lower flange of a plate girder by a suspension plate.

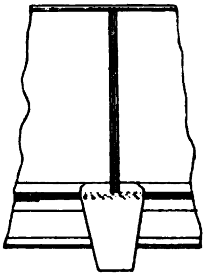


FIG. 217.

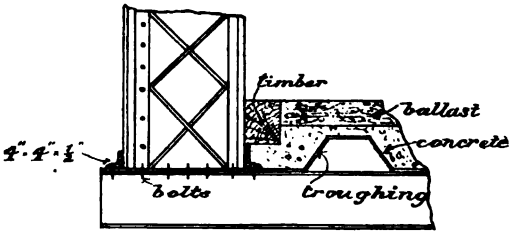


FIG. 218.

This suspension plate is on the outer edge of the main girder and is riveted to the end angles of the cross girder.

Fig. 218 shows the use of bolts and nuts.

These are permissible (although not very desirable) when it is impossible to arrange the joint without having rivets in tension. They must, of course, be properly set up, the ends of the bolts being turned up over the nuts so that they can never move.

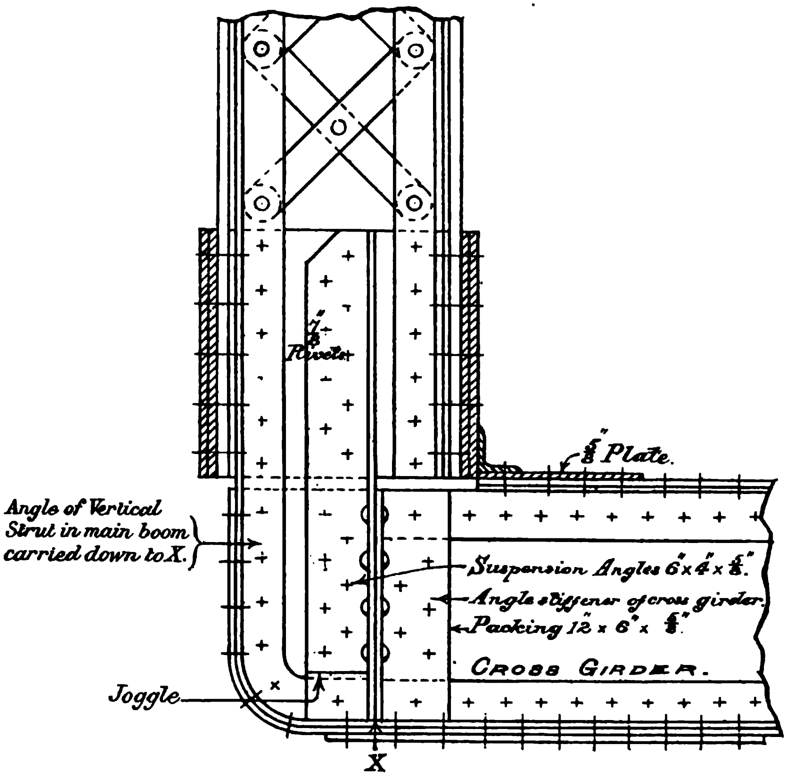

7/8" Rivets
5/8" Plate.
Angle of Vertical Strut in main boom carried down to X.
Suspension Angles 6"x4"x5/8".
Angle stiffener of cross girder.
Packing 12"x 6"x 5/8".
CROSS GIRDER.
Joggle
X

FIG. 219.

Figs. 219 and 220 show a cross girder hung underneath the lower flange of a large braced girder by four suspension angles. These transmit the weight from the cross girder end to the main girder by thirteen rivets in double shear forming a very strong joint. It should be noted that the flange plates in this example are vertical instead of horizontal, so that no other means of supporting the cross girders except by suspension plates or angles can be used.

Fig. 221 shows a cross girder with its end slotted so as to allow it to fit over the projecting flange of a channel, such as might form part of the lower flange of a braced girder.

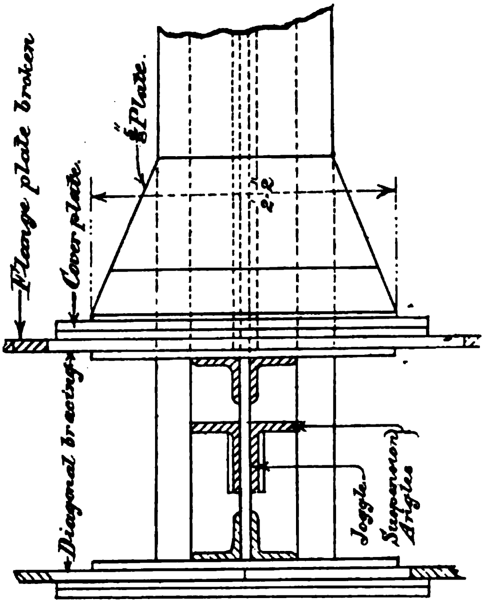


FIG. 220.

RAIL BEARERS.

These consist generally of I section rolled steel joists.

They need rarely be of any stronger section than is to be had of standard size from any makers.

They are, however, sometimes built up of angles and plates. When this is the case they rarely have a top flange plate, the angles at the top and bottom being sufficient to take the flange stresses.

They sometimes rest on top of the cross girders, but are usually fixed between the cross girders so that their top flanges are flush with the top of the cross girders.

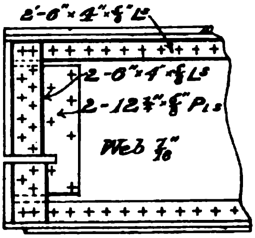


FIG. 221.

They are rarely as deep as the cross girders in section, and hence rest either on the lower flange of the cross girders, with a packing piece of suitable thickness to raise them up flush on top, or else on an angle bracket fastened to the web of the cross girder.

Fig. 222 shows various methods of attachment.

In order to allow the top flange of the rail bearer to be brought up flush, a rectangular notch or joggle is made in the rail bearer, either by cutting or by stamping when hot (Fig. 222).

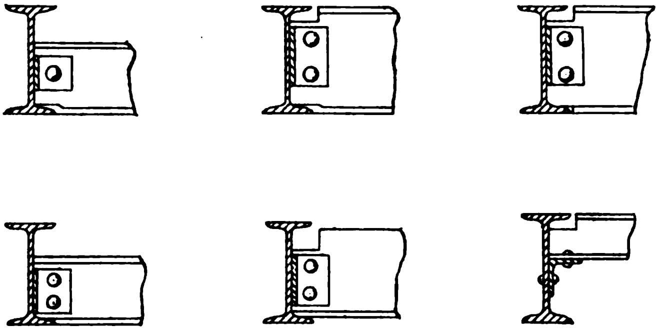


FIG. 222.

Angle brackets are fixed to the sides of the web of the rail-bearer and also to the web of the cross girder and help to take the weight.

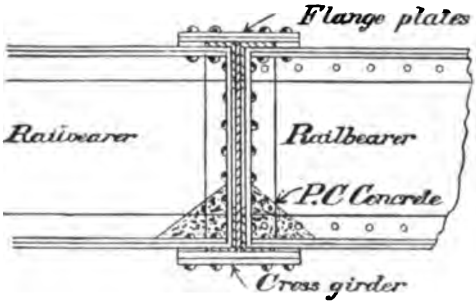


FIG. 223.

Fig. 223 shows the junction of two rail bearers with a cross girder.

Fig. 224 shows another join; the top flange angles of the rail bearers are bent downwards and are riveted to the web of the

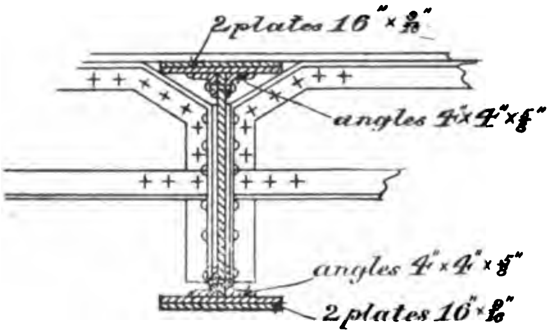


FIG. 224.

cross girder; they do not stop at the lower flange of the rail bearers but are carried on right down to the lower flange of the cross girder. This forms a good and stiff connection.

Example.

To design a floor for a plate girder bridge, with two main girders, for double line, 4 feet 8½ inches gauge, to carry English rolling stock.

Cross girders will be used with longitudinal troughing placed on top of the cross girders. (Refer to Plate IV.)

There are no restrictions as to headroom; the girders are 50 feet long, and they may be assumed to be 4 feet 8 inches deep and 16 inches across the flanges.

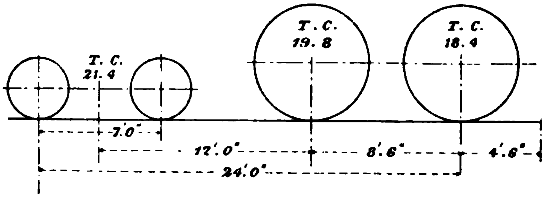


Fig. 225.

The cross girders will rest on the lower flanges of the main girders.

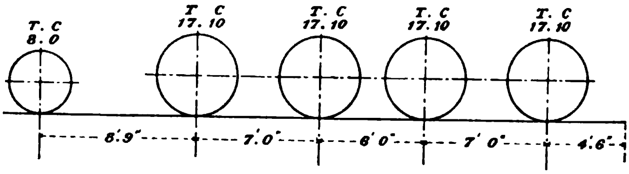


Fig. 226.

The ruling engines for the floor stresses are given in Figs. 225 and 226.

The first thing to consider is the troughing and its span, which depends on the spacing of the cross girders.

Spacing of Cross Girders.—The uniform equivalent load for a span of 50 feet clear or 53 feet between centres of bearings is 2·54 tons per foot run (Table XVI.).

Let l be the distance between two cross girders. Then the total uniform distributed load on one cross girder for one track

$$= l \times 2{\cdot}54.$$

This should be about equal to the heaviest axle load, viz., 19 tons 8 cwts., from which l is found to be 7·64 feet.

Make this spacing 8 feet, which will give 7 cross girders to the bridge, the end ones being 1 foot from the ends of the girders.

Troughing.—Hence the span of the troughing is 8 feet.

The heaviest axle is 19·4 tons or the heaviest wheel 9·7 tons.

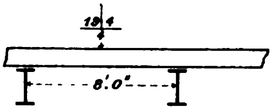


FIG. 227.

This will produce the greatest M_B in the troughing when at its centre (Fig. 227); also one trough only takes half the weight immediately above it. Hence:—

M_B on one trough

$$\begin{aligned} &= \frac{Wl}{4} \\ &= \frac{9{\cdot}7}{2} \times \frac{8 \times 12}{4} \\ &= 116{\cdot}4 \text{ inch-tons} \\ &= f\frac{I}{y} \end{aligned}$$

If we allow "f" to be $4\frac{1}{2}$ tons per square inch, then:—

$$\begin{aligned} \frac{I}{y} &= \frac{116{\cdot}4}{4\frac{1}{2}} \\ &= 25{\cdot}85. \end{aligned}$$

This is the modulus of the section required, calculated approximately.

A suitable trough section for this would be one of 1 foot 8 inches pitch, $7\frac{3}{8}$ inches deep, 6 inches table, weighing 32·97 lbs. per foot run, and having a section modulus = 30·60.

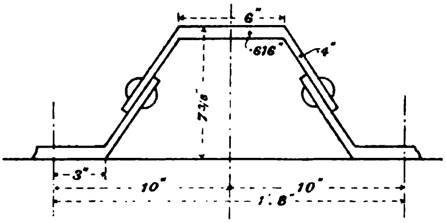


FIG. 228.

Now this must be recalculated including dead load.

The dead load is composed as under :—

Weight of trough, 8 feet long, at 32·97 lbs. per foot run = 264 lbs.

Weight of ballast on one trough up to 5 inches above the level of the top of the trough, at 110 lbs. per cubic foot = 1,045 lbs.

The weight of rails and track is 100 lbs. per foot run of track, or one quarter of this on each trough = 200 lbs.

Total, 1,509 lbs.

This has a central M_B on the trough of 8·085 inch-tons. This must now be added to the central *Max.* M_B found due to the accidental load, viz. :—

$$\begin{aligned} \therefore \textit{Max. } M_B &= 116{\cdot}4 + 8{\cdot}08. \\ &= 124{\cdot}5 \text{ inch-tons} \\ &= \ 27{\cdot}7 = \text{modulus of section.} \end{aligned}$$

The trough selected will, therefore, be strong enough and is shown in Fig. 228.

Cross Girders.– The cross girders are the next consideration.

Their span is the width of the bridge.

In this case the girders will not project more than 2 feet above the rail level, so that the reduced width may be given to the bridge, *i.e.*, 4 feet 7 inches from the inner edge of the girder flanges to the centre of the track.

Since there is no centre girder, the inner rails may be 6 feet apart in the clear. This fixes the total clear width between the two main girders as 20 feet 4 inches (Fig. 253).

But the pitch of the troughing will influence this slightly.

The centres of the two tracks instead of being only 11 feet 2 inches apart will be better spread out to 11 feet 8 inches, so that both tracks may rest symmetrically above troughs.

This will increase the whole width of the bridge to 20 feet 10 inches.

If then the width be made 21 feet 8 inches in the clear, this will be an even multiple of 1 foot 8 inches, and will be the best width to adopt.

Therefore the length of the cross girders will be 21 feet 8 inches + half the thickness of the main girder at each end, *i.e.*, 23 feet.

Twenty-two feet six inches may then be taken as the span for calculations.

Dead load.—The dead load on the cross girders is that due to the troughs, viz., 1,509 lbs. per trough, and there are 13 of them on one cross girder.

Therefore, total dead load = 13 × 1,509 lbs.

The weight of the cross girder itself is not yet known and may be omitted for the present.

The *Max.* M_B due to the dead load

$$= \frac{Wl}{8}$$

$$= 13 \times \frac{1{,}509}{2{,}240} \times \frac{45}{2} \times \frac{12}{8} \text{ inch-tons}$$

$$= 295{\cdot}55 \text{ inch-tons.}$$

The diagram for this loading is plotted in Fig. 230 by drawing a parabola with height of 295·55 inch-tons.

Live Load.—The live load to be selected for consideration on the cross girder requires to be chosen carefully. There are three alternatives :—

(1) The heaviest axle is shown in Fig. 225, viz., 19·4 tons. This may be immediately over the cross girder, in which case, as it is spaced 8 feet 6 inches from the next nearest axle, it will be the only load on the cross girder.

(2) The axles shown in Fig. 226 are not so heavy but are closer together. Consider the two axles weighing 17½ tons each, spaced 6 feet apart. When these stand on the troughing, 3 feet each side of the cross girder, as in Fig. 229, the total load on the cross girder will be :—

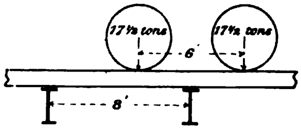


FIG. 229.

$$= \frac{5}{8} \times 17\tfrac{1}{2} + \frac{5}{8} \times 17\tfrac{1}{2} \text{ tons}$$

$$= 22 \text{ tons (approx.).}$$

(3) The equivalent uniform distributed load for a 50-foot span is 2·54 tons per foot run, making a total load of 2·54 × 8, or 20·32 tons per cross girder. Of these three loads that in (2) is the greatest and must be considered.

It will be noticed that in this case the loads are not actually on the cross girder, but are brought on to it by the rail bearers on either side of it.

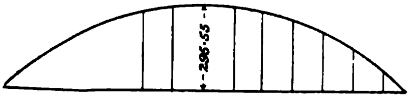


FIG. 230.

Distribution of Live Load on Cross Girder.—Referring to Fig. 234, it will be seen that the load of 22 tons brought on to the cross girder by each track has been divided up into 11 tons on each rail, and this again in turn into one half, or 5·5 tons under each trough under each rail, and one quarter or 2·75 tons under each adjacent trough. The M_B due to these loads must now be found graphically.

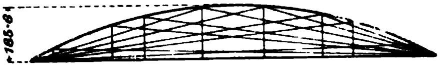


FIG. 231.

To do this the *Max.* M_B for each load on a span of 22 feet 6 inches must be drawn.

The *Max.* M_B of a load of 5·5 tons at the centre of a span of 22 feet 6 inches is:—

$$= \frac{Wl}{4}$$

$$= \frac{5\cdot5}{4} \times 22\tfrac{1}{2} \times 12$$

$$= 371\cdot24 \text{ inch-tons.}$$

Similarly the *Max.* M_B of a load of 2·75 tons is = 185·62 inch-tons.

The diagrams of M_B for these are parabolæ, and are drawn in Figs. 231 and 232 with central vertical ordinates of 185·6 and 371·2 inch-tons respectively. These two diagrams give the M_B that these two loads will produce wherever they are placed on the span.

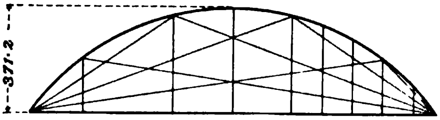


FIG. 232.

The various positions or points of application of these loads are projected up from Fig. 234, and their triangles of M_B are completed as shown.

Then Fig. 233 is plotted by adding together all the ordinates from Figs. 230, 231 and 232 at each point.

This gives a maximum M_B of 1,775 inch-tons at the centre of the cross girder.

Therefore the central M_R of the cross girder must be

$= 1{,}775$ inch-tons

$= fad.$

Suppose the cross girder to be 2 feet 4 inches in depth.

Then $d = 28$ inches

$\therefore fa = 63{\cdot}43$

$\therefore a = 14{\cdot}1$ square inches.

Suppose that two angle steels are used in the top flange of the cross girder, viz.:—

4½ inches × 4½ inches × ⅝ inch.

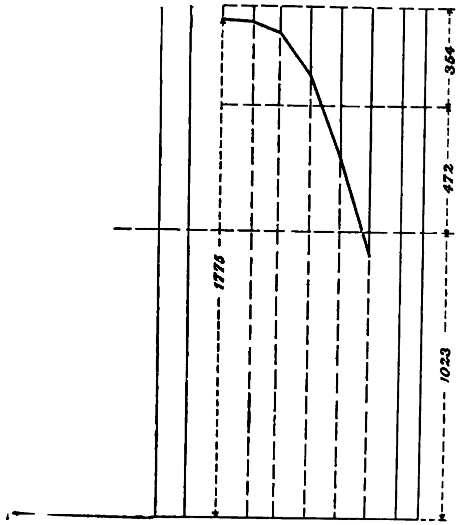


FIG. 233.

Deducting for two 1-inch rivet holes, the total cross-section of the two angles

$$= 2\,(8\tfrac{1}{2}\text{ inches} - 2\text{ inches}) \times \frac{5}{8}$$

$$= 8{\cdot}1 \text{ square inches}$$

and the total $M_R = fad = 1023{\cdot}75$ inch-tons. A flange plate ½ inch thick and 9½ inches wide will be suitable.

Deducting two 1-inch rivet holes, this plate would have a cross-section

$$= (9\tfrac{1}{2}\text{ inches} - 2\text{ inches}) \times \tfrac{1}{2}\text{ inch}$$

$$= 3{\cdot}75 \text{ square inches,}$$

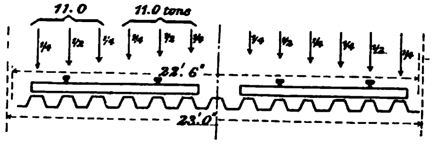


FIG. 234.

and a total M_R

$$= fad$$
$$= 472{\cdot}5.$$

These two values of M_R are plotted on Fig. 233.

An extra plate will be required over the central portion of the cross girder.

Fig. 233 shows a suitable length for this plate to be 12 feet, extending 6 feet either side of the centre of the span.

A plate ⅜ inch thick will have a cross-section of 2·75 square inches, and an M_R

$$= 354{\cdot}4 \text{ inch-tons.}$$

This makes up a total flange area of 14·6 square inches and a total M_R of 1,850 inch-tons.

This leaves sufficient margin to allow for the weight of the cross girder itself and avoids the necessity of recalculating.

Plate IV. shows the cross girder in elevation. The web is ⅜-inch plate, and the rivet holes in the vertical and horizontal faces of the angles of the flange are made to break joint, *i.e.*, not to occur at the same vertical section, so as only to reduce the cross-section by one rivet hole. This will give an extra margin of safety.

Details.—A ballast plate is fitted to the edge of the troughing to keep the ballast on the bridge floor from falling through between the troughing and the main girders.

This consists of a vertical plate 12 inches high and ½ inch thick, secured to the troughing at its bottom edge by a 3-inch × 3-inch × ½-inch angle, and held vertical by brackets of 3-inch × 3-inch × ½-inch angle steel fixed to the plate stiffeners of the girder where they occur. This prevents the ballast from the floor from touching the main girder, and thus keeps the main girder open for inspection and painting.

Stiffeners.—Four stiffeners are provided, being placed nearly under the rails.

They consist of vertical angles 4½ inches × 4½ inches × ½ inch, with a ⅝-inch packing piece underneath them, cut off square at the ends.

The ends of the cross girder are similarly formed; a piece of 4½-inch × 4½-inch × ⅝-inch angle is placed vertically against the web of the cross girder on each side, and is riveted to the web of the main girder with a ⅝-inch packing piece between. There is no end plate to the cross girder. The top and bottom flange plates and angles of the cross girder are cut off square and flush with the web.

The horizontal pitch of the rivets through the flange angles and the web of the cross girder near the ends is reduced to 3½ inches, so as to make them fit in with the length of the cross girder.

This reduced pitch extends from the ends as far as the first stiffener.

Countersunk rivets are required in the flanges above and below the stiffener angles where marked "*csk.*"

Drainage of rainwater would be arranged by holes through the hollows of the troughs near their points of support.

There are ⅜-inch packing pieces under the troughs where there is no ⅜-inch flange plate, *i.e.*, under the outer three troughs.

An alternative method of securing the cross girder to the main girder, combining a stiffener in the same piece, is shown in the next example.

Plate Floors.—A good floor, and one which until the introduction of troughing was almost universally used, is formed by cross girders, rail bearers, and a plate deck, with sufficient ballast to take the ordinary permanent way (Plate V.) The plate deck is made of flat plates, generally ½ inch thick. The drainage sometimes presents a little difficulty. When this floor is ballasted, the sleepers are usually laid transversely. This type of floor can also be used without ballast by laying the rails on longitudinal sleepers on top of the rail bearers.

The rail bearers are notched or joggled in order to allow them to come up flush with the cross girders, and the ½-inch plate is riveted over all. Different railway companies have different types of floors in general use, but this is one of the most usual.

EXAMPLE.

To design a floor for a plate girder bridge, with two main girders, 4 feet 8½ inches gauge, to carry English rolling stock.

Cross girders and rail bearers will be used with a flat plate floor (Plate V.).

There are no restrictions as to headroom.

The span is 50 feet, and the girders may be assumed to be 16 inches across the flange and 4 feet 8 inches deep.

The cross girders will rest on the lower flanges of the main girders.

The ruling engines for floor loads are given in Figs. 225 and 226.

Either transverse or longitudinal sleepers may be used, and the floor will be ballasted 9 inches deep.

RAIL BEARERS.

The rail bearers are the first consideration. Their span is the same as the spacing of the cross girders, and since the loads and span are the same in this example as in the preceding one, the spacing may be taken as the same, viz., 8 feet. The loads on the rail bearers are the same as in the preceding example, with this difference, that the rail bearers must take the whole of the load above them instead of only half as in the case of troughs. The ruling axle for $Max.\ M_B$ on two rail bearers weighs 19·4 tons (Fig. 225); hence the $Max.\ M_B$ on one rail bearer

$$= 232{\cdot}8 \text{ inch-tons}, = f\frac{I}{y}.$$

If we allow 4½ tons per square inch for "f," then

$$\frac{I}{y} = 51{\cdot}7.$$

Referring to tables, it will be found that a rolled steel I section beam, B. S. B. No. 22, 12 inches by 16 inches, would suffice for this purpose, having $\frac{I}{y} = 62{\cdot}58$.

In this case, however, I beams will not be used, and the rail bearers will be built up.

A depth of 12 inches will be adopted, with a web of ⅜-inch plate; the flanges will consist of two angles, 4 inches × 4 inches × ½ inch.

PLATE V.

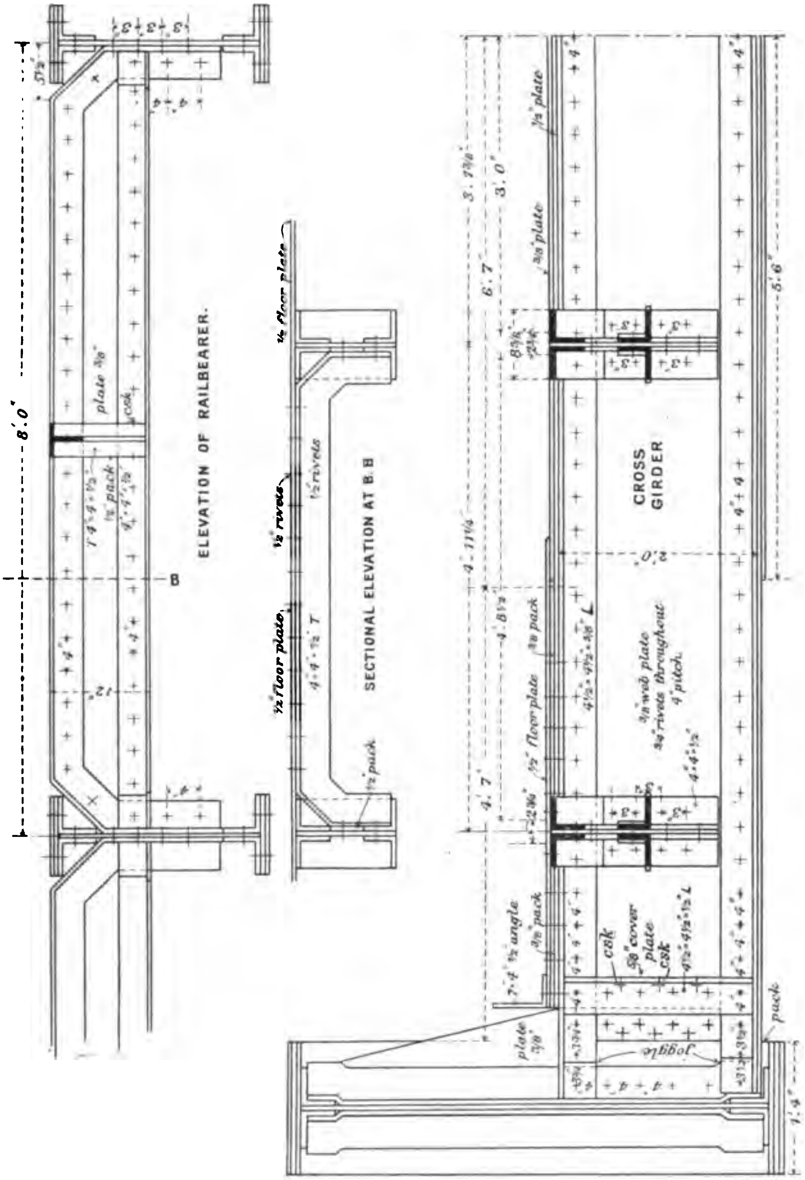

ELEVATION OF RAILBEARER.
SECTIONAL ELEVATION AT B.B
CROSS GIRDER
8'.0"
B
B
plate 3/8"
csk
7"·4"·4"·1/2"
1/2" pack
4"·4"·1/2"
12"
3 1/2"
1/2" floor plate
1/2" rivets
1/2" rivets
4"·4"·1/2" T
1/2" pack
1/4" floor plate
3/8" plate
1/2" plate
6'.7"
3'.0"
3'.7 3/8"
4'.11 3/4"
4'.8 1/2"
4'.7"
5'.6"
2'.0"
1'.4"
3/8" web plate
3/4" rivets throughout
4" pitch
4"·4"·1/2"
4 1/2"·4 1/2"·3/8" L
3/8" pack
1/2" floor plate
7"·4"·1/2" angle
3/8" pack
csk
5/8" cover plate
csk
4 1/2"·4 1/2"·1/2" L
plate 3/8"
joggle
pack

Fig. 235 shows a section through this rail bearer.

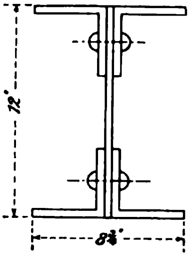


FIG. 235.

As the flanges of the rail bearer consist of angles only, the centre of gravity of each flange of the rail bearer is not at its upper edge, and it is therefore not correct to take the depth of the web, *i.e.*, 12 inches, as the depth of the section in the formula $M_R = fad$.

The distance between the centres of gravity of the top and bottom flanges should be taken.

This is approximately 10 inches, and has been taken as such for calculation.

Three-quarter-inch rivets will be used in this rail bearer, and the cross-section of the two angles in the top or bottom flange, deducting one rivet hole in each (⅞ inch diameter), is

$$= \frac{53}{8} \text{ square inches.}$$

Therefore the M_R of such a rail bearer, 10 inches deep

$$= fad$$

$$= \frac{9}{2} \times \frac{53}{8} \times 10$$

$$= 298 \text{ inch-tons.}$$

This greatly exceeds the *Max.* M_B, which was found to be 232·8 inch-tons; but the dead load has yet to be considered.

The weight of the rail bearer itself, taken piecemeal from tables, = 561 lbs.

The weight of the track taken at 100 lbs. per foot run for the whole track

$$= \frac{800}{2}$$

$$= 400 \text{ lbs.}$$

The weight of the ballast may be taken as 110 lbs. per cubic foot; each rail bearer must support half the ballast between itself and the next rail bearer and between itself and the main girder, *i.e.*, a total area of floor

$$= 8 \text{ feet} \times 4 \text{ feet } 9 \text{ inches,}$$

and with ballast 9 inches deep the total weight of same

$$= 8 \times 4\tfrac{3}{4} \times \frac{3}{4} \times 110 \text{ lbs.}$$

$$= 3{,}135 \text{ lbs.}$$

There is also the weight of the plate floor made of ½-inch steel plate.

The area to be supported by one rail bearer will be the same as in the case of the ballast, viz.,

$$= 8 \text{ feet} \times 4 \text{ feet } 9 \text{ inches};$$

this works out to a weight of

$$= 775 \text{ lbs.}$$

Therefore the total dead load on a rail bearer

$$= 4{,}871 \text{ lbs.}$$

This is a uniform distributed load, and will have a central M_B on the rail bearer

$$= \frac{4{,}871}{2{,}240} \times \frac{8 \times 12}{8}$$

$$= 26{\cdot}1 \text{ inch-tons.}$$

This added to the accidental load makes a total *Max.* M_B

$$= 232{\cdot}8 \times 26{\cdot}1$$

$$= 259 \text{ inch-tons};$$

and the rail bearer selected is well able to resist it.

The total actual load on the rail bearer is

$$= \frac{19{\cdot}4}{2} + \frac{4{,}871}{2{,}240} \text{ tons}$$

$$= 11{\cdot}87 \text{ tons.}$$

Cross Girder.—It is now necessary to calculate the cross girder.

The live load is the same as in the previous example, the engine in Fig. 226 being the ruling one.

The load is, however, more concentrated, occurring immediately under each rail.

The load under each rail = 11 tons, or 22 tons on each track.

In addition to this there is the dead load which is not uniformly distributed.

Now in considering the dead load on the rail bearers we have allotted the whole weight of the ballast and the plate floor to them, assuming that they take it all; in reality the floor is cut up into rectangles, the weight of which is taken by the members forming the sides of these rectangles, viz., the cross girders and the rail bearers, and also the edges of the main girders.

In this case our assumption errs on the safe side, since the rail bearers have been made strong enough to take all the weight, and they throw the concentrated weight of the flooring on to the cross girders at their points of junction, which also errs on the safe side.

The dead load, therefore, on the cross girders is assumed to be applied to them under the rails, and is equal to the dead load on the rail bearers, *i.e.*, 4,871 lbs. or 2·17 tons under each rail.

The actual weight of the cross girder itself must be left out until it has been approximately designed.

There is therefore a total load of 13·17 tons at each rail.

The width of the bridge will be the minimum possible, viz., 20 feet 4 inches in the clear between main girders. These are 16 inches wide, and the distance from centre to centre of main girders will be 21 feet 8 inches.

The span of the cross girders for calculation may be taken as 21 feet.

The M_B of a load of 13·17 tons at the centre of a 21-foot span

$$= \frac{W l}{4}$$

$$= 13{\cdot}17 \times 21 \times \frac{12}{4}$$

$$= 829{\cdot}71 \text{ inch-tons.}$$

In Fig. 236 a parabola is drawn with this as vertical height, and verticals are drawn at the positions of the rails giving the M_B due to all the loads at these four points. Fig. 237 is drawn giving the total combined M_B at these four points due to all the loads.

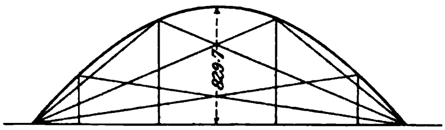


FIG. 236.

The *Max.* M_B at the inner rails is 1,520 inch-tons, and at the outer rails 800 inch-tons.

Assume the cross girder to be 24 inches deep.

The flange might be made up of one ½-inch plate, 9⅜ inches wide, and two angle steels 4½ inches × 4½ inches × ⅝ inch.

Then, the M_B for the angles $= fad$

$$= \frac{9}{2} \times 24 \times \frac{65}{8}$$

$$= 877{\cdot}5 \text{ inch-tons.}$$

And the M_R for the ½-inch plate = 398·25 inch-tons.

In both of these two rivet holes have been deducted.

The total M_R = 1275·75 inch-tons.

An extra ⅜-inch plate will have to be added in the centre of the cross girder.

This will give an extra M_R of 298·7 inch-tons, making a total of 1,574·5 inch-tons, which is ample to include the dead weight of the cross girder itself.

This extra plate is found to be 10 feet 8 inches long, from Fig. 237.

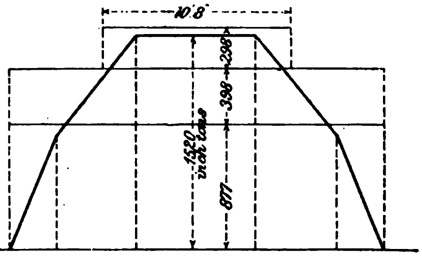


FIG. 237.

Joints. — Junction of rail bearers and cross girders.

The rivets for this joint depend upon the vertical shear. The maximum shear on the end of a rail bearer will occur when the load of the heaviest wheel is just coming on to the rail bearer.

The heaviest wheel is 9·7 tons, and will cause a vertical shear of 9·7 tons at the end of the rail bearer.

To this must be added half the dead load of one rail bearer, viz.:—

$$\frac{2{\cdot}17}{2} \text{ tons}$$

$$= 1{\cdot}09 \text{ tons.}$$

This makes a total shear of 10·8 tons approximate.

The web of the cross girder is ⅜ inch thick, and the rivets used to make this joint are ⅞-inch rivets. The rivets of this joint will be weakest in their bearing against the web plate of the cross girder. The resistance to bearing of one rivet

$$= \tfrac{7}{8} \times \tfrac{3}{8} \times 6{\cdot}4 \text{ tons,}$$

6·4 tons being the safe resistance to bearing for rivets in floors, when a reduced value of constants is taken to allow for live load; 4·8 tons per square inch is the resistance to shearing under similar conditions.

Let n be the number of rivets required.

Then

$$n \times \tfrac{7}{8} \times \tfrac{3}{8} \times 6{\cdot}4 = 10{\cdot}8$$

$$\therefore n = 5\tfrac{1}{7} \text{ or } 6 \text{ rivets.}$$

For resistance to shearing:—

$$n \times \pi \times (\tfrac{7}{16})^2 \times 4{\cdot}8 = 10{\cdot}8$$

$$\therefore n = 3{\cdot}75 \text{ or } 4 \text{ rivets.}$$

Six rivets $\frac{7}{8}$ inch diameter will therefore have to be used. The equivalent number of $\frac{3}{4}$-inch rivets would also be six. The $\frac{3}{4}$-inch rivets would be the best to use, since all the other rivets in the rail bearers are $\frac{3}{4}$ inch diameter.

Now Plate V. shows the end of the rail bearer, in which the angles of the upper flange have been bent down so as to clear the top flange of the cross girder, and in consequence there is not room for six rivets in these flange angles, where they rest against the web of the cross girder. They are therefore prolonged and carried down the web of the cross girder far enough to obtain room for six rivets.

It should also be noted that the rail bearers are raised sufficiently high up as to bring their upper surfaces flush with the top flange of the cross girder.

This top plate does not go the whole length of the cross girder, and packing pieces will be required in places to make up the level for fixing the flooring plates.

Ends of Cross Girders.—The cross girders are to rest on the lower flanges of the main girders.

An excellent method of stiffening the main girders at the point of attachment to the cross girders is that which will be adopted here.

The vertical angle steels that are placed on both sides of the web of the cross girder, in order to fasten it to the web of the main girder, are carried right up to the top of the web of the main girder in a similar manner to the angles of a plate stiffener.

The flange angles of the cross girder are bent outwards enough to allow these angles to pass between or underneath them.

The web plate of the cross girder is also carried upwards between these vertical angles, to act as the plate of a plate stiffener.

For this, the top flange plate of the cross girders must be stopped as shown in the drawings (Plate V.), so as to allow the web plate to be carried up. The M_B in the cross girder at this point is so small that the flange plate can easily be dispensed with.

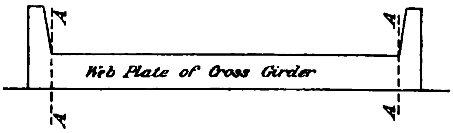


FIG. 238.

Now this carrying up of the web plate means a joint in the web where shown, otherwise a single whole plate of the shape shown in Fig. 238 would have to be used for the web plate of the cross girders.

This would mean much cutting out of the plate. Joints would therefore be made as shown in Fig. 238 at *AA*.

Referring again to Plate V., it will be seen that a stiffener to the web of the cross girder is made at this point of the web.

The vertical shear at this joint is half the total load on the cross girder, *i.e.*, 13·17 tons. Hence the rivets in the cover plate to this joint must be able to take 13·17 tons.

With ¾-inch rivets, let n be the number required. Then, for bearing on the web plate

$$n \times \tfrac{3}{4} \times \tfrac{3}{8} \times 6{\cdot}4 = 13{\cdot}17.$$

$$\therefore n = 7{\cdot}33 \text{ or } 8 \text{ rivets.}$$

As the cross girder is 24 inches deep, the rivets at their usual pitch will not suffice. Seven rivets are provided in the cover plate on each side of the join, and these with the rivets of the top and bottom flange angles will be sufficient.

The stiffener at this joint is made of one 4½-inch × 4½-inch × ½-inch angle steel, and the cover plates to the joint in the web are ⅝ inch thick, so as to come flush with the face of the flange angle of the cross girder.

No additional stiffeners are required since the rail bearers themselves provide ample stiffening under the loads, where they are riveted to the cross girders.

Rivets and Pitch.—The pitch of the rivets in the cross girders is 4 inches throughout, with the exception of the last two rivets at each end.

In these the pitch must be altered slightly.

In the upper flange it is reduced to 3¾ inches and in the lower to 3½ inches.

This just fits the end of the cross girder.

The total length of the cross girder is 21 feet 7½ inches along the top, and 21 feet 6¼ inches along the bottom flange. The pitch of the rivets in the rail bearers is also 4 inches, but this pitch is broken at the bends of the rail bearers.

Also the rivets forming the joint to the cross girders are placed out of pitch so as to fit them in conveniently.

Plate Floor.—This consists of ½-inch plain steel plates.

They rest at their edges on the rail bearers and cross girders. They are riveted to the rail bearers and the cross girders, the rivet holes in the latter being made to break joint with those in the flange angles of the rail bearers.

The maximum stock sizes for ½-inch plates are very large, and there will be no difficulty in obtaining plates of the sizes required.

The plates on the centre of the floor between the two middle rail bearers are 8 feet × 6 feet 2¾ inches.

Those between the rail bearers of each track are 8 feet × 4 feet 9¼ inches.

Those between outer rail bearers and the main girders are 8 feet × 1 foot 9½ inches.

These are supported at their outer edge by an angle steel 7 inches × 4 inches × ½ inch, riveted to them, extending from one cross girder to the next, and forming a ballast plate to keep ballast on the floor.

These outer plates will be strong enough without any other support.

The other rectangular plates, however, 8 feet × 4 feet 9¼ inches, and 8 feet × 6 feet 2¾ inches, are not strong enough to carry the weight of the ballast, platelayers, etc., without some further support.

A Tee steel is therefore introduced between rail bearers under the plate floor to give it support. This Tee steel is shown in

Plate V., and is riveted to the middle of each rail bearer and also to the plate of the floor.

The section used is 4 inches × 4 inches × ½ inch and the rivets joining it to the floor plate are ⅝-inch rivets, and are disposed alternately on opposite sides of the stem of the Tee, *i.e.*, so as to break joint.

These Tees also help to stiffen the whole framework of the floor.

Buckle Plate Floors.—The ordinary cross girder, rail bearer, and plate floor may be varied by using buckled plates instead of flat plates.

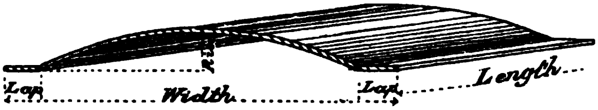


FIG. 239.

These have the advantage of strength, and easy surface water drainage, since a hole in their lowest part will allow all water to escape. But troughs have all these advantages as well.

Buckled plates are not always easy to obtain, and they are not very convenient to use, since they must either be made of special sizes, or else, if a standard size is adopted, the design of the floor is hampered thereby.

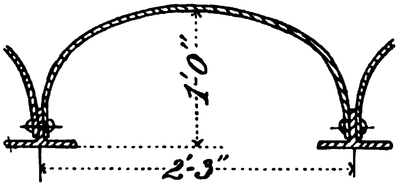


FIG. 240.

Buckled plates are shown in Fig. 239 having two flat edges. They can also have four flat edges, if required.

Buckled plates are usually made 3 feet square, ¼ inch to ½ inch thick.

When these plates are only supported on two sides, the edges where they join one another should be supported by a T or L steel.

Buckled plates with the curve downwards are about three times as strong as with it upwards.

The strength of a buckled plate is given roughly thus :—

$$W = 4fht.$$

Where $W =$ total weight on the plate in lbs.

$f =$ permissible stress in the plate in lbs., in floors say 8,000 lbs. per square inch.

$h =$ depth of the buckle in inches.

$t =$ thickness of plate in inches.

This is independent of the size of the plate.

Other curved plate systems have been used, but have now entirely gone out of practice.

Fig. 240 shows the most common of these, known as the arched plate system.

Such a floor would practically never be used where any other type would serve.

It is not strong for its depth, nor economical of headroom.

The curved plates are difficult to obtain; their ends are difficult to deal with; and the drainage of water is as bad as it could be.

TIMBER DECK FLOOR.

Another kind of floor is that made with timber decking.

This is light and also cheap where timber is easily obtainable.

It does not last very long, but is noiseless and easy to fix.

Plate VI. shows such a floor in which the portion between the rails is covered with buckled plates, which can withstand the fall of the ashes, etc., from passing trains better than timber.

Longitudinal wood sleepers must be securely bolted to the rail bearers when they rest immediately upon them, to keep them firmly in position; otherwise when they begin to rot or work loose, the gauge will become very bad.

In some localities where good timber is very plentiful, the rail bearers are omitted and timber stringers are used instead. These must be firmly bolted to the cross girders to preserve the gauge.

Special attention should be given to this.

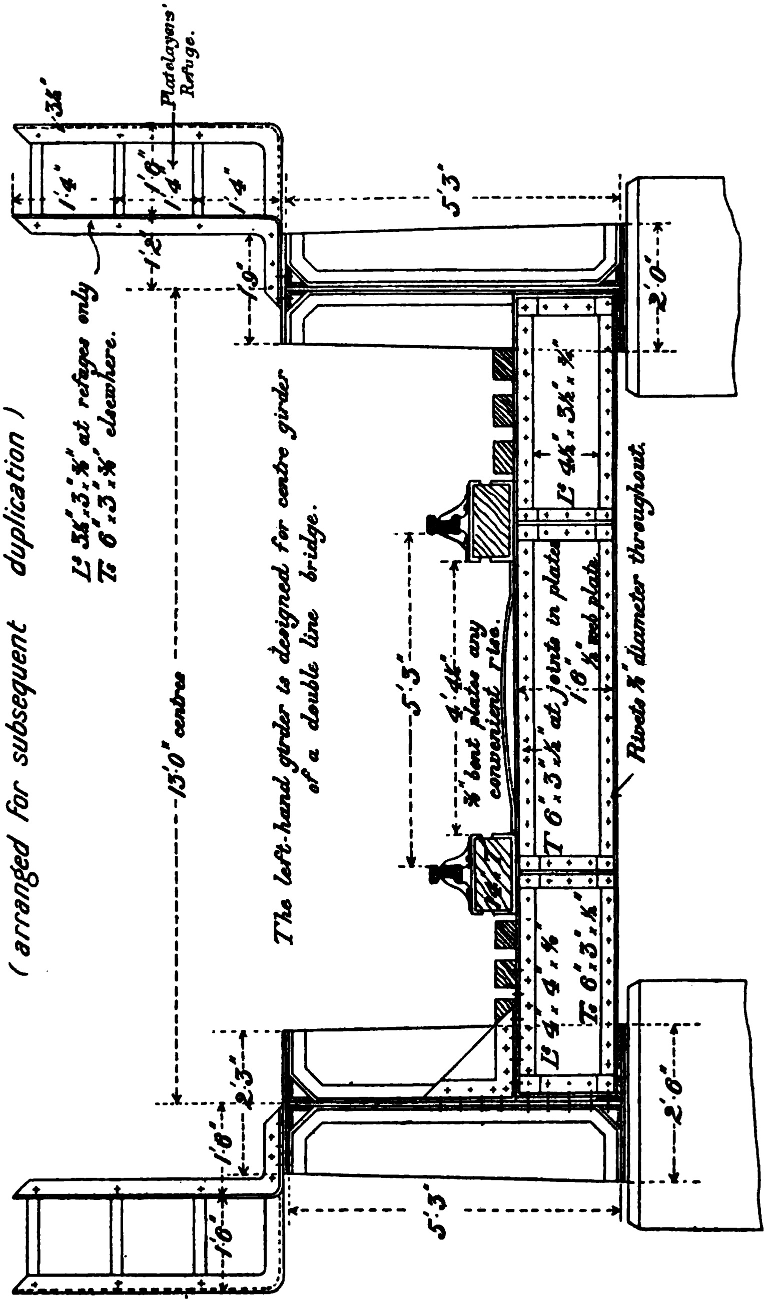
GENERAL.

Ballast Board.—Every bridge that is ballasted must have some means of preventing the ballast from falling through between the

PLATE VI.

MAIN AND OUTER GIRDER FOR SINGLE LINE BRIDGE.

(arranged for subsequent duplication)


3½"
Platelayers' Refuge.
1'4"
1'6"
1'4"
1'4"
1'2"
1'0"
5'3"
2'0"
Ls 3½" x 3" x ⅜" at refuges only
Ts 6" x 3" x ⅜" elsewhere.
13'0" centres
The left-hand girder is designed for centre girder of a double line bridge.
5'5"
4'4½"
⅜" bent plates any convenient rise.
Ls 4½" x 3½" x ⅜"
T 6" x 3" x ½" at joints in plate
1'6"
⅜" web plate
Rivets ⅞" diameter throughout.
Ls 4" x 4" x ⅜"
Ts 6" x 3" x ½"
2'3"
1'8"
1'6"
5'3"
2'6"

floor and the main girders. In trough floors if the troughs join up to the main girder, then no special precautions are necessary. But otherwise a ballast board must be provided.

The ballast board may be of timber or steel, and is generally riveted or bolted to the floor.

Plate VIII. and Plates IV. and V. show various forms of ballast boards employed.

In the case of transverse troughing the main girder has to serve as ballast board, which is not a very desirable arrangement, as it prevents easy access to the main girder for inspection or painting; it also allows water to be in constant contact with the main girder, unless the drainage is very carefully attended to.

A floor having its own ballast board some little distance away from the main girder is preferable, as it allows easy access to the latter.

Ends of Floors.—At the ends of the bridge, where the floor lands on the abutments, the ballast and track are kept continuous without any break if possible.

The ordinary track is ballasted right up to the ballast wall, a light low wall which is built across the top of the abutment between the bedstones of the main girders, high enough to retain the ballast from falling off the abutment, but not as high as the underneath of the rails.

The rails pass over the ballast wall from the sleeper behind it to the sleeper in front of it which is resting on the bridge floor.

Drainage of Floors.—Drainage is very important. If water stands on a steel floor and is kept from drying for a long time, by the presence of the porous ballast, it may prove exceedingly injurious to the floor. Needless to say, water enters into all angles and joints, and by capillary attraction enters all the smallest cracks, *e.g.*, underneath rivet heads or between plates, etc., causing rust action.

There are therefore two main problems to deal with:—

(1) To get the water off as soon after it has fallen on the bridge as possible.

(2) To prevent it getting into contact with the steel at the joints, etc., by means of some artificial covering.

The first is done as follows.

With transverse troughs in a two-girder bridge, one main girder is bedded ½ inch to 1 inch higher than the other, so that troughs slope one way.

With three main girders the same is done, the centre girder being set 1 inch lower than the outside ones.

All the water runs down the troughs towards the centre girder and out through holes bored in the lowest part of the troughs.

Some water of course finds its way on to the lower flange of the main girder and tries to stand there. The water on passing through the holes through the troughs is caught by an ordinary simicircular rainwater gutter and is led away to the abutments, and down.

With longitudinal troughing the centre cross girders of the bridge are set a little higher than those near the abutments; 1 inch to ½ inch is sufficient for all ordinary spans.

There is the camber of the main girders also to assist the slope of the troughs towards the abutments. The rainwater is collected and disposed of at the abutments.

A flat plate floor is difficult to drain, but generally speaking the centre of each plate should have a weep hole bored in it; or if the water may not be permitted to drip through the bridge, the floor must be sloped down towards the ballast boards.

With regard to the second problem, the matter is mainly one of painting, but other means are sometimes resorted to.

Asphalte and concrete are the two best fillers of joints and inconvenient holes.

Painting is sometimes resorted to in floors, but this is not satisfactory if the floor is to be ballasted, as the metal cannot easily be got at for renewing the paint.

It is with this in view that every effort should be made to provide a ballast board to the edge of the floor, which shall at any rate keep the ballast away from the main girders and allow them to be seen and easily got at for painting.

When using trough floors, if there is no ballast, paint will serve admirably.

When there is ballast to be used and there is no easy way of keeping the ballast away from the main girders, the following is a good expedient for protecting them from water.

Lay a ¾-inch layer of asphalte in the hollows of the troughs and in all joints, horizontal spaces where rain might lodge, etc. The ballast will rest over this. Thus where the water is likely to stand there will be an impervious covering for it to rest on.

Sometimes a layer of 1½ inches to 2 inches of concrete may be laid over this before the ballast is put on.

Some engineers use this covering right over the whole of flat plate floors, viz., 1 inch to 1¼ inch of bituminous covering and 2 inches of concrete over it.

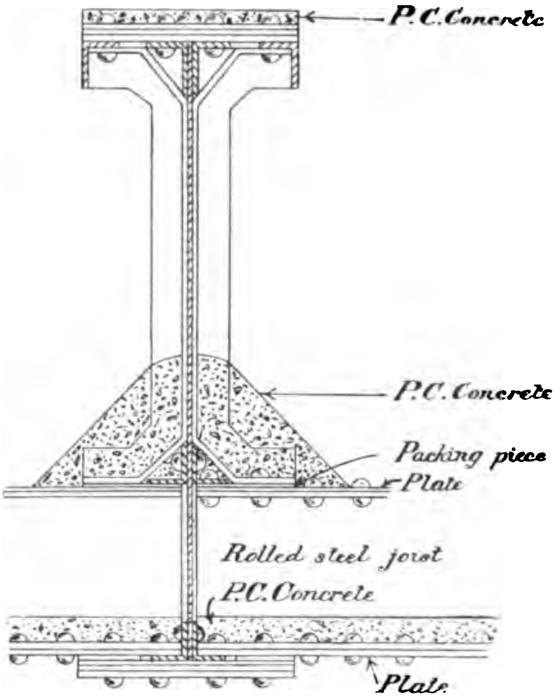


FIG. 241.

The concrete is a physical protection to the asphalte, and prevents the waterproof covering being injured by platelayers with their picks, etc.

Concrete itself is an excellent covering adhering fairly well to steel, and not having any special tendency to flake off or peel, if properly applied. Its coefficient of expansion is very nearly the same as that of steel.

It may either be laid all over the floor, a method which has not been found very satisfactory, on account of lack of adhesion, or may be utilised in angles, joints, junctions, etc.

For instance, troughs are sometimes filled up solid with concrete to a flush surface on top, and only sloped a little for drainage.

Fig. 241 shows the use of concrete in some joints of a floor where water might lodge.

It must be remembered that unless the steel of the floor is perfectly clean, free from all mill scale, dirt, remains of paint, or boiled linseed oil, adhesion will be imperfect, and leakage and rust will set in.

The steel should be cleaned by the sand blast before applying the concrete, and the latter must be thoroughly rammed in and must not be too coarse or too dry.

In the floors of bridges abroad, across gorges or stormwaterways, where the drip from a bridge does not matter, there is frequently no flooring provided at all, and the rain drops right through.

A few steel strips are fixed from one cross girder to another to form a platelayer's walking plank, and the rails on longitudinal sleepers rest directly on the rail bearers.

Handrails.—A parapet of some kind must be provided to every bridge.

Plate girders form their own parapet if they are high enough above the floor.

If they are more than 2 feet above rail level they will also be fully 2 feet in the clear from any rolling stock; but this is hardly enough for a man to stand in comfortably. In long bridges, therefore, some form of platelayer's recess should be provided at intervals along the bridge. In towns, overhead bridges crossing over roads must have a close parapet 4 feet 6 inches high, by Board of Trade regulations, *i.e.*, one which cannot be seen through.

If the plate girder is not high enough, an additional parapet can be got by securing bent angle steels to the top flange of the girder, or the outer edge of plate stiffeners of the girder, and fixing the necessary width of plate to these to form the required parapet.

If, however, the girders are low, and consequently are close to the rails, *i.e.*, 4 feet 7 inches from the centre of the track, then the parapet and handrail should include a narrow side walk for a man, generally on top of the flange of the girder, and of course must project outside the main girders in accordance with Board of Trade regulations, being not less than 6 feet 7 inches from the centre of the track.

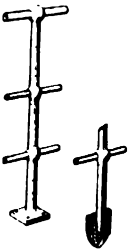


Fig. 242.

Such a parapet might be formed of angle steels and a light floor plate if necessary, outside the edge of the top flange of the main girders.

Plate VI. shows such a parapet rail with platelayers' recesses at intervals.

A parapet plate ¼ inch thick is secured to the outer side of the upright Tees and angles.

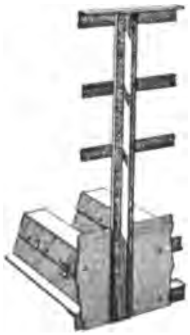


Fig. 243.

Large span braced girders hardly ever occur over roads, in which case they do not need a close parapet, and the girders themselves form ample parapet for safety when used in the through type of bridge.

In over-bridges the edge of the bridge floor usually projects outside the main girders, and a handrail of some kind is required for the safety of platelayers, etc. This may be of open type.

Figs. 242 and 243 show two types of upright, and no difficulty should be experienced in arranging such a handrail so as to be both light and strong and also sightly.

Some more types are shown under road bridge floors.

CHAPTER XV

ROAD BRIDGE FLOORS

In connection with this subject it should be remembered that this book only deals with steel work, and the theory of stresses in it, and not with the masonry or details of the road surface.

Road bridges differ from railway bridges in many essentials, the two principal of these are:

(*a*) Loading.

(*b*) Type of construction of floor.

The general loading has been discussed under "loads" in Chapter XII., and no further reference need be made thereto here except in working out examples.

TYPES OF FLOORING.

The following are some types of flooring in common use:

(1) Transverse troughing.
(2) Cross girders and longitudinal troughs.
(3) „ „ and jack arches.
(4) „ „ and buckled plates.
(5) „ „ and reinforced concrete slabs.

(1) *Transverse Troughing.*—Such a floor is very similar to that in use in railway bridges. It is very common indeed for bridges for foot passengers or light loads, as for instance overhead passages or covered ways over railway stations or roads; or for country bridges over railways, canals, etc., or anywhere where it might prove economical.

Transverse troughs would usually be used with a narrow bridge made with two plate girders, which would serve as part parapets, or two heavy I beams, and would rest on the lower flanges of these girders.

These troughs are fixed in exactly the same way as in railway bridges (Fig. 244).

They must be protected first of all from wet, either by asphalte or concrete.

The usual practice is to clean them and then to fill them up solid with concrete.

The top of the concrete is sloped as required to drain off any water that may leak through the road surface.

The concrete should be turned up against the main girder and carried up above the road surface to keep the road metal from touching the main girder, and to protect it in the part where it will never be accessible for painting.

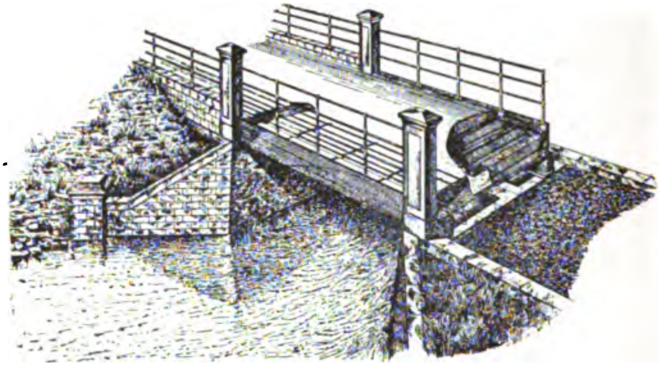


FIG. 244.

On top of this concrete surface the road metal or paving may be laid.

The chief limitation of this type of floor is the enormous size of trough required for any but a narrow bridge and light loads.

(2) *Longitudinal Troughing.*—This is much used for short spans by itself, merely laid across the span (Fig. 245).

The troughing is laid across the span and requires no main girders.

This is frequently the cheapest and best bridge for small spans.

Each trough must of course be designed to carry the maximum load, since the load may be anywhere on the bridge, and is not confined to rails, as in a railway bridge; and this leads to waste of material and heavy trough sections.

But where these can be got, and transport and handling are easy, they are economical.

The troughs are filled with concrete as before, and the road surface is laid on top.

The outside edges are generally provided with a face plate to give a finish to their appearance, and a handrail of open type is fixed by using special standards of Tee or angle steel.

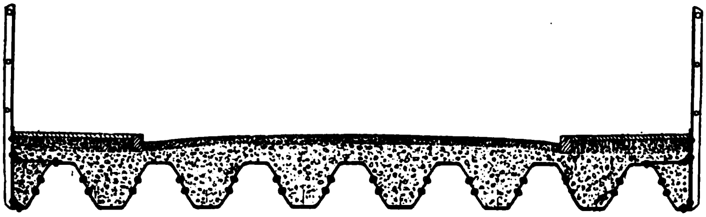


FIG. 245.

They are especially useful in narrow foot bridges.

Longitudinal Troughs with Cross Girders.—When the span is too long for the direct use of these troughs unsupported, they may be used with cross girders.

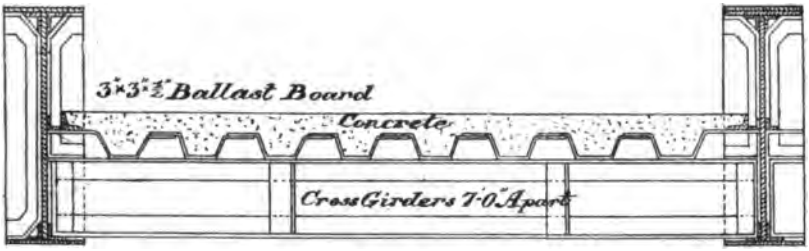


FIG. 246.

This of course implies two main girders (or more), cross girders, and the troughs on the top of them.

This and all other types of floor have cross girders, with some form of flooring resting upon them.

In this type of floor the troughs have only a span equal to the spacing of the cross girders.

The filling of concrete and general road formation is the same as before, and the concrete base to the road should be turned up against the main girders.

The drainage from the surface of the concrete will be towards the abutments, and the camber that is given to the bridge will help to throw off any water that may soak through the road.

Fig. 246 shows such a bridge.

(3) *Cross Girders and Jack Arches.*—This type also implies main girders and cross girders.

The floor is placed on the cross girders by building jack arches in between them.

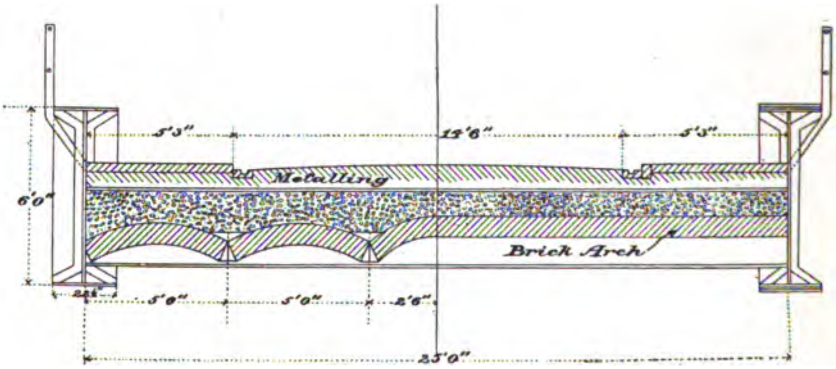


FIG. 247.

This is a very usual type of floor, almost universal before troughs came into common use.

The jack arches may be of brick, or concrete, and generally spring from the lower flanges of the cross girders.

The cross girders are spaced at a suitable distance apart, depending on the design of the arches, 4 feet to 6 feet, not more.

If concrete is used, a mixture of 5 to 1 or 6 to 1 is good enough if properly made.

The haunches should be filled in with 8 to 1 concrete up to the level of the crown of the arches.

The axis of these arches is transverse to the bridge, and their thrust is lengthwise with the bridge.

Some arrangement must therefore be made to take this thrust at the last cross girders, close to the abutments.

The end series of jack arches cannot be turned against the abutments because of expansion and contraction.

One device employed is to turn the arches between the last two cross girders at each end of the bridge at right angles to the remainder, inserting between the cross girders short steel joists for the arches to spring from (Fig. 247, left half).

This makes the end bays of jack arches rigid, to take the thrust of the other arches in the bridge.

Another and simpler way is to provide strong steel tie bars running through the bridge from end to end.

These pass through the webs of the cross girders and are built into the masonry of the arches. Where they pass through the web of a cross girder, they are secured to the web by backnuts on both sides of the web, and they can be adjusted accurately to the spacing of the girders.

The thrust in jack arches is given by :—

Pressure in lbs. per lineal foot of arch

$$= \frac{15wS^2}{h}.$$

w = load in lbs. per foot super.
S = span of arch in feet.
h = rise in inches.

This pressure will give the stress which the tie rods have to take, and will decide their diameter and spacing.

The tie rods should be placed as low as possible in the cross girders.

Diagonal ties are also useful to steady the bridge.

In building jack arches the skewbacks are formed of concrete, which should be thoroughly well packed against the cross girder, after cleaning the latter of all rust, scale, etc.

The spandrel filling over the haunches of the arches is important, and should be of concrete or masonry filled in to a level with the top of the crown of the arch.

(4) *Cross Girders and Buckled Plates.*—These are not often used.

Road bearers or joists are required between the cross girders to support the buckled plates at their edges.

They are covered with concrete in the ordinary way.

They may be secured to the bottom or top of the cross girders or half-way up their web.

(5) *Cross Girders and Reinforced Floors.*—The cross girders for these require no special remarks, and the slabs themselves do not come into the scope of this discussion.

General.—The various common types of floor have now been enumerated, and a few examples are here given.

The type of floor to be selected must depend on the nature of the circumstances and the type of girder chosen for use.

Plate girders when set low enable parapet walls to be built up on their upper flanges.

When set high up they serve as parapets themselves.

EXAMPLE.

To design a floor for a narrow road bridge, having two main girders and transverse troughing.

The floor is to be 14 feet wide with a 2-foot path on each side of a 10-foot road.

The loads to be carried are all ordinary vehicular traffic.

The road is made of concrete laid 3 inches deep above the upper surface of the troughs, and good macadam above it.

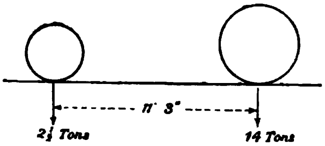


FIG. 248.

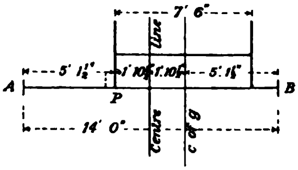


FIG. 249.

The axle loads for a heavy traction engine are shown in Fig. 248.

The back axle of this engine will be taken as the ruling load on one transverse trough.

Live Load.—Fig. 249 shows an elevation of one trough over the span, with the wheels of the back axle of the engine placed upon it in the position of *Max.* M_B, *i.e.*, with the centre of the span half-way between one wheel and the c. g. of the two wheels.

It is necessary to find the *Max.* M_B on the span for this live load.

The reaction at A

$$= 14 \text{ tons} \times \frac{5 \text{ feet } 1\tfrac{1}{2} \text{ inch}}{14}.$$

Hence the M_B which occurs under the wheel which is nearest the centre of the span, *i.e.*, P,

$$= R_A \times 5 \text{ feet } 1\tfrac{1}{2} \text{ inch} \times 12$$

$$= 14 \times \frac{5 \text{ feet } 1\tfrac{1}{2} \text{ inch}}{14} \times 5 \text{ feet } 1\tfrac{1}{2} \text{ inch} \times 12 \text{ inch-tons}$$

$$= \frac{5{,}043}{16} \text{ inch-tons.}$$

Now the road surface is to be of good macadam on concrete, but it is shallow, being only about 9 inches above the top of the trough.

With a cone of dispersion of pressure having an apex angle of 90°, the pressure still only distributes itself over one trough.

The troughs to be used will probably have a pitch of 2 feet to 2 feet 8 inches, and although they are riveted together, it is hardly fair to suppose that much of the load on the one trough is taken by adjoining troughs, certainly not until the one trough has deflected to a dangerous extent (Fig. 250).

Hence we must assume that one trough is to take all the load itself.

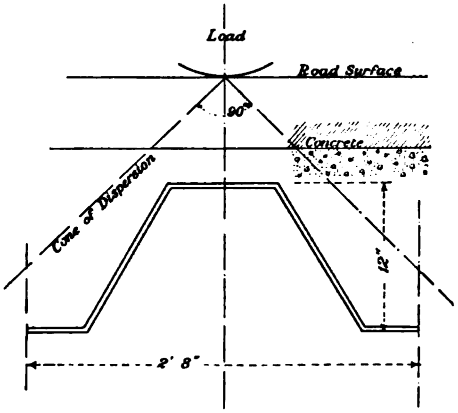


FIG. 250.

Compensation for Live Load. — The span of the trough is 14 feet, and Table XV. gives a percentage addition for this of 30 per cent. for main girder calculations.

The road surface in this case is to be good, but the troughs are to be transverse (to the bridge), and are placed in the worst possible position for impact.

An extra addition to the percentage given in the Tables will therefore be made, *i.e.*, 15 per cent. addition, making 45 per cent. in all.

Hence *Max.* M_B due to live load

$$= \frac{5{,}043}{16} \times \frac{145}{100}$$

$$= 451 \text{ inch-tons nearly.}$$

Dead Load.—This consists of the concrete and road metal on one trough, and the weight of the trough itself.

Assume the trough to be 2 feet 8 inches wide and 1 foot deep, weighing 30 lbs. per foot run.

Then the weight of the trough = 420 lbs.

Weight of the concrete from the bottom of the trough to 3 inches above the top

= 9 inches × 14 feet × 2 feet 8 inches × 120
= 3,360 lbs.

Weight of road metal over the concrete 6 inches deep, at 110 lbs. per foot cube,

= 6 inches × 14 inches × 2 feet 8 inches × 110 lbs.
= 2,053 lbs.

Therefore the total dead load = 5,833 lbs.

Therefore M_B due to the dead load

$$= \frac{Wl}{8}$$

$$= \frac{5{,}833}{2{,}240} \times \frac{14 \times 12}{8} \text{ inch-tons}$$

$$= 54{\cdot}70 \text{ inch-tons.}$$

This is at the centre of the span.

But the M_B at P, the point where the greatest M_B due to the live load occurs, may be taken as 54 inch-tons.

Therefore total *Max.* M_B = 505 inch-tons

$$= M_R$$

$$= f\frac{I}{y}.$$

Allowing $f = 7$ tons per square inch,

then $\frac{I}{y} = 72.$

Reference to Tables shows that a trough 2 feet pitch, 1 foot deep, with ½ inch flange thickness weighing 28·78 lbs. per foot run, has a section modulus of 72·67, and this will suffice.

EXAMPLE.

To design an overhead passenger footbridge, with longitudinal troughs resting on cross girders which are supported from two open lattice girders (Fig. 251).

The span is 39 feet.

The bridge is approached by steps at one end.

The width of the footway is 8 feet.

Live Load.—The load to be considered here is only that of people, since one approach is by steps which will not permit any other form of traffic.

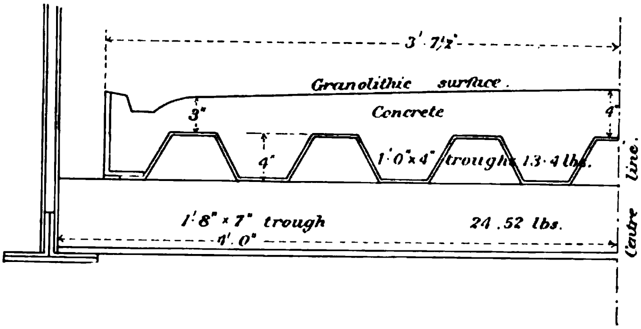


FIG. 251.

The actual weight of people here will be very great, and we shall take it as 80 lbs. per square foot, which must be corrected for impact.

Now the spacing of the cross girders may be assumed to divide the span into three, giving a span of 13 feet to the troughing.

For a span of 13 feet, the percentage for compensating live load from Table XV. is 39 per cent.

This is the addition for main girders; 50 per cent. might be taken as a fair total percentage addition here, for the floor calculations.

Hence the equivalent dead load = 120 lbs. per foot.

Therefore the total load on the bridge 8 feet wide per bay of 13 feet

$$= 8 \times 13 \times 120 \text{ lbs.}$$
$$= 12{,}480 \text{ lbs.}$$

Dead Load.—A very light section of troughing is assumed.

Reference to Tables shows a trough 1 foot wide 4 inches deep, weighing 13·4 lbs. per foot run, which seems suitable.

Its section modulus is 4·92.

The weight of such troughing for the whole width of bridge is

$$= 8 \times 13{\cdot}4 \times 13 \text{ lbs. per bay of 13 feet}$$
$$= 1{,}394 \text{ lbs.}$$

The troughing is to be filled in with concrete to 3 inches deep over the top of the troughing, with a rise of 1 inch in the middle of the pathway to throw water off.

The average depth of concrete will be 6 inches approximate. Hence the weight of the concrete at 120 lbs. per foot cube

$$= 13 \text{ feet} \times 8 \text{ feet} \times 6 \text{ inches} \times 120 \text{ lbs.}$$
$$= 6{,}240 \text{ lbs.}$$

Therefore the total dead load = 7,634 lbs.

And total load altogether = 20,114 lbs.

$$\therefore \ Max.\ M_B = \frac{20{,}114}{2{,}240} \times \frac{13 \times 12}{8} \times \frac{1}{7} \text{ on one trough}$$

$$= 25{\cdot}0 \text{ inch-tons} = M_R = f\frac{I}{y}$$

$$\therefore \ \frac{I}{y} = 3{\cdot}6 \text{ if } f \text{ be taken as 7 tons.}$$

Hence the trough selected will suffice; it is nearly the lightest made, and is on the safe side.

Reference to Fig. 251 will show that only seven troughs are really used, not eight, although the bridge is taken as 8 feet in width.

In the construction a ballast board should be provided for the edge of the concrete, about 4 inches clear of the main girders, so as to allow access to them for painting. These would consist of 7 inch × 3½ inch × ⅜ inch angles, riveted to the troughing with ½-inch rivets.

Rainwater gutters would be formed in the sides of the pathway in the concrete, inside the ballast boards.

There should be a slight camber in the floor from the centre towards the ends, which would be obtained by setting the two central cross girders 1 inch higher than the two outer ones, by the use of packing pieces under them. The camber of the main girders also assists the slope for the rapid flow of water off the bridge.

The rainwater gutters should end in small circular gratings set in the concrete, and rainwater pipes led down the upright supports of the bridge.

Fig. 251 shows a section through the floor.

The surface should be granolithic, or similar.

Cross Girder.—The total load on one cross girder is that due to one complete bay of flooring.

$$= 20{,}114 \text{ lbs.}$$

This is distributed along the cross girder.

Hence
$$Max.\ M_B = \frac{Wl}{8}$$
$$= \frac{20{,}114}{2{,}240} \times \frac{8 \times 12}{8}$$
$$= 107{\cdot}7 \text{ inch-tons.}$$

This requires a section of cross girder having

$$\frac{I}{y} = \frac{107{\cdot}7}{7}$$
$$= 15{\cdot}4.$$

A trough might be used for this, 8 feet long, 1 foot 4 inches wide × 6¼ inches deep, weighing 28 lbs. per foot run and having a section modulus 17·50.

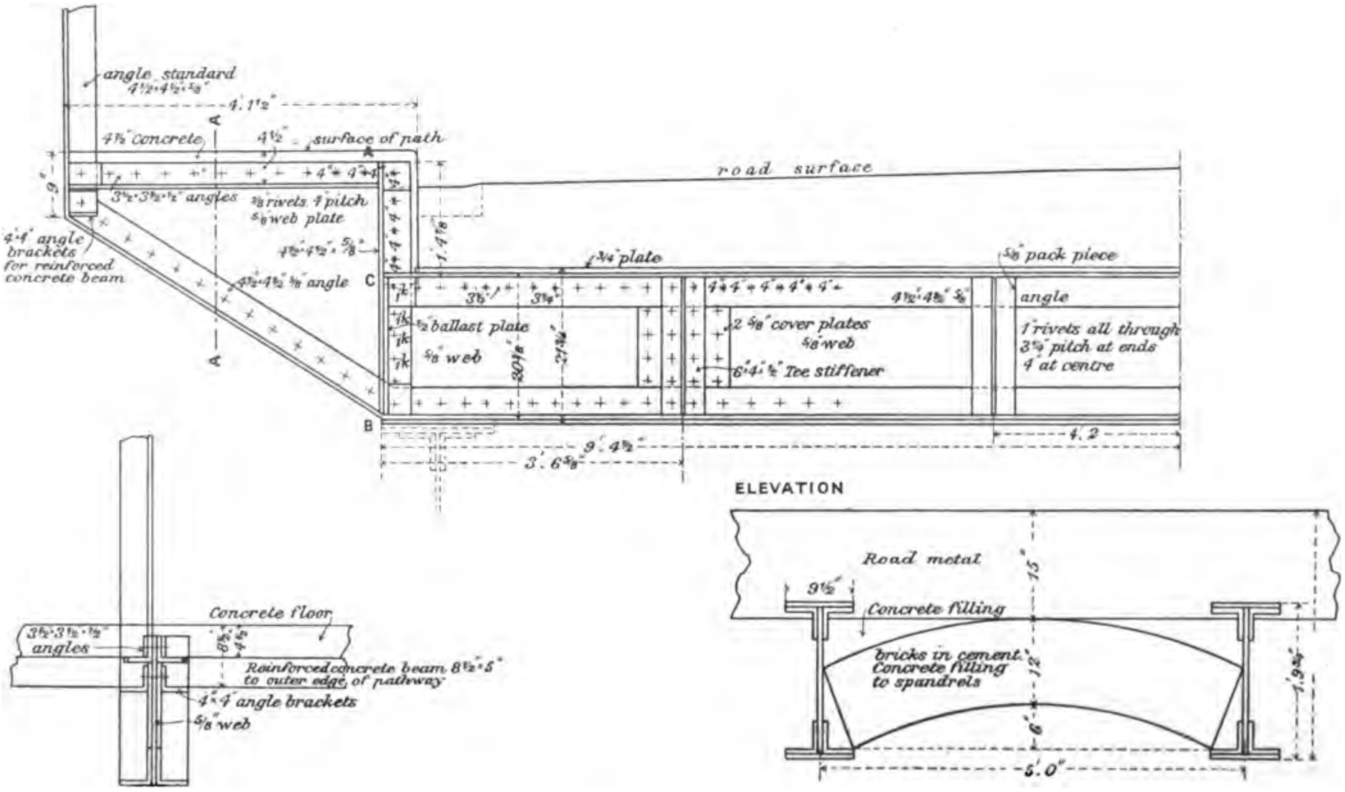
EXAMPLE.

To design a road bridge with an 18-foot roadway, and two pathways of 4 feet each, for a span of 45 feet.

The bridge will be an overbridge, the pathways projecting over the main girders as cantilevers (Plate VII.).

The bridge will take all ordinary vehicular traffic. The floor will be of cross girders and jack arches of brick in cement mortar, with a macadamised roadway on the top. The spandrel filling will be of concrete.

PLATE VII.



ELEVATION

SECTION AT A.A

SECTION THRO' CROSS GIRDER AND ARCH

The two main girders will be under the inner edges of the pathway, so that the span of the cross girders will be 18 feet.

These will be spaced 5 feet apart centre to centre.

CROSS GIRDERS.

Live Load.—The ruling live load on cross girders will be the heavy axles of traction engines or loaded trucks.

These may be taken as 16 tons per axle with a wheel track of 7 feet.

The bridge is wide enough to allow two streams of traffic, so that two such axles may be on the cross girder at the same time.

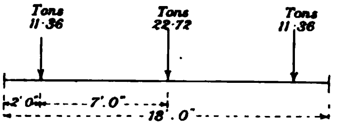


FIG. 252.

The percentage to be added to live loads for girders of 18-feet span (from Table XV.) is 27 per cent., and since these girders are in the floor, an extra 13 per cent., or a total of 40 per cent., must be added.

This makes an equivalent dead load of 22·72 tons for each axle.

The loading is shown in Fig. 252.

$$R_A = 22{\cdot}72 \text{ tons.}$$

$$\begin{aligned} \textit{Max. } M_B &= (22{\cdot}72 \times 9 - 11{\cdot}36 \times 7) \times 12 \\ &= 124{\cdot}96 \times 12 \\ &= 1499{\cdot}52 \text{ inch-tons.} \end{aligned}$$

Dead Load.—The jack arches are assumed to be 12 inches deep at the crown, with 6-inch rise (Plate VII.).

The spandrel filling is of concrete and is made level with the crown of the arch.

Therefore the weight of brickwork and concrete, taking them at 120 lbs. per foot cubed, on one cross girder

$$= 18 \text{ feet} \times 5 \text{ feet} \times 1 \text{ foot } 3 \text{ inches} \times \frac{120}{2{,}240} \text{ tons.}$$

And the road surface is 15 inches above this in the centre, hence the metal of the road will weigh, taken at 110 lbs. per foot cubed,

$$18 \text{ inches} \times 15 \text{ inches} \times 5 \text{ feet} \times \frac{110}{2{,}240} \text{ tons.}$$

The weight of the cross girder itself may be neglected. Hence the total dead load

$$= 11{\cdot}56 \text{ tons.}$$

$$\text{The } Max.\ M_B = \frac{W l}{8}$$

$$= \frac{11{\cdot}55 \times 18 \times 12}{8}$$

$$= 311{\cdot}8 \text{ inch-tons.}$$

Therefore the total *Max.* M_B due to dead and live loads

$$= 1{,}811 \text{ inch-tons.}$$

This is too large for a rolled section of standard size, and a built-up beam will be used, having:

$$M_R \text{ or } fad = 1{,}811 \text{ inch-tons}$$

$$\text{If } f = 7 \text{ tons}$$

$$ad = 258{\cdot}8.$$

Let $d = 20\frac{1}{4}$ inches (the odd depth is to suit the pitch of rivets),

then $a = 12{\cdot}8$ square inches.

For the flange use two $4\frac{1}{2}$-inch × $4\frac{1}{2}$-inch × $\frac{5}{8}$-inch angles, and deduct for two $\frac{17}{16}$-inch rivet holes for 1-inch rivets.

The *a* for the two angles

$$= 2 \text{ inches} \times \tfrac{5}{8} \text{ inch} \times 6\tfrac{1}{4} \text{ inches}$$

$$= 7{\cdot}82 \text{ square inches.}$$

Also use a $9\frac{1}{2}$-inch × $\frac{3}{4}$-inch flange plate; then deducting two rivet holes, $a = 5{\cdot}44$ inches.

This makes a total area of 13·26 square inches.

This will suffice for the whole cross girder.

Shear.—It is necessary to test for the number of rivets required to resist horizontal shear, *i.e.*, to connect the flanges to the web.

The vertical shear at the end of the girder

$$= \tfrac{1}{2} \text{ dead load} + \text{weight of one engine}$$

$$= \frac{11{\cdot}55}{2} + 22{\cdot}72$$

$$= 28{\cdot}49.$$

Assuming 1 inch rivets at $3\frac{1}{4}$ inch pitch, there will be six of them in the depth of the cross girder.

The web plate is $\frac{5}{8}$ inch thick, and therefore the resistance to bearing of these rivets will be

$$= 6 \text{ inch} \times \frac{5}{8} \text{ inch} \times 1 \text{ inch} \times 8 \text{ tons } (r = 8 \text{ tons})$$
$$= 30 \text{ tons, which is sufficient.}$$

This will also have to be the pitch horizontally.

This pitch will be carried to a point about 3 feet 6 inches from the end of the girder, *i.e.*, to the fourteenth rivet.

At this point there will be a stiffener and a joint in the web plate. The reason of this joint will be stated later; the stiffener is about under where the outer wheel of a passing engine will generally be.

Beyond this point the pitch may be increased to 4 inches in a horizontal direction.

Stiffeners.—One is provided near the end of each girder, and two in the middle portion of the girder.

These are made of 6-inch × 4-inch Tee steels.

They are merely standard stiffeners.

The masonry, &c., will stiffen the girders a great deal.

PATHWAYS.

These are to be 4 feet wide.

The load is people crowded.

The table of floor loads gives the load due to people as 100 lbs. to 150 lbs. per foot super, for public halls, etc.

This is already converted into equivalent dead load.

It will here be taken as 120 lbs. per square foot.

Therefore the total equivalent dead load due to people

$$= 120 \times 4 \times 5 \text{ lbs.}$$
$$= 2{,}400 \text{ lbs. on the end of each cross girder.}$$

The $M_B = \frac{Wl}{2}$

$$= \frac{2{,}400}{2{,}240} \times \frac{4 \times 12}{2}$$
$$= 26 \text{ inch-tons.}$$

Detail of Design.

The cross girder is carried out to form a cantilever to carry the path. The depth of this cantilever is increased so as to bring the pathway up to the required level.

The pathway is to consist of a slab of concrete 4½ inches thick with a curb of concrete 4 inches thick resting on the top of the arch crown and spandrel filling, and an outer facing lintel of concrete 9 inches deep and 4 inches thick, reinforced with steel bars, and supported at the ends on steel brackets, 4 inches × 4 inches × ½ inch.

The floor slab rests on two angles, 3½ inches × 3½ inches × ½ inch which form the upper flange of the cantilever and are placed inverted so as to be of use as supports to the concrete slab.

The web of this cantilever portion of the girder is the same as the web of the cross girder. The same plate is carried through between the end angles of the cross girder and is continued to the end of the cantilever.

This involves cutting a re-entrant angle at *C* (Plate VII.), and it is therefore desirable to make a joint in the web at some point near to the end of the cross girder so as to avoid a lot of cutting to waste.

A joint is, therefore, made at the first stiffener, 3 feet 6 inches from the end.

The lower flange angles of the cross girder are also carried through beyond the end of the main cross girder, and are bent upwards to form the lower flange of the cantilever portion. At the end they are again bent up into a vertical position and one of them is carried up as a post for a handrail, the other being cut off at floor level.

The end of the cross girder is formed by two upright angles 4½ inches × 4½ inches × ⅝ inch, which lie outside the angles of the lower flange of the cross girder, pass between the angles of the upper flange of the cross girder, and are then carried up to embrace between them the 3½ inches × 3½ inches × ½ inch angles of the upper flange of the cantilever portion.

This cantilever portion is amply strong in itself.

Calculations.

It has been found that the M_B at A is 26 inch-tons.

The depth of the cantilever at the section $A\ B$ is 32 inches.

Now $M_R = fad = 26$

$$\therefore fa = \frac{26}{32} \text{ tons.}$$

This is the horizontal tensile force in the upper flange of the cantilever.

This is resisted by the strength of the upright angles forming the end of the cross-girder to resist cross breaking at C.

This force at A will cause a M_B round C:—

$$= \frac{26}{32} \times 16 \text{ inches}$$
$$= 13 \text{ inch-tons.}$$

The strength of two 4½-inch × 4½-inch × ⅝-inch angles to resist cross breaking is 42·42 inch-tons.

They are, therefore, strong enough.

It is necessary also to ensure that these two angles are held in an upright position sufficiently firmly by the rivets which attach them to the web of the cross girder.

In Plate VII., the pull to the left on the upright angles is applied at A and is $\frac{26}{32}$ inch-tons.

There are four rivets that may be considered as holding these angles upright to the web, so as to resist such a pull, *i.e.*, those marked k, k, k, k.

These have a mean lever arm round B of 12 inches. Hence, if F_t be the maximum safe pull to the left at A the top of the upright angles,

$$\text{then } F_t \times 32 \text{ inches} = \text{strength of four rivets in bearing} \times 12 \text{ inches}$$
$$\therefore F_t \times 32 \text{ inches} = 4 \text{ inch} \times 1 \text{ inch} \times \tfrac{5}{8} \text{ inch} \times 8 \text{ tons} \times 12 \text{ inches}$$
$$F_t = 7{\cdot}5 \text{ tons nearly.}$$

This is much in excess of what has to be resisted, and the uprights are therefore safe.

A ½-inch ballast plate is secured from one cross girder to the next at the ends, to cover the ends of the jack arches. It is riveted to the end angles of the cross girders.

The cross girder will be fixed to the girder underneath it by two bent plates or angle brackets, which will serve to keep it in position.

The rivets on the main girder under the cross girder end will be countersunk, so as to give a flat bearing.

Tie bars through the arches are not shown. These will have to be calculated from the thrust of the arches, and present no difficulties (formula, p. 247).

The design is shown complete in Plate VII.

CHAPTER XVI

PRINCIPLES OF DESIGN OF BRIDGES

All bridges are divided into two general classes, over-bridges and through-bridges.

Over-bridges are those in which the flooring for a railway, or roadway for a road bridge, is carried on top of the girders. These are sometimes called "under-bridges."

Through-bridges are those in which the flooring and roadway are carried between the girders.

The choice as to which of the two is to be used depends entirely on the circumstances of the case, and principally on the question of headroom, or height.

By headroom is meant the clearance required under the bridge or over it.

A railway bridge crossing a road must have a certain amount of clearance underneath the lowest part of the bridge, *i.e.*, under the main girders.

In such a case the floor would be placed between the girders so as to avoid carrying the track or roadway higher than necessary, involving higher banks for the approaches to the bridge, etc.

The same would apply to a canal or river crossing, or over another railway.

It sometimes happens, too, in large cities that there is another line of railway crossing above the bridge on another bridge which makes it necessary to save every inch of headroom, and to keep the bridge floor as low as possible, so that its underneath is flush with the underneath of the girders.

On the other hand, a road or railway line may be descending in a cutting to a river crossing, in which case the road or floor would be carried on top of the girders in order to save going lower than necessary.

Generally speaking, the main girders of under-bridges or over-bridges, which are the same, are closed in together somewhat, coming more immediately under the heaviest loads, and thus greatly reducing the span and weight of flooring, especially in double-line or four-way bridges.

TYPES OF BRIDGE FOR DIFFERENT SPANS.

For very small spans troughs may be laid longitudinally across the interval to be spanned, with a light face plate at each side for sake of appearance (Plate VIII.). This is in reality merely one bay of flooring laid across the gap.

For slightly larger spans, where there is enough headroom, I section rolled steel beams might be placed under the rails, suitably braced up against overturning. A light plate floor attached to the top flanges would complete the bridge.

For spans of 20 feet to 80 feet, such as are the majority of bridges on a line of railway, plate girders are the most economical and the best.

For spans of 60 feet upwards braced girders are common, being used almost entirely for spans of 80 feet to 250 feet.

Other types exist, each having its own particular advantage.

MOMENT OF RESISTANCE.

The M_B diagram of loads is a parabola (approximate).

The M_R is always $= fad$.

Now f is fixed for the material under consideration, and a and d are the two variables.

As the design of bridges grew, the practice of increasing d to meet the increase on M_B towards the centre of the span became very common.

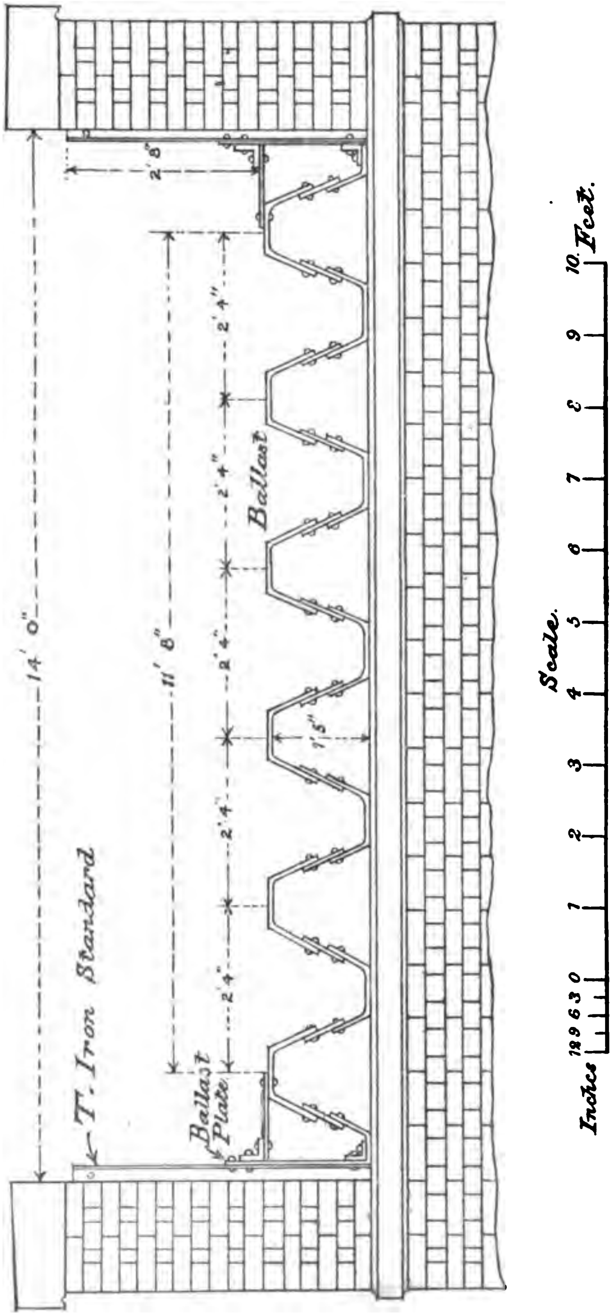
It is obvious, that if a be kept constant, then d will vary as the ordinates of a parabola in order to keep the M_R exactly equal to the M_B.

The variation of d results in the type of bridge known as the "bowstring girder," in which the shape of the girder is the same as that of the M_B diagram.

This is theoretically the most economical design, no web or bracing being necessary at all under a uniform load.

PLATE VIII.

Bridge without Main Girders.



THREE MAIN GIRDERS.

In railway bridges for double line three main girders are generally used up to spans of 40 or 50 feet.

This is more particularly the case when the headroom is limited, as with three girders a comparatively shallow transverse trough floor resting on the bottom booms of the main girders is sufficient to take the traffic of each line.

When more headroom is available, an excellent bridge is formed with three high main girders and cross girders, and light longitudinal troughs.

It is evident that for spans up to 40 or 50 feet a saving will always be effected by using three main girders in preference to two, as although the total weight of the main girders is increased, there is more than a corresponding diminution in the weight of the floor.

Beyond these spans it is generally more advantageous to use only two main girders with trough flooring, or with cross girders and longitudinal troughs, or with cross girders and rail bearers and plate floor.

It must be remembered that in using three main girders the width of the bridge will have to be increased if the main girders extend more than 2 feet above rail level. They should be kept down below this if possible.

POSITION OF FLOORS.

The circumstances of the case will influence and limit the design very much, leaving little choice to the engineer.

Either headroom is all important, in which case the floor members will be on the lower flange of the girders; or, as in the Colonies, or in any new country, when crossing a gorge or river, or on an ordinary high viaduct, headroom does not matter and the roadway may be carried on the top of the booms.

In the former case, whatever the floor consists of, the ends of the troughing or cross girders will nearly always rest on the lower flange of the main booms.

This does undoubtedly produce a torsional stress on the flange of the boom, which is met by making the flange amply strong.

Sometimes an extra large angle steel is used in the lower flange, and where cross girders are used a stiffener is brought down on to the top of each.

The cross girders may sometimes be fixed a part of the way up the main girders, being attached to the web by riveting their ends on to angle brackets (Fig. 254).

When the floor rests on the top of the main booms, there is no difficulty in obtaining a bearing, and this is the most satisfactory method when it is possible to use it.

It is sometimes found necessary to fix cross girders underneath the main boom. This is done by "hangers" or "suspension plates" or bolts (Chapter XIV., Floors).

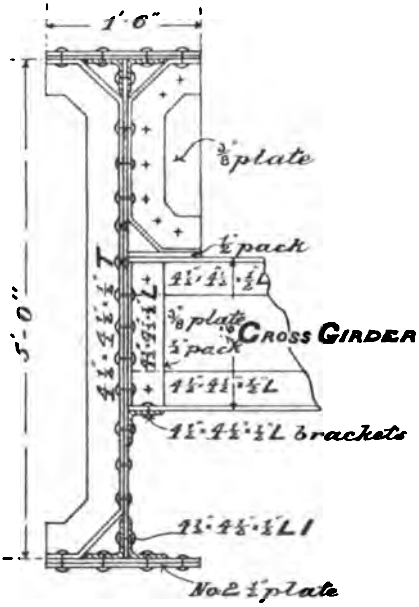

1'·6"
5'·0"
$\frac{3}{8}$" plate
$\frac{1}{2}$" pack
$4\frac{1}{2}$"·$4\frac{1}{2}$"·$\frac{1}{2}$" T
$4\frac{1}{2}$"·$4\frac{1}{2}$"·$\frac{1}{2}$" L
$3\frac{1}{2}$"·$3\frac{1}{2}$"·$\frac{1}{2}$" L
$4\frac{1}{2}$"·$4\frac{1}{2}$"·$\frac{1}{2}$" L
$\frac{3}{8}$" plate
$\frac{1}{2}$" pack
CROSS GIRDER
$4\frac{1}{2}$"·$4\frac{1}{2}$"·$\frac{1}{2}$" L
$4\frac{1}{2}$"·$4\frac{1}{2}$"·$\frac{1}{2}$" L brackets
$4\frac{1}{2}$"·$4\frac{1}{2}$"·$\frac{1}{2}$" L 1
No 2 $\frac{1}{2}$" plate

FIG. 254.

ENGINE LOADS.

In a previous chapter the relative advantages of the two methods for finding the *Max.* M_B due to engine loads have been considered.

These are:—

(a) The uniform equivalent load method, in which the equivalent load is taken from Table XVI. for any modern engine; or if this table is unsuitable, and any particular type of engine has to be considered, the uniform equivalent load can be worked out for that particular engine for all spans.

(b) The method of separate wheel loads.

In both the foregoing methods the calculations are made as if the wheel loads or rails were immediately above the main girders, which is not the case.

We have therefore to consider how the deductions are affected thereby.

Transverse troughing transfers the loads to the main girders very much as a real uniform load.

Cross girders close together do so also, though not to such a great extent.

Cross girders far apart do not; in fact, in such cases the main girders should theoretically be considered as under a uniform dead load, due to their own weight, and a number of concentrated loads due to the cross girders.

This is done in the case of braced girders, the loads occurring at the cross girders, or panel points.

If the case be treated thus, of course the stresses can be dealt with more economically than by any other method.

In practice the difference is small, and for all ordinary cases the load may be considered as transferred very much as a uniform load to the main girders.

SPACING OF CROSS GIRDERS.

With braced girders the cross girders will nearly always be at the panel points and their spacing will therefore be a matter depending on the design of the girder to a great extent.

With plate girders, there are no panel points to consider, and the spacing of cross girders will depend entirely on the spacing of the wheel loads.

In the first place it is always undesirable to place a cross girder at the centre of the span, since this tends to give an increased M_B on the girders.

Further, the object of spacing cross girders close together is to reduce the weight on each; hence it is clearly useless to place them closer together than the spacing of the axles of the heaviest locomotive, since in any case each cross girder has got to carry the heaviest axle load.

Let L be the distance apart of the heaviest axles of the heaviest locomotive.

Then if the cross girders are L feet apart, they will transfer a weight equal to the heaviest axle to the main girders at points L feet apart.

Now let W be the weight per foot run of the engine, *i.e.*, the equivalent load per foot from Table XVI.; then the weight on each cross girder due to this load will be $W \times L$, and each cross girder will transfer a load of $W \times L$ to the main girders at points L feet apart.

Hence if this load $W \times L$ be not greater than the heaviest axle load, L will be the minimum spacing for the cross girders.

Assume that two pairs of driving wheels of 18 tons each are 7 feet apart, and the uniform equivalent load for the span, is 2·67 tons per foot run.

Then we equate the two:—

$$L \times 2{\cdot}67 = 18$$
$$\therefore L = 6{\cdot}7 \text{ feet.}$$

This spacing thus found is less than the spacing of the heaviest axles.

Seven feet is therefore the most economical spacing.

This is not quite accurate, but very nearly so.

If the uniform equivalent load is very small, *i.e.*, in the case of long spans, the spacing might be increased, for increasing the spacing will only slightly increase the load on one cross girder, due to the actual axle loads, but will bring up the uniform equivalent load per cross girder, so as to equal it.

But as a general rule, considerations of skew, length of span and spacing of axles of engine will decide this matter.

In practice 1 to 1½ times the spacing of the heaviest axles is the best arrangement.

SKEW.

When the skew is a very flat one, *i.e.*, the obstacle to be crossed is nearly at right angles to the line, there is no need to make any special arrangements.

The gap is treated as a square gap; that is, although the face of the abutment is not at right angles to the centre line of the bridge, still the one girder is given a little more bearing than the other, either on one abutment or the other, so as to bring the bridge floor square.

This is more particularly so in the case of narrow or single line bridges (Fig. 255).

Of course, the wider the bridge the more waste of girder is entailed by this method.

This is even more common in the case of braced girders than of plate girders. In braced girders it is of very great importance to get panel points opposite to one another, and the method of giving one girder more bearing than the other is the only method of doing so in cases of slight skew.

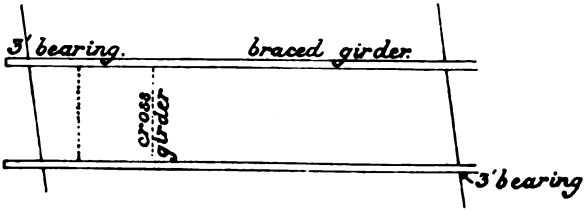


FIG. 255.

Plate girders are easier to deal with inasmuch as they have no panel points and cross girders may be fixed to them anywhere (Fig. 256).

They may therefore either be given an unequal bearing, as described above, or an equal bearing (Fig. 256), in which case, whether troughs or cross girders are used, they may be put straight into the floor, without reference to the skew, *i.e.*, at right angles to the main girders.

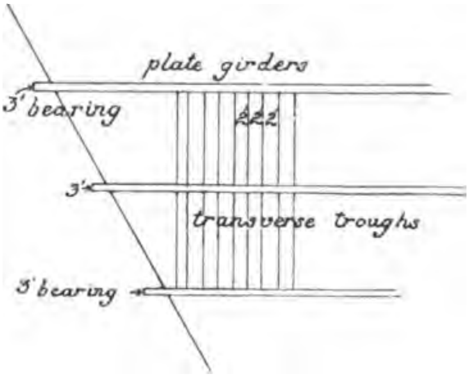


FIG. 256.

When the skew is steeper, or more accentuated :—

It may be possible in this case, with braced girders, to get No. 2 panel of one girder to fall opposite to No. 3 panel of the other girder (Fig. 257).

This arrangement may necessitate moving either girder a little forward or back on its bearings so as to bring the panels exactly opposite to one another (Fig. 260).

Where a three-girder bridge is under consideration, it may happen that No. 2 panel of one outer girder comes opposite to No. 3 panel of the other outer girder, in which case the cross girder will obviously come on to the centre girder exactly half-way between two panel points (Fig. 258).

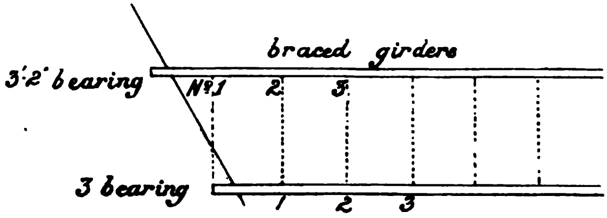


FIG. 257.

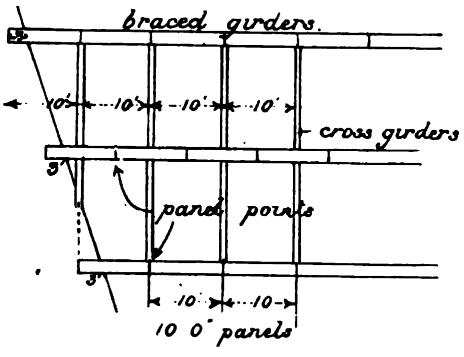


FIG. 258.

The centre girder may therefore either be designed to carry the cross girders in this position, or may be made with a different series of panel points to the outer girders.

For example, in the former case, suppose the girder were a Warren type, as in Fig. 259, then obviously midway between two panel points of the lower boom is exactly underneath a panel point in the upper boom, so that if a tie bar were introduced to transfer the weight of the cross girder to the upper boom, the object in view would

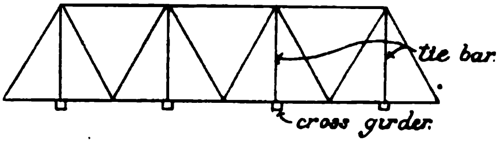


FIG. 259.

be achieved, and there would be no bending moment on the lower flange.

Introducing a cross breaking stress on to the lower flange is, of course, most undesirable.

In other cases a double system of triangulation might be used, or the centre girder might have a half-sized panel at either end. This would bring the panel points of the centre girder opposite those of the outer girders.

To summarise, therefore, for short spans up to 60 feet, it is simplest to use plate girders and then no further difficulties occur. All girders have an equal bearing, and the floor is just the same as for square abutments.

There will be a few members of the floor that do not run the whole width of the bridge, just near to the abutments, but which will have one end resting on the abutment.

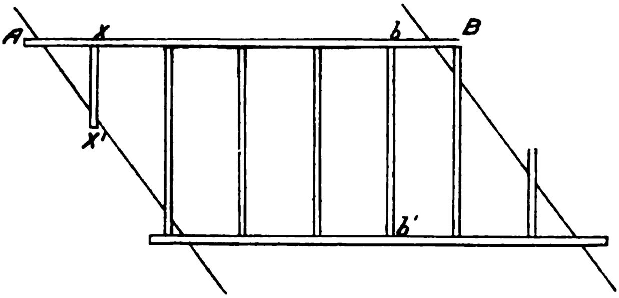


FIG. 260.

But for braced girders for longer spans, exceeding 60 feet, either a little of each girder must be wasted by giving extra bearing at one end or the other, and getting them level in this manner, or the type of girder and the panel length must be arranged so as to bring the panel points opposite to one another.

For spans of 100 feet and over it is very seldom that skew spans will occur.

In all cases of skew it is obvious that one end of each girder will be slightly less heavily loaded than the other and should the skew be at all steep this will have to be allowed for; *e.g.*, in Fig. 260, girder AB has half the total weight on cross girder bb'

to carry, near B, whereas at X it has only half the total weight on the cross girder XX'. This causes a certain amount of reduction of stress at one end of each girder.

Unless the amount of metal saved by considering this reduction of stress is considerable, it is more economical to disregard it.

In a populated country such as England, the numbers of square and skew bridges are about equal. In new countries skew bridges are exceptional.

TROUGHING.

The various kinds of troughing may be divided into "transverse" and "longitudinal," a distinction depending entirely on the method of their use and not on their form.

Transverse.—When the width of the bridge is not great, troughs may be used transversely to the bridge. That is to say, that if the span of the troughs is not great, they are useful, but if the span is great, the depth of section required would lead to too heavy troughs, and they would not be economical.

Broadly speaking, troughs, transverse to the bridge, are used in the floors of bridges with three main girders, the span being small, and they are not used with two main girder bridges, as the span is too great.

Some railway companies do not use troughs at all, or very rarely.

Road on Troughs.—When transverse troughs are used, a great diversity of opinion exists as to whether an ordinary ballasted road with cross sleepers should be laid or whether the rails should be laid without ballast resting on longitudinal timbers, secured on the top of the troughs.

A floor laid without ballast has the following advantages:—

(1) There is less dead weight to carry.

(2) The load of the axles of the locomotives is a little better distributed on to the troughs through longitudinal timbers than on a ballasted road with cross sleepers.

(3) Rainwater runs away more quickly.

(4) The condition of the floor can easily be seen, and it can easily be painted or attended to as required.

The advantages of a ballasted floor are:—

(1) There is no break in the ordinary permanent way, which is a great advantage, especially in the matter of the creep of track.

(2) It is not quite so dangerous as an unballasted floor in the case of a locomotive leaving the track.

Flexible Road and Rigid Road.—A rigid road is one in which the rails are absolutely fixed to the floor of the bridge, and cannot creep with the rest of the permanent way.

When longitudinal timber sleepers are used, the rails are secured through them to the members of the floor, and cannot move; this is a rigid road. Care should be taken to see that it really is rigid if this type is adopted.

The alternative is to lay ballast on the bridge, and to lay the cross sleepers on the ballast in the same way as off the bridge. The rails have the same expansion allowance as the rest of the track, and they can also creep with the rest of the track. In the case of rigid track, platelayers are continually pulling the track back at the ends of the bridge, and much trouble and risk of bad work is caused.

For these reasons it will be found that nearly all engineers prefer the flexible road, *i.e.*, one having no rigid connection to the bridge.

DISTRIBUTION OF LOAD.

The question of the amount which each trough has to carry is a very important one.

For instance, suppose that there is 17·5 tons on an axle which is over the centre of a trough, how much of that load comes on to that trough, and how much on to adjacent troughs?

It is evident that if the axle were placed on the trough without the intervention of rails or sleepers, the one trough would have to carry almost all the load, as the connection between troughs is not stiff enough to allow adjacent troughs to help much.

But when there is a rail between the load and the trough, and also a longitudinal sleeper, the question is quite changed.

The amount of load which the trough will then have to carry varies inversely as the stiffness of the rail and the longitudinal sleeper.

The rail and sleeper distribute the load over adjacent troughs. The amount of this distribution varies with the kind of rail and trough, and depends a good deal on the state of the permanent way.

Not very much has been written on the subject of this distribution, probably because it is rather indefinite.

But many engineers assume that with cross sleepers the load is distributed over three sleepers, and it can be proved that with a locomotive on a transverse trough floor, with longitudinal sleepers, and troughs at 2-feet pitch, not more than three-tenths of the load on an axle can come on any trough even in the worst case, and in the case of a ballasted transverse trough floor, with troughs 1 foot 8 inches pitch, not more than about seven-twentieths of the weight of one axle can come on a single trough.

It has, therefore, been assumed for the bridges that are considered in this book, that half the load on one axle is the maximum that need ever be provided for on any one trough, when the pitch of the trough is not less than 1 foot 8 inches and not more than 2 feet 4 inches, and this assumption will probably in most cases be on the safe side of what actually takes place.

While making this assumption it should not be lost sight of, however, that the amount of the distribution depends greatly on the nature of the troughs, and the rail and sleepers, and is evidently greater with longitudinal sleepers than with cross sleepers.

When a ballasted floor with transverse troughs is adopted the ballast is generally spread so that there shall be a minimum of 3 inches of ballast between the surface of the trough and the bottom of the sleeper.

Some bridges, when the headroom is extremely limited, have the sleepers laid in the troughs with about 3 inches of ballast between the bottom of the trough and the bottom of the sleepers,

By this means about 8 inches or 9 inches can be saved in headroom, but unless absolutely necessary this method ought not to be adopted, as if there is any defect in the drainage of the troughs the sleepers rot quickly; they are also more difficult to renew, as they cannot be drawn out sideways; a rail has to be lifted to get the sleeper out or in.

One great difficulty in transverse trough floors is to make them watertight at their junction with the main girders.

LONGITUDINAL TROUGHS.

These can be used on the top of cross girders when headroom is not of importance. They are generally of small span, and are therefore very shallow in section.

With cross sleepers the distribution of loads upon these is much the same as that in the case of transverse troughs with longitudinal sleepers. It is taken as the same in this book.

DRAINAGE.

The drainage of floors is very important.

It is essential to collect the water that soaks through the ballast and lodges on the floor in rainwater pipes and to take it away properly. If it be allowed to drip it will ruin any road underneath the bridge.

In towns, a light iron roof is sometimes hung under the bridge to take away drippings due to rain or condensation from the bridge floor to the sides of the road.

Trough floors are drained by sloping them one way and collecting the water in ordinary guttering.

It is, however, found that floors leak in every possible place, and recent experiments in covering floors with concrete, asphalte mastic, felt, etc., have not been very successful. The concrete adheres well but is liable to fracture by cracks. Asphalte adheres badly, unless the steel is scrupulously clean.

In English practice the troughs are usually covered with $\frac{3}{4}$ inch of bituminous compound, if they are to be ballasted.

This impervious layer should be turned up at the sides to prevent water that collects from reaching the naked steel.

Plate floors are often covered with a 2-inch layer of concrete and $\frac{3}{4}$ inch of asphalte, and then again $1\frac{1}{2}$ inches of concrete on top of this. The latter is merely a physical protection to the asphalte underneath.

To obtain the best condition of steel for adhesion the sand blast should be used.

Reinforced concrete floors, which are now being largely tried in America, are giving better results than previously, and are cheap, noiseless, and watertight.

In both single- and double-track bridges the concrete has been used by forming it into trough shape across the tops of the main girders, and in some cases it has been cantilevered out beyond the girder to form a sidewalk.

In through-bridges, or bridges where the floor rests on the lower flange of the main girders, the concrete has been carried on the top of rolled steel joists, spaced about 18 inches apart and resting on the bottom flanges of main girders, the concrete being from 4 inches to 5 inches thick, and having in it ½-inch rods running transversely to the cross girders.

In the former case the trough is filled with ballast and the track is laid in the usual way.

In the latter the concrete floor is splayed up against the main girders to form a ballast board, and is covered with ballast in the ordinary manner, a layer of asphalte being usually placed over the concrete before ballasting.

Such floors seem satisfactory, and are quite watertight. There is undoubtedly a great future for this type of construction in floors.

STRESSES.

Good steel shows very little grain. It will stand the same tensile test across the direction of rolling in the mills as with it.

Plate steel for bridges is usually specified to break with a tensile stress varying from 27 tons per square inch to 32 tons per square inch.

The elastic limit for steel is reached at about 15 tons per square inch.

Some engineers allow as low a factor of safety as 2 in the calculations of main girders; 3 is, however, better practice.

When live load is compensated for by the addition of a percentage from Table XIV., the following safe stresses may be adopted for calculations :—

Main Girders.—In tension or compression, 7 to 8 tons per square inch.

In shear, 6 to 7 tons.

In bearing, 8 to 10 tons.

The lower values are usually taken for rivets.

Cross Girders.—In tension or compression, 6 to 7 tons per square inch.

In shear and bearing, as for main girders.

When no percentage is added to the live load, but actual wheel loads are taken, the following safe stresses should be used:—

Main Girders.—In tension or compression—

Up to 20 feet span, 5 to 6 tons.

From 20 to 50 feet span, 5½ to 6½ tons.

From 50 upwards, 6 to 7 tons.

In shear, 4·8 tons per square inch.

In bearing, 6·4 tons per square inch.

Cross Girders.—In tension or compression, 4½ tons per square inch.

In shear and bearing, as for main girders.

Two values are given above in each case.

Which is to be adopted depends on the opinion of the engineer concerned.

Whether 5 or 6 tons per square inch be accepted as correct means a difference of 20 per cent. in the weight of the girder.

The engineer will frequently have to be guided by local regulations or laws, or by the nature of workmanship expected.

If the gauge of the line be very narrow, high speeds are not possible, and the higher value may, perhaps, be taken, since the shocks are not very great. If, however, the bridge is near the entrance to a station where the engines will cross it with the brakes hard on, the shocks on the bridge will be much increased and the lower value should be used.

Each case must be decided on its own merits.

STIFFNESS OF FLANGE.

When flanges are very wide, it is desirable to have an extra L steel under the outer edges of the flange plates to prevent them buckling, as their edges are somewhat weak.

Plate stiffeners tend to promote stiffness of the flange as well as the web.

Rivets also, by keeping the different plates that they join close together, promote stiffness.

Rivets should never be more than 4 inches apart in the compression flange of any girder.

STIFFENERS.

These are required to support the web of plate girders and to prevent it buckling; also to give general rigidity to the girders and to their junction with the bridge flooring.

Stiffeners consist practically of vertical T steel stiffening brackets carried on both sides of the web of a girder.

They are sometimes made of T steels alone bent to fit the shape of the web and flanges, and sometimes of two angle steels placed back to back and similarly bent.

A stronger class of stiffener called a plate stiffener is often used, consisting of two angles bent as described above, but having a plate between them which extends out flush with the edges of the girder and adds greatly to their strength.

SPACING.

In small spans up to 50 or 80 feet, stiffeners are generally put in without making any calculations. They are seldom, except in girders of very short span, put more than 6 or 7 feet apart, and are, of course, much closer together near the abutments, where the shear is greatest, than at the centre.

The general rule is "that the spacing of stiffeners should not exceed ⅜ of the depth of the girder near the abutments and ⅝ of the depth towards the centre."

A number of practical limitations exist in the spacing of stiffeners; *e.g.*, the pitch of rivets, the suitable symmetrical division of the span or length of girder, the position of cross girders in floors where such are used, or the pitch of troughs, etc. Stiffeners are always placed over cross girders; they are stopped ½ inch clear of their top surface, so as to allow the cross girders to be slid underneath them during erection. A ½-inch packing piece is then inserted and the stiffener is riveted to the cross girder.

The same applies to transverse troughing.

These stiffeners when riveted to the floor provide a very strong through bracing to the whole bridge.

When three girders are used the stiffeners on the outer girder must correspond with those on the centre girder, so as to secure continuity of bracing through the floor to the three girders.

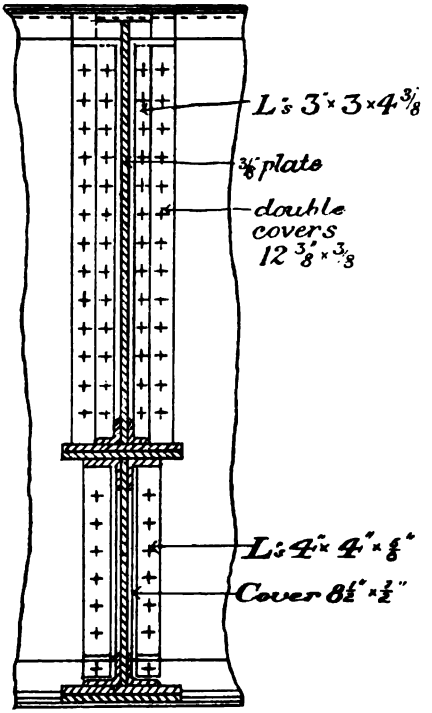


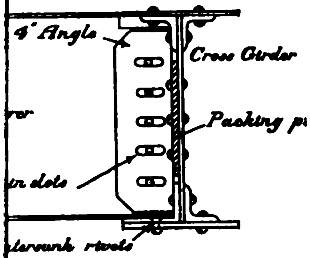
Fig. 261.

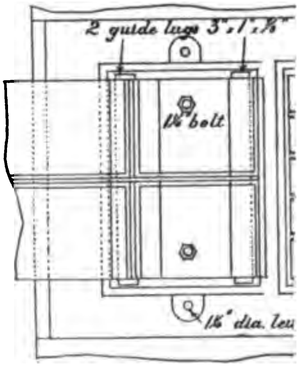
If the bridge be on the skew this will entirely govern the design of the outer girders in this respect.

There is nearly always a plate stiffener immediately over the edge of the bearing plate, and one also over the middle of the bearing plate. These stiffeners together make a strong column at the girder end, which helps to transfer the weight on to the bearings.

Joints in the web are usually made at a stiffener (Fig. 261).

No satisfactory formula has so far been found for calculating the spacing of stiffeners; many attempts have been made, but without success, owing to the indeterminate nature of the stresses in the web. It should be remembered that a web plate is merely an economical arrangement or means of connecting the top and bottom plates of a girder, and it enables the flanges to be built up upon it, and renders them convenient in form and shape for riveting. This device of a continuous web plate is utilised in small spans up to 50 feet, or even 80 feet, on account of its economy and


4" Angle
Cross Girder
rer
Packing p
r slots
tersunk rivets


2 guide lugs 3"×1"×⅜"
1¼" bolt
1¼" dia. lew

convenience. A girder with such a web is called a "plate girder."

But in reality the web is only doing what for larger girders is done by the bracing.

The use of bracing between the top and bottom flanges instead of a continuous plate is due solely to reasons of economy and efficiency.

The web plate of a girder performs in a lesser degree the same functions as the bracing, and it may be considered as a large number of separate strips or laminæ, acting upwards towards the centre of the girder as struts, or downwards towards the centre as ties, while the stiffeners complete the bracing series by acting as ties or struts respectively.

Figs. 262 and 263 give two ideas, the latter being the most likely, of what may be actually taking place in the girder web, the stresses shown representing two forms of N girder.

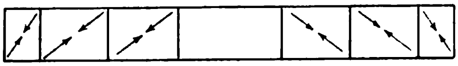


FIG. 262.

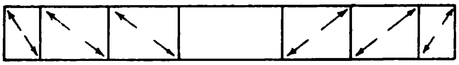


FIG. 263.

It is probable that the tie action of the web plate is what is mostly called into play, while the stiffeners act as vertical struts. In this way the web and the stiffeners together make up a whole and complete series of bracing exactly similar to that in a braced girder, without the trouble of constructing separate members and riveting them up together, as is done in braced girders. The waste of web plate and the indeterminate nature of the web stresses is the only reason for not using this system for all spans.

ENDS OF GIRDERS.

These may be square or round.

Square ends are nearly always used. Rounded ends are only used when the girder is the middle one of three or more, and being in the middle of the track or permanent way, a square end might cause serious hurt to platelayers, etc., at night. They are sometimes used in large girders for appearance. Square ends are finished like a half stiffener.

If the ends are rounded, the flange angles have to be bent, and the flange plates also. The long angles and plates of the girder are very inconvenient to bend, as, owing to their length, they are difficult to handle. A joint should therefore be made in the angles and plates near the end of the girder, so that it will only be short lengths of plate and angles that have to be bent.

The ends of girders should never be encased in masonry, but should be left free of access for inspection and painting. If this is impossible the steel should be thoroughly cleaned and then encased first in cement.

BEARINGS.

In all girders some arrangements have to be made for expansion.

As a rule, for small spans up to 40 feet, the ends of the girders are shod with a shoe of flat steel plate ($\frac{3}{4}$ inch to 1 inch thick) about 3 feet long.

This is bolted to the underneath of the lower flange of the girder end, the flange being riveted with countersunk rivets at this point (Plate IX.).

This plate rests direct on the bedstone of the abutment and is free to slide.

Both ends of the girder may be fixed by Lewis bolts into the bedstones, so as to prevent travel, or the movement of the whole bridge on its abutments. These bolts are free to move in slots through the bearing plate of the girder, at one or both ends.

If desired, 10-lb. sheet lead or felt may be introduced between the girder and the bedstone.

For larger spans up to 60 or 70 feet a similar steel plate is used, but it rests on a cast iron or cast steel shoe, which is fixed to the bedstone.

This gives a smooth surface to slide upon.

Plate IX. gives typical examples of bearings.

For larger spans sliding rocker bearings would have to be used; one end only of the girder would be free to slide.

CAMBER.

Camber is given to a girder to neutralise the deflection, and prevent a locomotive as it runs on to a bridge from running down hill into the middle of the span and then having to run up hill to get off again.

The shock action thus brought on to the girder would be considerable.

It also prevents the girder from appearing to sag. A straight horizontal girder which has no camber always appears to sag.

There are two cambers given to a girder.

(a) The permanent camber.

(b) The initial camber before erection.

(a) The permanent camber is generally 50 per cent. more than the maximum calculated deflection.

Half an inch per 20 feet of span is a good rule of thumb for plate girders.

The camber for plate girders is never shown on drawings but is specified.

For braced girders of small span, there is generally a little more camber given than for plate girders, since there is more yield in the girders.

In the case of braced girders, of less span than 60 feet, camber need not necessarily be shown on drawings, but the accurate lengths of the members of the girder will be worked out again by the makers who are constructing the bridge.

For spans over 60 feet it is desirable to show the camber in the drawings.

(b) Initial camber is given to a girder during construction by the makers, to allow of settlement when the girder is placed on its bearings; with this the designing engineer has nothing to do.

The initial camber is small in the case of plate girders, but in braced girders of large span, may amount to as much as 4 or 5 inches.

This camber is useful for pulling plates and bars into position.

During erection, tension plates may be found to have their holes drilled $\frac{1}{1000}$ inch wrong; this will cause them to sag at their middle, which means that they are not taking their full share of stress. The only cure for this is to take a little of the camber out of the girder so as to pull it tight, and it is for this purpose that the camber is given.

While it is being put together the girder usually rests on skids, and by raising or lowering centre skids the camber can be increased or diminished.

When these skids are finally removed, the girder settles down a good deal, and should then have the exact amount of permanent camber originally specified by the designer.

LINE OF MEAN FIBRE.

Every member or plate may be said to have a line of mean fibre. This is the line joining the centres of gravity of all cross sections through the member.

The line of mean fibre of a uniform plate lies along its centre; that of an equal angle lies along the angle; that of an unequal angle lies somewhere in the longer arm of the angle.

In connecting any member at its end to any other member:—

(a) The rivets or bolts should be disposed symmetrically round the lines of mean fibre of both if possible.

(b) The theoretically best junction point between any two members is the intersection of their lines of mean fibre.

(a) This is not always easy to attain, especially in the case of angles, tees and channels, but it should be striven after as far as possible, since it transmits stresses to or from a member easily and symmetrically.

(b) This is of the utmost importance in all structures, especially braced girders. Thus the junction point of a member of the bracing, and a flange of any girder, is the point of intersection of their lines of mean fibre.

Further, all members meeting in one joint should be so arranged that their lines of mean fibre meet in one point.

This also is not always easy to attain, but must be striven after.

The line of mean fibre of a composite member is of course the mean of the lines of mean fibre of the component parts, *i.e.*, the line joining the centres of gravity of sections through the whole member.

In the design of girders of pin and link type construction, it is of course essential that the pin forming the joint should be on the intersection of the lines of mean fibre of all the members meeting at that point.

CHAPTER XVII

PLATE GIRDERS

EXAMPLE.

To design a three-girder bridge, plate girders being used, for a double line of ordinary standard gauge line.

The clear span is 52 feet, and the floor is to be a plate floor on rail bearers and cross girders; ballast will be 8 inches deep, and longitudinal sleepers will be used, bolted to the rail bearers, making a rigid floor.

The bridge will be a "through" bridge.

The bridge will be made as narrow as possible. With this end in view, the girders must all be kept below the 2 feet limit above the rail level.

The loads to be carried are the heaviest that can be met with in ordinary English practice.

Live load is to be corrected in floors by using $f = 4\frac{1}{2}$ tons.

Refer to Plate X.

RAIL BEARERS.

Live Load.

On these the heaviest axle load is the ruling factor. The heaviest load is an axle weighing 19·4 tons, with the nearest other axles 8 feet 6 inches and 9 feet away.

The cross girders are to be spaced 8 feet apart.

Hence the span of the rail bearers is 8 feet.

$$\begin{aligned} Max.\ M_B &= \frac{Wl}{4} = \frac{19{\cdot}4}{4} \times 8 \times 12 \times \tfrac{1}{2} \\ &= \frac{465{\cdot}6}{2} \\ &= 232{\cdot}8 \text{ inch-tons per rail bearer.} \end{aligned}$$

Dead Load.

Weight of ballast 8 inches deep over plate at 110 lbs. per foot cube on an area 5 feet by 8 feet = $5 \times 8 \times 110 \times \frac{2}{3}$ . . . . = 2,933 lbs.

Weight of $\frac{1}{2}$ inch plate floor at 20·4 lbs. per foot super = $5 \times 8 \times 20·4$ . . . . . . = 816

Weight of half track at 100 lbs. per foot run = 400 lbs.

Weight of rail bearers, assuming B. S. B. 22, 12 inches by 6 inches, by 54 lbs. per foot run, section modulus 62·58 = 54×8 . . . . . . = 432

Total weight . . . . . . = 4,581 lbs.

Therefore the *Max.* M_B due to the dead load

$$= \frac{4{,}581}{2{,}240} \times \frac{8 \times 12}{8}$$

$$= 24·54 \text{ inch-tons}$$

Therefore the total $M_B = 232·8 + 24·54.$

$$= 257·3 \text{ inch-tons.}$$

But $f = 4\frac{1}{2}$ tons,

$$\therefore \frac{I}{y} = 57·2$$

Therefore the beam selected is strong enough.

To test this for deflection :—

$$\Delta = \frac{n' l f}{E\ y} \; (n' = \tfrac{1}{12} \text{ for central load})$$

$$= \frac{1}{12} \cdot \frac{8^2 \times 12^2 \times 7}{13{,}400 \times 6}$$

$$= ·066 \text{ inches}$$

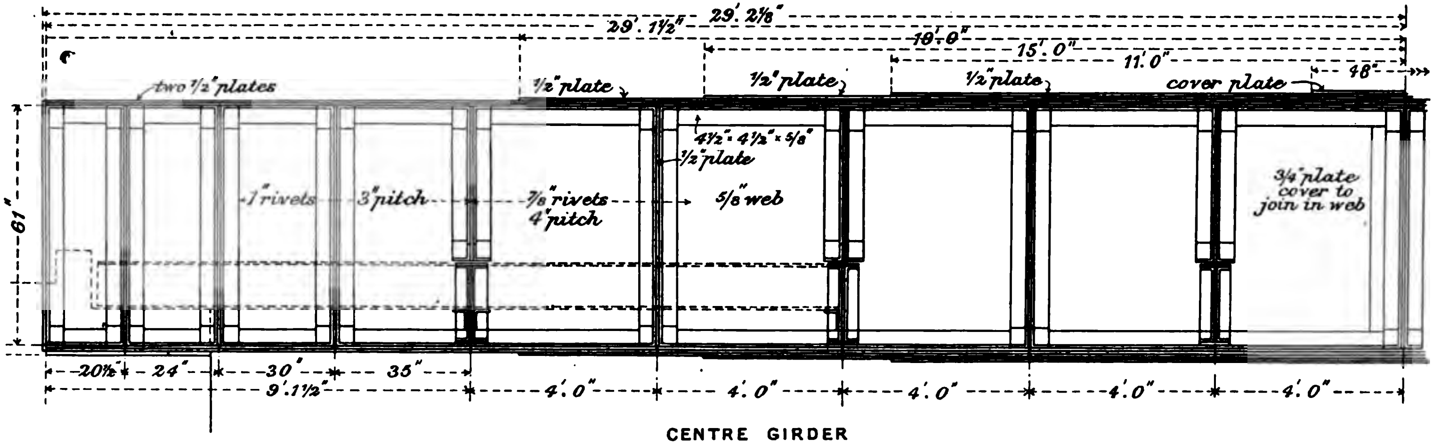
or $\frac{1}{1400}$th of the span, and is safe.

CROSS GIRDERS.

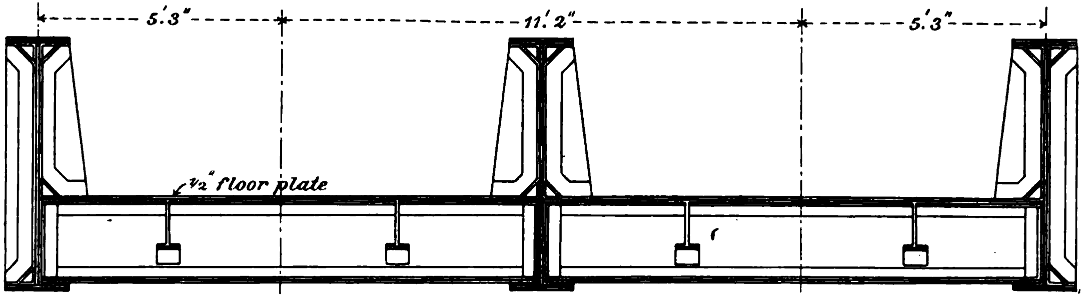
The width of the floor is to be the minimum possible.

Therefore, supposing that the girders do not project more than 2 feet above the rail level, the total width of the bridge will be 20 feet 4 inches between inner edges of girders. Only 6 feet will be left in the clear between the inner two rails.

PLATE X.



CENTRE GIRDER



The flanges of the girders are estimated to be 16 inches wide, and therefore the length of the cross girders will be

$$= 8 \text{ inches} + 20 \text{ feet } 4 \text{ inches} + 8 \text{ inches}.$$

$$= 21 \text{ feet } 8 \text{ inches},$$

and this will be the distance between the centres of girders.

Each cross girder will be 10 feet 10 inches long.

The rails and wheel loads will be spaced as in Fig. 264.

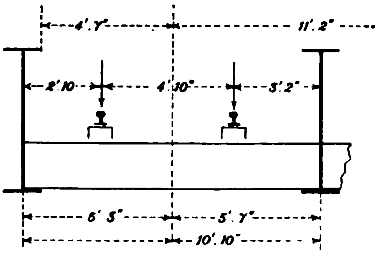


FIG. 264.

All the live loads that the cross girder has to carry are applied through the rail bearers, and all the dead load is assumed to be concentrated there as well.

Dead Load.

The dead load from each rail bearer is $\frac{5,000}{2}$ lbs. approx., and since there are two such at each rail point, the load on the cross girder at each rail point is 5,000 lbs.

Live Load.

The cross girders have been spaced 8 feet apart. This has been done with regard to convenience, and also to the fact that the spacing of the engine wheels is just less than 8 feet.

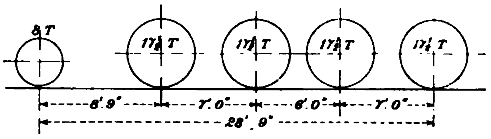


FIG. 265.

The equivalent uniform distributed load for a span of 55 feet is 2·50 tons per foot run; and on a cross girder, this amounts to 20 tons.

The heaviest engine loads on the cross girder amount to about 24 tons, which would point to a slightly wider spacing being better; but this would bring rather a long span and heavy M_B on the rail bearers.

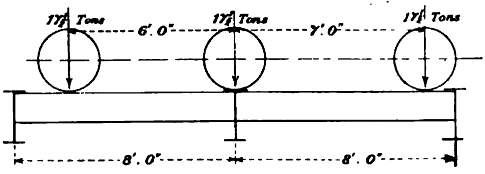


FIG. 266.

The heaviest engine is that shown in Fig. 265.

Fig. 266 also shows how it would be placed so as to get the greatest M_B on the cross girder.

One axle is over the girder, while the other two adjoining axles are on the adjoining rail bearers and bring a portion of their weight on to the cross girder.

The total load is

$$= 17\tfrac{1}{2} + 17\tfrac{1}{2} \times \frac{2}{8} + 17\tfrac{1}{2} \times \tfrac{1}{8}$$

$$= 24{\cdot}06 \text{ tons.}$$

This gives 12·03 on each rail.

Therefore the total load on each rail on the cross girder

$$= 12{\cdot}03 \text{ tons} + 5{,}000 \text{ lbs.}$$

$$= 14{\cdot}26 \text{ tons.}$$

The spacing of the positions of these loads is shown in Fig. 264, quarters of inches being omitted.

The M_B under the outer rail due to its own load only,

$$= 14{\cdot}26 \times \frac{8 \text{ feet}}{10 \text{ feet } 10 \text{ inches}} \times 12 \text{ feet } 0 \text{ inches} \times 12$$

$$= 358 \text{ inch-tons.}$$

M_B under inner rail due to its own load only

$$= 14{\cdot}26 \times \frac{3 \text{ feet } 2 \text{ inches}}{10 \text{ feet } 10 \text{ inches}} \times 7 \text{ feet } 8 \text{ inches} \times 12$$

$$= 383{\cdot}5 \text{ inch-tons.}$$

These values are plotted and the two diagrams are combined (Fig. 267).

The $Max\ M_B$ under the outer rail is 529·5 inch-tons.

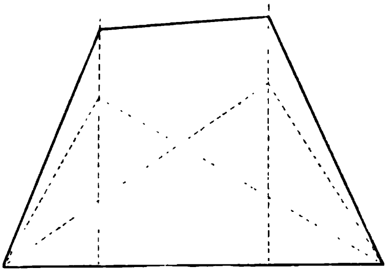


Fig. 267.

The M_R of the cross girder must be equal to this.

If $f = 7$ tons

$ad = 117{\cdot}7$

therefore if $d = 20$ inches, then $a = 5{\cdot}9$ square inches.

We shall therefore use a $\tfrac{3}{8}$-inch plate $7\tfrac{1}{2}$ inches wide, and two angles $3\tfrac{1}{2}$ inches $\times$ $3\tfrac{1}{2} \times \tfrac{1}{2}$ inch.

Deducting two 1-inch rivet holes from the plate and each of the angles, the combined area is

$$= 6{\cdot}56 \text{ square inches.}$$

SHEAR.

The shear on the outer end of the cross girder will be greater than that at the inner end, since the load is nearer to the outer girder.

The shear at the outer end will therefore be

$$= 14{\cdot}26 \times 2 \times \frac{5 \text{ feet } 7 \text{ inches}}{10 \text{ feet } 10 \text{ inches}}$$

$$= 14{\cdot}7 \text{ tons.}$$

The cross girder is 20 inches deep, and at a pitch of 4 inches there will be five rivets in a vertical line at the end of the girder. The intensity of vertical shear will therefore be

$$= 14{\cdot}7 \div 5$$

$$= 2{\cdot}94 \text{ tons per rivet.}$$

$\frac{7}{8}$-inch rivets will be sufficient, using reduced constants to allow for the live load, viz., shearing 4·8 and bearing 6·4 tons,

These will therefore be used, and the pitch of 4 inches will be adopted throughout.

JOINTS (FLOOR).

Rail bearer to cross girder.

The greatest shearing load is brought on to the cross girder by the rail bearer and its loads when one 17½-ton axle is just coming on to the cross girder from the rail bearer, and another 17½-ton axle is 6 feet behind it, on the same rail bearer. The shear then is 22 tons, or 11 tons on one rail bearer. To this must be added the dead load, which is equal to half the dead load on one rail bearer, *i.e.*, 2,500 lbs., making a total of 23 tons (approx.).

Allowing reduced stresses in rivets for live load compensation, viz., same as above, 8 rivets will suffice.

The rail bearer will be secured with two angle pieces on each side of its web, having 3 rivets in each, and also by an angle bracket underneath it having 2 rivets in it. Total 8 rivets.

MAIN GIRDERS.

The load to be taken for the calculations of the main girder is that obtained from the table of uniform equivalent loads.

It will be instructive to compare this with the loads that the cross girders appear to bring on to the main girders, which have been arrived at in the calculations for the cross girders and floor.

The outer girders take slightly more than half the load due to one track, and the centre girder takes slightly less than half the load due to two tracks.

First consider the load imparted by the cross girders.

This is 14·26 tons on each rail, and is made up of 12·03 live load and 2·23 dead load.

The former must now be corrected to reduce it to equivalent dead load. The percentage addition is 14 per cent.

The total load on one rail becomes therefore

$$12{\cdot}03 \times \frac{114}{100} + 2{\cdot}23$$

Therefore the load per cross girder on each outer girder;

$$= \frac{\text{5 feet 7 inches}}{\text{10 feet 10 inches}} \times 2 \times \left(12{\cdot}03 \times \frac{114}{100} + 2{\cdot}23\right)$$

$= 16{\cdot}6$ tons dead load.

and on the centre girder;

$$= \frac{\text{5 feet 3 inches}}{\text{10 feet 10 inches}} \times 2 \times 2 \times \left(12{\cdot}03 \times \frac{114}{100} + 2{\cdot}23\right)$$

$= 31{\cdot}0$ tons.

Now the girders are 58 feet long and have 7 panels, making 6 cross girders. Therefore the total load imparted through cross girders is 31 × 6 tons.

= 186 tons, on the centre girder.

Consider the centre girder only.

This total load is composed roughly of 159 tons equivalent dead load and 27 tons dead load, and is found from the cross girder loads.

In comparing this with the uniform equivalent distributed load, the dead load is of course the same in either case.

The uniform equivalent load for this span is 2·50 tons per foot run.

The theoretical span is 55 feet.

Therefore the total equivalent uniform load is 137·50 tons.

This has now to be corrected for live load by adding 14 per cent.

This amounts to 156·75 tons per track.

This makes the load on the outer girders

$$= \frac{5 \text{ feet } 7 \text{ inches}}{10 \text{ feet } 10 \text{ inches}} \times 156{\cdot}75$$

$= 81{\cdot}80$ tons, and on the centre girder,

$$= \frac{5 \text{ feet } 3 \text{ inches}}{10 \text{ feet } 10 \text{ inches}} \times 2 \times 156{\cdot}75$$

$= 151{\cdot}95$ tons, found from uniform equivalent load for the span.

This is about one ton less per panel than the load arrived at by the cross girders.

The reason of this discrepancy is that the cross girders are assumed to be loaded with the heaviest axle and adjoining loads, and such loading could not occur on every cross girder at the same time. The load as found from the table is the more correct and is the one to take.

These loads are only the live loads.

The dead loads are the same as those brought by the cross girders as stated above, and also the weight of the girders themselves.

Assume the weight of the outer girders to be 12 tons each and the centre girder 16 tons in weight (Table XIII.).

Then the total loads are:—

OUTER GIRDER.

$= 81{\cdot}80 + 13{\cdot}50 + 12$ tons.

$= 108$ tons.

CENTRE GIRDER.

$= 151{\cdot}95 + 27 + 16$ tons.

$= 195$ tons.

OUTER GIRDER.

$$Max.\ M_B = \frac{Wl}{8}$$

$$= 108 \times \frac{55 \times 12}{8}$$

$= 8{,}910$ inch-tons.

CENTRE GIRDER.

$$Max.\ M_B = 195 \times \frac{55 \times 12}{8}$$

$= 16087{\cdot}5$ inch-tons.

The parabolic curve of M_B for both the outer and centre girders is now drawn (Fig. 268).

OUTER GIRDER FLANGES.

Assume two angles 4½ inches × 4½ inches × ¾ inch, and deduct from each two rivet holes 1 inch diameter for ⅞-inch rivets.

The net sectional area of the two is 9·37 square inches.

The M_R due to this is . . . . 3,940 inch-tons.

Assume one plate 16 inches wide by ½ inch thick, and deduct four 1 inch rivet holes.

The M_R due to this is . . . . 2,520 inch-tons.

This gives an M_R . . . . . . = 6,460 inch-tons.

Plot this on the parabola of M_B.

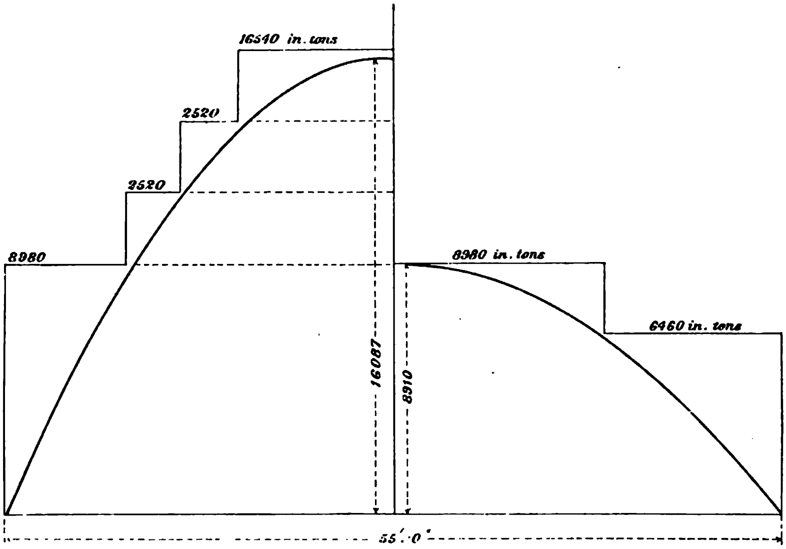


Fig. 268.

From 15 feet on either side of the centre of the span an extra plate will be required.

Assume another ½-inch plate.

This will give an additional M_R of 2,520 inch-tons, making a total of 8,980 inch-tons, which is enough.

CENTRE GIRDER FLANGES.

The above sections will be used throughout from end to end in the centre girder.

They should be plotted on the M_B curve. This will show that from a point 18 feet 6 inches on either side of the centre an extra plate is required.

The extra plates will be three ½-inch plates, making up the required M_R, *i.e.*, 16,540 inch-tons.

These plates are respectively 38 feet, 30 feet, and 22 feet long.

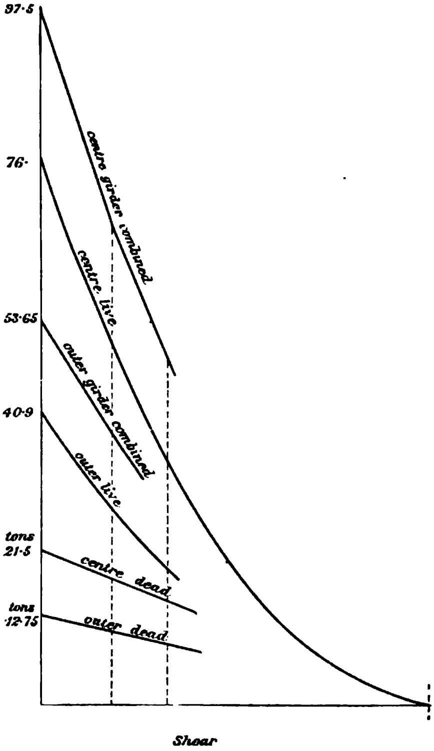


FIG. 269.

SHEAR.

The vertical shears at the ends of the girders are:—

OUTER GIRDER.

$$= \frac{81\cdot8}{2}\,(\text{live}) + \frac{25\cdot5}{2}\,(\text{dead})$$

$$= 40\cdot9\,(\text{live}) + 12\cdot75\,(\text{dead})$$

$$= 53\cdot65 \text{ tons.}$$

CENTRE GIRDER.

$$= \frac{151\cdot95}{2}\,(\text{live}) + \frac{43}{2}\,(\text{dead})$$

$$= 76\,(\text{live}) + 21\cdot5\,(\text{dead})$$

$$= 97\cdot5 \text{ tons.}$$

The shear stress diagram must be drawn for both girders, the curve for the equivalent live load being parabolic, and that for the dead load a straight line (Fig. 269).

CENTRE GIRDER.

The end shear is 97·5 tons.

The shear 5 feet from the end is 67·6 tons.

The mean shear on the end 5 feet of the girder is = 82·6 tons.

The girder is 5 feet deep, therefore there is 16·5 tons of vertical shear per foot run.

With rivets at 4-inch pitch, this gives 5·5 tons per rivet, which is too much.

Reduce the pitch to 3 inches, which gives four rivets per foot run, and the stress per rivet is 4·12 tons.

In bearing, 1-inch rivets will take 5 tons in a $\frac{5}{8}$-inch plate, and for shear 1-inch rivets are safe.

These 1-inch diameter rivets and the 3-inch pitch will be carried as far as 9 feet from the end of the girder.

At this point, 9 feet from the end, the vertical shear is measured from the diagram and is found to be 48·5 tons.

This in a depth of 5 feet is equivalent to a shear of 9·7 tons per foot run. At a pitch of 4 inches there will be three rivets per foot; if these are $\frac{7}{8}$-inch rivets they will be able to take a stress of

4·37 tons each in bearing, and
3·62 tons in shearing.

They will therefore be strong enough.

It should be noted that 1-inch holes have been deducted from the sectional area of the plates and angles, allowing for $\frac{7}{8}$-inch rivets. This is not enough near the ends where the rivets are made 1 inch diameter and the holes $1\frac{1}{16}$ inches.

But near the ends the M_B is very small and the plates employed are ample for the M_B in any case.

Near the centre, where the M_B is greatest, the rivets are only $\frac{7}{8}$ inch as allowed for.

OUTER GIRDER.

End shear = 53·65 tons.

Shear 5 feet from the end is = 37·6 tons.

Mean shear over end 5 feet of girder = 45·62 tons.

This is 9·1 tons per foot run.

And $\frac{7}{8}$-inch rivets at 4-inch pitch are sufficient, if the web is $\frac{1}{2}$ inch thick.

END OF GIRDER.

The end plate is made vertical and is of ⅝-inch plate. There is a bearing plate 1 inch thick, 3 feet × 16 inches, riveted to the underneath of the lower flange over the bearing; the rivets on its underside are countersunk so as to lie flush.

LENGTH.

The whole span is 58 feet, *i.e.*, 52 feet + 3 feet + 3 feet.

This allows 3 feet bearing at either end.

There are several other points, however, which require attention, and which will necessitate lengthening the girder slightly.

If the length be only 58 feet, the last rivet at the ends of the girder will occur with its centre 2 inches from the back of the angle steel which forms the end of the girder.

This will not allow of the rivet being put in; there will not be room for forming the head.

The length of the girder will therefore be increased by ½ inch so as to give more room.

This applies to all three girders.

The end plate is ⅝ inch thick, and this amount must be added to the overall length of the girder, making it 58 feet 2¼ inches long.

The centre girder has also the change of the pitch of rivets to be considered.

Stiffeners must occur at cross girders, and there is one cross girder at a point 20 feet from the centre of the girder. This is, assuming the length found above, 9 feet 1⅛ inches from the outside of the end plate, or 9 feet ½ inch from the outside of the vertical angle at the end of the girder.

The pitch is to change, in the case of the centre girder, at this cross girder. To fit this 3-inch pitch in, it will be found necessary to lengthen the centre girder another 1 inch.

The length from the stiffener to the end will then be:—

= 2½ inches + (3 inches × 35) + 2 inches.
= 109½ inches.
= 9 feet 1½ inches.

This makes a total length of girder 58 feet 3 inches without the end plates and 58 feet 4¼ inches with the end plates.

The centre girder will be 2 inches longer than the two outer girders.

STIFFENERS.

The rule is that they should be spaced about ⅜ of the depth at the ends and about ⅝ the depth near the middle of the span.

They will therefore be 4 feet apart in the middle, those over cross girders being returned on to the cross girders, and the spacing near the ends will be, measuring from the back edge of the vertical angles at the ends, 20½ inches, 24 inches, 30 inches, 35 inches.

This will fit the change of pitch and the spacing of the cross girders.

All stiffeners will be plate stiffeners, with a ½-inch plate.

DEFLECTION.

This works out to ·790 inches, which is safe.

A camber of 1·25 inches, might be given to the girder.

WEIGHT.

The weight of these girders has been worked out and comes to:—

Centre girder = 14·83 tons.
Outer girder = 11·37 tons.

These weights are less than those taken for calculations.

Depth.—On account of the pitch of rivets being 4 inches and the angles being very thick, the depth has been increased from 5 feet to 5 feet 1 inch so as to allow an extra ½ inch at the top and bottom of the girders for the rivets. The rows of rivets in the top and bottom angles will therefore be 2½ inches from the outer edges of the angles.

The details of the floor and connections are not shown in the plate as they are the same as in the examples of floors.

CHAPTER XVIII

PARALLEL BRACED GIRDERS

GENERAL.

Braced girders are girders that have not got a continuous plate web, or, those in which the shearing stresses are borne by diagonal members, called "bracing," instead of by a continuous plate web as in the case of plate girders.

Almost all large girders are braced girders, and these form by far the largest class in use in bridge work.

A braced girder is sometimes called a truss; but in this book the word "truss" is used to express any system of bracing or members, so joined together as to form a complete braced frame.

Braced girders may be of any shape, *e.g.*, arch shape or segmental, with the curved boom placed either on top or underneath, or rectilinear, having the top and bottom flanges either parallel to one another or not parallel.

Parallel braced girders, *i.e.*, those having their top and bottom booms straight and parallel to one another, are by far the commonest in ordinary practice, for spans from 75 feet to 250 feet.

Figs. 270 to 277 show eight types of parallel braced girders.

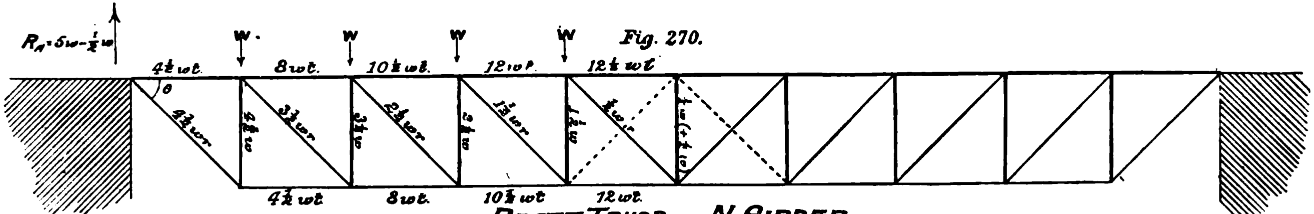
PANEL LENGTH—PANEL POINTS.

The distance apart of the points where the members of the bracing meet the flanges is called the "panel length" and these points are called the "panel points."

The space between two adjacent panel points is called a "panel."

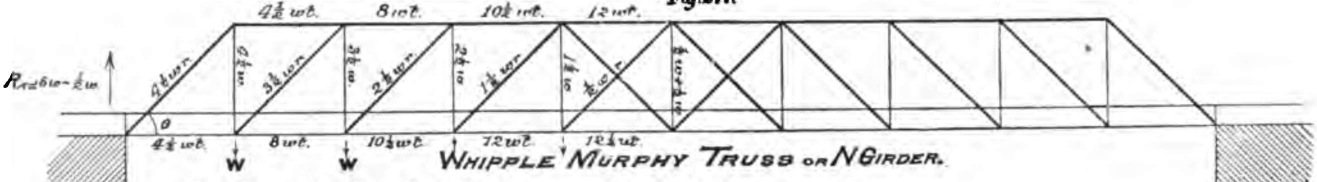
The cross girders of the floor are nearly always fixed at the

Fig. 270.



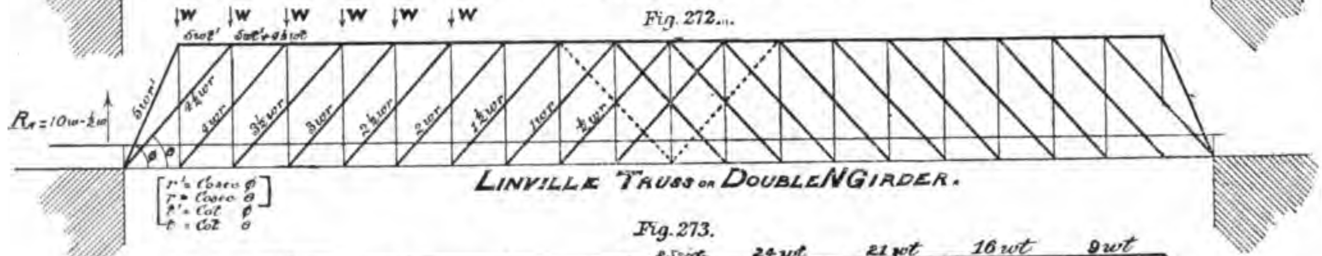
PRATT TRUSS OR N GIRDER.

Fig. 271.



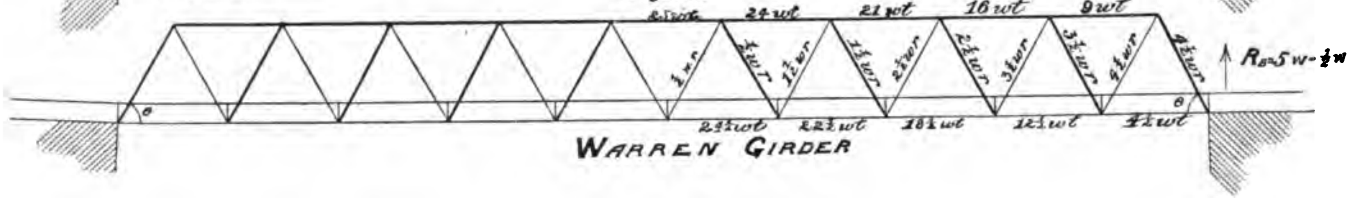
WHIPPLE MURPHY TRUSS OR N GIRDER.

Fig. 272.



LINVILLE TRUSS OR DOUBLE N GIRDER.

Fig. 273.



WARREN GIRDER

panel points so that the loads on any truss are applied at the panel points.

Refer to Figs. 270 to 277.

Figs. 270 and 271 are known as "N girders," the one being for an under-bridge, sometimes called a Pratt truss, and the other being for a through-bridge, sometimes known as the Whipple-Murphy truss.

In the former (Fig. 270), the longer members of the bracing are in tension, while in the latter (Fig. 271), the longer members are in compression.

Fig. 270 is therefore the most economical of the two, when headroom under the bridge allows of its use.

Fig. 272 is a combination of two girders similar to Fig. 271, superimposed on one another, sometimes called a Linville truss.

In order to bring the diagonal members at the ends of the girders on to the same bearing plate, the end struts of one series must be made steeper than the rest.

The Warren girder, shown in Fig. 273, is not so common for small spans, but it is economical in large spans, about 150 feet, when the depth of the girders is considerable. Fig. 275 shows the same girder again, but vertical bars are introduced from the apices of the triangulation or panel points in the upper flange to support the lower flange against cross bending, at the centres of panel points. This is only done when cross girders of the floor must unavoidably occur at unsupported points of the lower flange, *i.e.*, between panel points. Fig. 274 shows a double Warren girder, in which two girders similar to that in Fig. 273 are superposed.

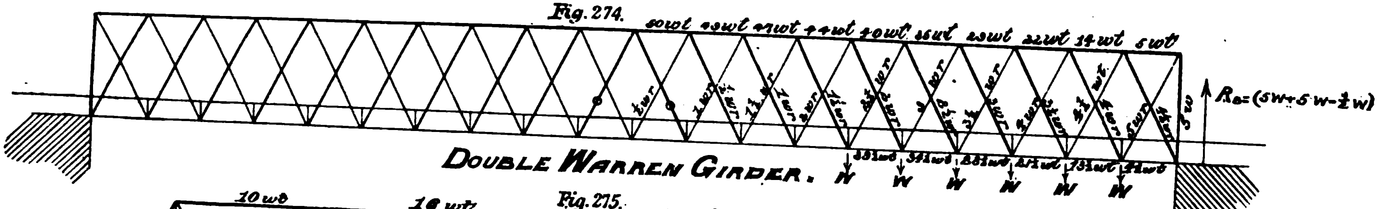
The ends of the upper superposed girder require vertical struts to take the weight from the girder on to the abutment.

Fig. 277 is also a common form.

It is the same as two girders of N-type (Fig. 270) superposed one on the other, and is the exact converse of Fig. 272.

The ends are supported by two vertical members which communicate the weight of the girders on to the abutments.

Fig. 274.


50wt 49wt 47wt 44wt 40wt 35wt 29wt 22wt 14wt 5wt
½wr
1wr
1½wr
2wr
2½wr
3wr
3½wr
4wr
4½wr
5wr
5½wr
3w
W W W W W W
$R_B = (5W + 5w - \frac{1}{2}w)$

DOUBLE WARREN GIRDER.

Fig. 275.

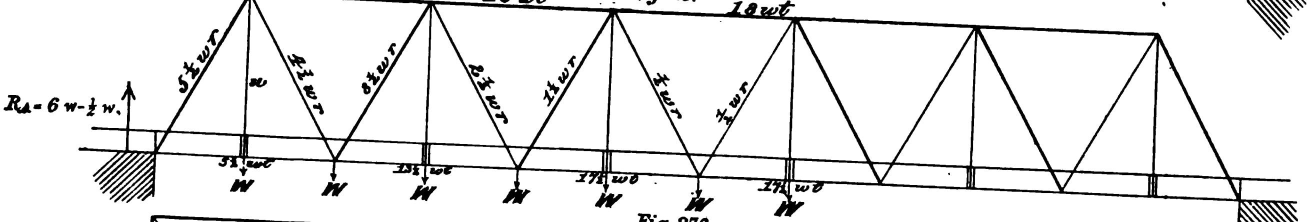
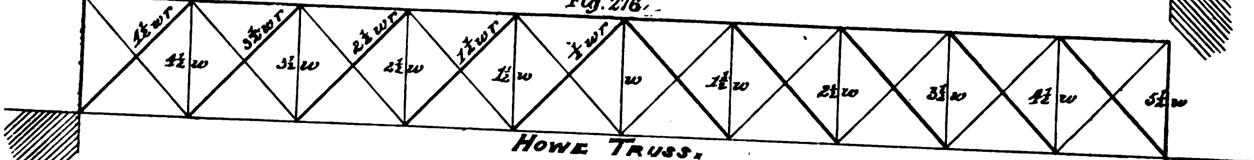
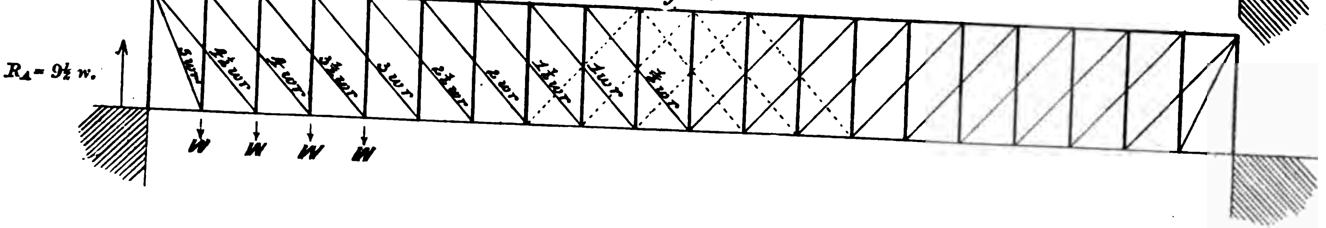


Fig. 276.



HOWE TRUSS.

Fig. 277.



CHANGE OF STRESS IN MEMBERS AT PANEL POINTS.

Consider Fig. 271, which represents diagrammatically a Whipple-Murphy truss, for a certain span, and suppose Figs. 278 and 279 to be the M_B and shear stress diagrams for this span, under a uniform distributed load.

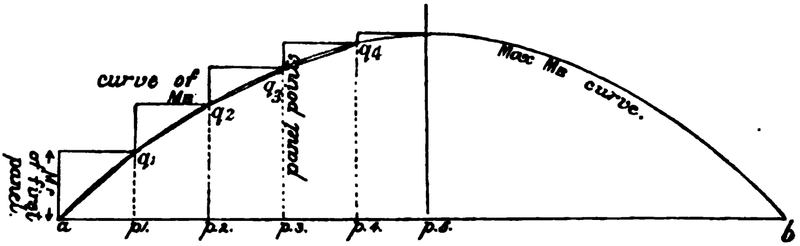


FIG. 278.

The M_R of the girder is constant throughout any one panel, that is, between any two panel points, and any increase or decrease in the value of M_R will occur at panel points only.

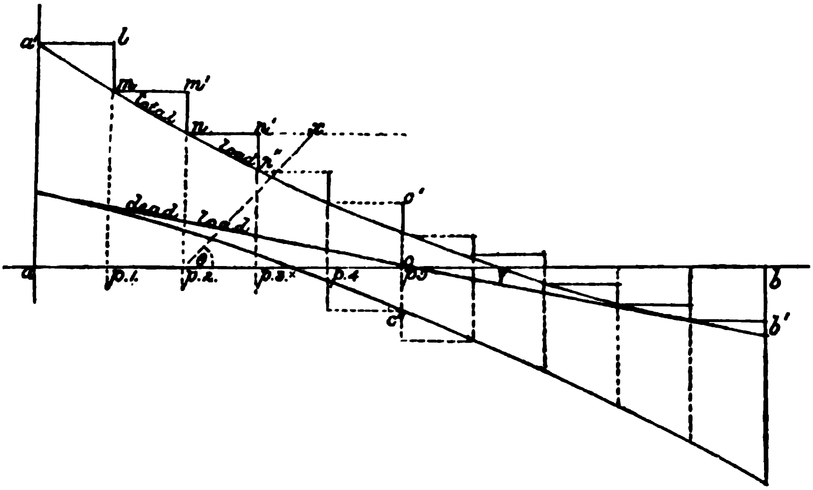


FIG. 279.

For, in Fig. 278, let p_1, p_2, p_3, etc., be panel points. Then p_1, p_2 is one panel of the flange, and the tension in the member

p_1, p_2 of the lower flange must be the same throughout its length; for no additional stress is taken from it or brought on to it, except at its ends where the bracing joins it, *i.e.*, at panel points p_1 and p_2.

The same remarks also apply to the top flange and the diagonal bracing in any panel.

Therefore ordinates of M_R must be constant between these points, or throughout any panel.

If the panel points p_1, p_2, p_3, etc., be projected upwards as in Fig. 278 to cut the M_B curve at q_1, q_2, q_3, etc., then the M_R throughout any whole panel p_1, p_2, must be sufficient to resist the greatest M_B occurring in that panel, which is represented by the ordinate p_2, q_2, in this case, in Fig. 278.

The diagram of M_R for the panel p_1 p_2 must therefore be the horizontal drawn through the point q_2; or in other words, the diagram of M_R throughout the span must be composed of a series of horizontal lines, the ordinates of which change at panel points only.

It should be noted further that the *Max.* M_B at the panel points is all that is required for the purposes of calculation, since we are not concerned with variations of M_B except at panel points.

Now Fig. 279 shows the shear stress diagram, and here again the resistance to shearing of the beam must remain constant throughout each panel, changing at panel points only, and the strength at each panel to resist shear must be made equal to the maximum shearing stress which can occur in that panel.

Hence the shear diagram also is composed of a series of horizontal lines, the ordinates increasing at panel points only.

Therefore, in order to obtain the diagram of M_R or shear resistance, for which a girder is to be designed, it is necessary first to draw the M_B and shearing stress diagrams, and then to determine the size of panels that is to be adopted and mark them off on these diagrams. Then with the aid of these points, the M_B and shearing stress diagrams may be "stepped" by drawing horizontal lines from the points where the vertical ordinates at panel points cut the curves, maximum values only of ordinates being taken.

The complete diagram will be rectilinear or "stepped" in shape, and will just enclose the M_B or shear diagram.

ECONOMY OF BRACED GIRDERS.

In braced girders the top and bottom flanges provide the fa in the formula $M_R = fad$ in the same way as in a plate girder.

The lever arm d is a variable; in plate girders it is best to keep d small, so as to economise the plate of the web, but in braced girders d may be made very large in order to gain M_R.

The shearing stresses are taken by the bracing instead of by the web plate.

For small spans where the shearing stresses are not very large, a plate web is found to be more economical than bracing owing to saving in labour in joints and in plating up, and a plate girder is therefore used; the web also serves as a parapet to the bridge in some cases.

But for long spans and heavy girders the waste metal in plate girders becomes very great, owing to the greatly increased thickness of the web required to obtain sufficient bearing area for rivets, and the large number of vertical stiffeners required to ensure stiffness against buckling.

This is exceedingly wasteful, and great economy is effected by the use of bracing instead of a plate web, and the dead weight of the girder is much reduced thereby. For spans of 60 feet and over, therefore, the braced girder becomes more economical than the plate. The bracing also possesses another great advantage over a plate web in that the stresses are determinate.

As has been previously stated, the stresses due to shear in a plate web are indeterminate and must remain so, whereas the stresses in each member of the bracing can be accurately determined and the member can be designed in minute accordance with them.

The importance of this point in large structures hardly needs any emphasis.

LOADING.

All loads on braced girders should be concentrated and applied at panel points whenever possible.

When the loading rests on a floor this is nearly always built up on cross girders, which are fixed to the main girders at panel points.

It is only in exceptional cases that any loads will be brought upon the flanges of a braced girder between the panel points.

When this occurs, cross bending in the flange itself is produced and must be allowed for, the vertical load, of course, distributing itself proportionately between panel points.

The dead load of the floor itself is also applied through the cross girders at the panel points.

For the calculation of stresses in members the dead load of the main girder itself must be considered as being divided up and applied at panel points, the load at each panel point being that due to half a panel on each side of it.

The dead weight of the members of the main girder will also produce a cross bending tendency in themselves, which although negligible in small or medium spans may be most important in large spans of 250 feet or more.

Briefly stated.—All loads must be considered as concentrated at panel points, and in the case of those which are clearly not so in reality, additional allowance must be made for their cross bending effect on the members on which they rest.

The whole loading may, therefore, be generally treated as a series of concentrated loads applied at the panel points. If then the parabolic M_B curve be drawn as in Fig. 278, it must be analysed thus:—

Divide up the base line, or span line, into the proposed number of panels for the girder at p_1, p_2, p_3, p_4, etc. Erect vertical ordinates at p_1, p_2, p_3, etc., and join aq_1, q_1q_2, q_2q_3, q_3q_4, etc.

This diagram of straight lines is the theoretical diagram of M_B, the M_B decreasing uniformly from one point of application of a load to the next, being a M_B diagram for a number of concentrated loads.

At the same time the maximum value of M_B in any one panel, say p_2q_2, in panel p_1p_2, is what has to be met by the M^R of the truss, and hence the M_R in panel p_1p_2, must be equal to p_2q_2 throughout the panel.

Similarly if Fig. 279 be the diagram of maximum shearing stress on the girder, it may be analysed by dividing up the span line into panels and projecting them upwards as before.

Then the greatest shear that may occur at any point p_3 is represented by p_3n'', and at p_2 by p_2n; each of these occurs when the head of the load is advancing across the span, from right to left, and has just reached the point p_3 or p_2.

Hence the shear stress in panel p_2p_3 varies in magnitude between p_2n and p_3n'', and the girder must be able to resist the maximum value (p_2n) throughout the panel p_2p_3. Draw nn' horizontally; this line with other similar lines will give the stepped diagram of shear resistance required.

M_B AND SHEAR DIAGRAMS.

The methods of drawing the M_B and shear stress diagrams due to the loading given, require more careful elaboration than those for a simple plate girder of small span, especially as regards shear stresses, but the general principles are exactly the same.

METHOD OF ACTUAL WHEEL LOADS.

The best and most accurate way to draw both the *Max.* M_B and shear stress diagrams is to select by trial and error the combination of engines that will produce the greatest M_B or shear upon the span and to draw the diagrams direct from them, when they are placed in the most disadvantageous position on the span.

For the *Max.* M_B diagram the most disadvantageous position must be found by trial and error, for each point on the span.

That position of the load which will produce the greatest M_B near the centre of the span is not necessarily the same as that which will produce the greatest M_B at other points of the span.

For the shear stress diagram the greatest shear occurs at every point of the span, as the head of the advancing load reaches that point, and decreases again as the head of the load passes over that point.

This process is in reality the process which has been termed the "method of actual wheel loads," and is the most reliable method for any span.

METHOD OF UNIFORM EQUIVALENT LOADS.

But in the design of bridges or structures for ordinary rolling stock it is not always easy for the designer to obtain the weights of all engines, etc., and it means a great deal of labour to select the ruling combination of engines and their most disadvantageous position on the span; this has, therefore, been done and the results have been tabulated in Table XVI. in the form of "uniform equivalent loads" in tons per foot run, which if distributed along the span under consideration, will give the same *Max.* M_B and M_B at all points as would actually be produced by the heaviest engines, and will also give the greatest shearing stress at all points during the passage of the load over the span.

In using Table XVI. the diagrams of M_B and shear are drawn of parabolic shape, as if the load were uniformly distributed, and the values of M_B and shearing stress found at panel points are those which are required for use.

As the design of engines alters, Table XVI. may, and must, of course be altered in order to keep it up to date, and for this reason is liable to prove a snare unless used with circumspection.

The equivalent distributed loads given in Table XVI. only apply to heavy rolling stock, on the standard 4 feet 8½ inches gauge, and are consequently of limited application, even when they may be relied upon as being up to date and abreast of modern locomotive design. Table XVII. contains the uniform equivalent load for metre gauge and 5 feet 6 inches gauge.

When the diagrams have been drawn either by the method of actual wheel loads or by using the uniform equivalent distributed load, the span lines must be divided into panels and the diagrams must be stepped, as explained above.

Max. M_B DIAGRAM.

The ordinates of the stepped M_B diagram represent the values of M_B that have to be met in each panel, and

$$M_B = M_R = fad.$$

Hence if d, the depth of the girder be known, then fa is also known and is the stress in the longer or "*bearing*" flange in the panel.

By the term longer or "bearing" flange is meant that flange from which the end diagonal, or bracing member, springs.

In Fig. 270 the upper or compression flange is the bearing flange; in Fig. 271 the lower is the bearing flange; in Fig. 273, also, the lower is the bearing flange, and in Fig. 277 the upper is the bearing flange.

To obtain the stresses in the other flange which does not extend the whole length of the span, a lesser M_B diagram is quite simply drawn.

In Fig. 285 the outer diagram $a\ l\ m\ n\ o\ r\ s\ t\ .\ .\ .\ b$, represents the M_B in the bearing flange, while the inner stepped diagram $a\ p_1\ m\ v\ o\ w\ .\ .\ b$ represents the M_B in each panel of the shorter flange.

In composite girders each system of bracing must be considered separately, and the stresses found must be superposed or added together for each portion of the flange.

In the case of a Warren girder (Fig. 273), the panel points of the shorter flange are different to those of the longer or bearing flange and the diagram must be divided up accordingly.

SHEAR DIAGRAM.

The maximum shear stress in the verticals and diagonals is obtained from the shearing stress diagram.

The ordinates of the shearing stress diagram represent the vertical shear in any panel, and also, therefore, the stress in any vertical member in that panel.

To obtain the stress in an inclined or diagonal member it is only necessary to multiply the vertical shear by cosec θ, where θ is the angle of inclination of the member to the horizontal.

This may be done arithmetically, or graphically as in Fig. 279, where p_2x represents p_2nX cosec θ and is the stress in the diagonal member of the panel p_2p_3.

Generally stated diagonal members inclined upwards from the abutment towards the c.g. of the load or loads are struts, and those inclined downwards from the abutments to the c.g. of the load or loads are ties.

ANOTHER METHOD.

If the panel loads are known time may be saved by writing down the stresses in each member direct by the rules stated below.

This will give the stresses in the flanges and the bracing under a uniform or steady load, considered as concentrated at panel points; but the stresses in the bracing so found will not be the maximum stresses that will occur in the bracing as the load advances on to the span, but are only the stresses that will occur in the bracing when the whole span is loaded.

The shearing stress diagram will, therefore, have to be drawn as before, but the flange stresses written in by the rules below are the maximum values and may be used for the design of the flanges.

Let *w* be the load per panel point consisting of dead load and live load.

w may be found either by taking half the load which has been used for the calculation of any cross girder on the bridge floor, as the weight per panel point of the main girder, or by taking the load per panel point due to the uniform equivalent distributed load from Table XVI. and adding to it the dead load of the main girder and flooring per panel.

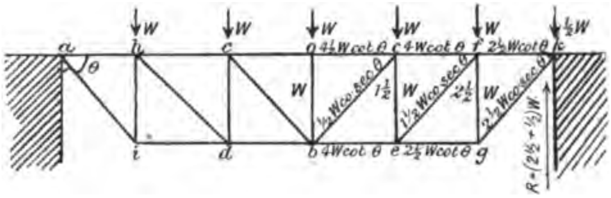


FIG. 280.

The latter is the more accurate of the two, if the Table is correct for the case under consideration, but not otherwise.

Let *w* be the load at each panel point of the girder.

Refer to Fig. 280 showing an N-type girder.

RULES.

(1) Write on each inclined bar or member, cosec θ or some symbol such as "r" and on each horizontal bar cot θ or some such symbol as "t."

(r = cosec θ and t = cot θ).

(2) For the stresses in the bracing start with the reaction at either abutment due to the girder and its loads, and work towards the centre of the span, writing along each bar in the bracing "the reaction minus any loads which come between such bar and the abutment."

(3) On any horizontal bar, write the sum of the horizontal components of all the bars meeting it at its end nearest the abutment.

(4) All bars sloping upwards towards a vertical through the c.g. of the live and dead loads are struts; those sloping downwards towards this line are ties.

Vertical bars are ties or struts according to the stresses in the sloping bars.

It should be noted that half the load on the end panels of any girder is borne by the abutment, and not by the girder.

Thus if a girder (Fig. 280) has 6 panels there will be 7 panel points, *i.e.*, two ends, and 5 intermediate points.

$\frac{1}{6}w$ will rest on each of the five intermediate panel points, and $\frac{1}{12}w$ will rest on each end panel point, *i.e.*, on the bearing plate of the girder on the abutments. These two end loads need not be considered further, since resting on the abutments they do not concern the design of the girder.

The girder really only carries $\frac{5}{6}$ of the total load.

The reaction, therefore, under each end of the girder is half of $\frac{5}{6}W$ or $\frac{5}{12}W$, though at the same time the total weight on each abutment is $\frac{6}{12}W$ or $\frac{1}{2}W$.

These rules apply to all parallel girders supported at both ends.

They do not apply to cantilevers or continuous girders. They give the stresses that do occur in the flanges and bracing of a girder under a uniform load resting on the bridge.

Now since the equivalent uniform load has been used from

Table XVI. the M_B stresses or flange stresses found are the maximum that can occur under any rolling stock, as stated in Table XVI.

But they do not give the maximum stresses in the bracing.

For the stresses in the bracing are shear stresses, and we know that the maximum shear stress at any point does not occur when the bridge is fully loaded, but when the head of the advancing load reaches that point. Hence if maximum stresses in the vertical and diagonal members are required, the rules are of no use, and a maximum shear stress diagram must be drawn for a uniform load advancing on to the span.

This uniform load is of course the same as that used for M_B.

The shear stress diagram must be stepped in panel steps as described before.

The stresses so found will in every case be greater than those arrived at by the rules.

Girders with a double series of triangulation such as shown in Fig. 272 would be treated in a similar manner, each system being dealt with by itself.

In writing the stresses on the members of such girders, the reaction at the abutments due to each series of triangulation or panels must be found separately, and used as the basis for writing down the stresses.

The flange stresses would of course be taken as the sum of those in the two systems.

In fact the two systems are analysed quite separately, each system taking its own proportion of the load, and the stresses so found are added together, where they co-exist in the girder.

In some types there will be one bar or member of the bracing at the end which is at a different inclination to the others; this merely means a different value of θ in that particular bar (Fig. 272).

Double series of triangulation give greatly reduced stresses in the members of the bracing and for large spans are very common.

All the types of girders given in Figs. 270 to 277 have their solutions written on them; they are all perfectly straightforward.

Where double systems of triangulation are employed, w the load per panel has only half the value it would have with a single series.

PROOF OF THESE RULES.

The proof is very simple being based on the consideration of a trussed beam.

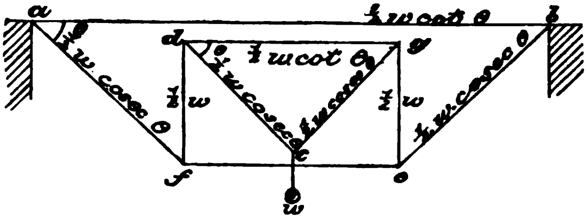


FIG. 281.

Consider a truss as in Fig. 281.

w produces $\frac{1}{2}w$ cosec θ in cd and cg, and the stresses are passed on to the abutments at a and b as shown.

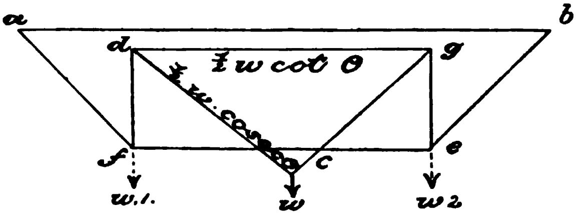


FIG. 282.

Now suppose two more loads w_1 and w_2, at e and f as in Fig. 282.

Then there are still the forces $\frac{1}{2}w$ cosec θ in cd and cg, and

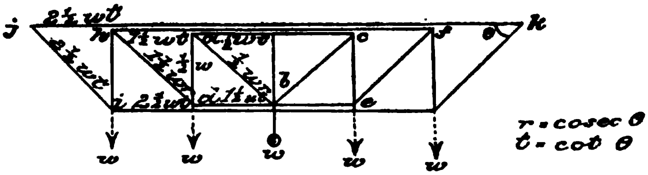


FIG. 283.

$\frac{1}{2}w$ cot θ in dg, just as before (these remain unaltered since w_1 and w_2 would not be affected if the whole middle portion dcg were removed).

Also stresses in *fd*, *eg*, remain $= \frac{1}{2} w$.

But stresses in *af* and *eb* become $(\frac{1}{2} w \operatorname{cosec} \theta + w_1 \operatorname{cosec} \theta)$
and $(\frac{1}{2} w \operatorname{cosec} \theta + w_2 \operatorname{cosec} \theta)$
respectively.

Now if $w_1 = w_2 = w_3$:

Then stress in *af* and *eb* $= 1\frac{1}{2} w \operatorname{cosec} \theta$
and stress in *ab* $= 1\frac{1}{2} w \cot \theta$.

Fig. 283 goes one step further.

Now suppose all the parallel bars at top and bottom to be united into one of varying thickness, it then becomes the case from which the rules are derived.

REVERSAL OF STRESS AND COUNTERBRACING.

When the shear stress diagram has been drawn for any girder and divided into panel points and panel steps, it will be found that near the ends of the girder the shear is either "always positive," or "always negative," although at one time it may be great and at another it may be small, depending on the direction in which the load is moving. But between certain points, x, y, in Fig. 279 the shear may be either positive or negative, the magnitude being given by the diagram.

If these points x and y occur in the middle of a panel, the whole panel is of course affected thereby; *e.g.*, in Fig. 279 reversal occurs in the four centre panels. Stated in plainer words, it means that the vertical and diagonal members in these four panels may be struts at one time, and ties at another time, depending on the direction of motion and position of the load.

Some members may have at one time a heavy compressive load to carry, changing at another moment to a light tensile load or *vice versâ.*

Now the usual way to deal with this is:—

(1) When struts may be called upon to carry a light tensile load, they should be designed of such section that they have a large factor of safety when in compression, and may be allowed to take the tension when required.

(2) When ties may be called upon to take compression, they should not as a rule be designed as struts, but counterbracing

should be resorted to, so as to relieve them of the compressive stress.

(1) This course is simple and needs no further explanation.

(2) This means the introduction of extra members into each panel, which will act as ties across the other diagonal of the panel.

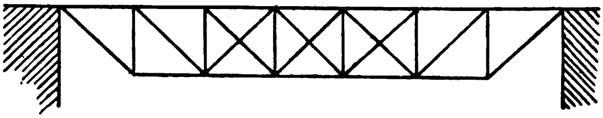


FIG. 284.

For example, in Figs. 270 and 284, the sloping bars are ties, and would have to take compression, in the centre panels, under certain loading.

This is avoided by introducing tie bars crossing the panels along the other diagonals, and thus preventing distortion of the panel, and also preventing any compression being thrown on to the sloping tie bars.

GENERAL REMARKS.

One most important point in design must here be emphasised. All members, whether diagonal members or portions of the flanges are subject to direct stresses only, communicated to them at the joints at their ends.

Any transverse stress placed upon them between their ends is most injurious and undesirable.

For instance, suppose that a portion of the upper flange which is a strut in direct compression, is loaded transversely at its middle point with some load, causing M_B in it; such a condition of loading would be undesirable, the bending tendency helping the strut to fail by buckling. For this reason all loads must be gathered and applied at panel points, and the rivets or connections of joints must be arranged round the line of mean fibre of the members so as to apply their stresses symmetrically and without lateral bending moment.

As has been pointed out before, the M_R and shearing strength of the girder do not increase gradually as in the case of a plate girder, but by jumps at each panel point, and the shearing stress diagram gives the vertical shear at any point, and not the actual shear in the bracing.

On reference to Fig. 270 (N girder), it will be seen that the horizontal stress in the lower flange in any one panel is less than that in the upper flange in the same panel. The difference between these two amounts is equal to the horizontal component of the stress in the bracing in that panel.

For example take No. 2 panel from the left in Fig. 270.

The compression in the upper flange is 8 wt.

The tension in the lower flange is $4\frac{1}{2}$ wt ($t = \cot\theta, r = \text{cosec}\,\theta$).

But 8 wt − $4\frac{1}{2}$ wt = $3\frac{1}{2}$ wt which is the horizontal component of the stress in the bracing, viz., $3\frac{1}{2}$ wr.

This may be seen graphically in Fig. 285.

There, *a l m n o r s t . . . b* represents the diagram of stress in the upper flange.

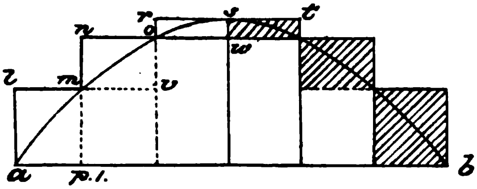


FIG. 285.

Now from a to p_1 there is no lower flange (from the nature of the girder), and therefore the stress is nil in the lower flange.

At p_1 the stress in the lower flange begins, and it is represented by a curve $a\ p_1\ m\ v\ o\ w$ . . etc.

Now the difference at any point between the stress in upper and lower flanges, or in other words the difference between the ordinates at any point gives a value at the same scale of the horizontal shear at that point.

The shaded portion in Fig. 285 is virtually a shearing stress diagram for a uniform load, and the value of this horizontal shear in any panel must be multiplied by sec θ to obtain the stress in the bracing in that panel.

RULES FOR CANTILEVERS.

The foregoing rules can be applied to cantilevers with parallel flanges thus :—

1. As before.
2. As before, but for " centre of span " read " free end of span."
3. In horizontal bars start at the free end and work towards the abutment, and write on each horizontal bar the sum of all the horizontal components of all the bars meeting it at its end furthest from the abutment.

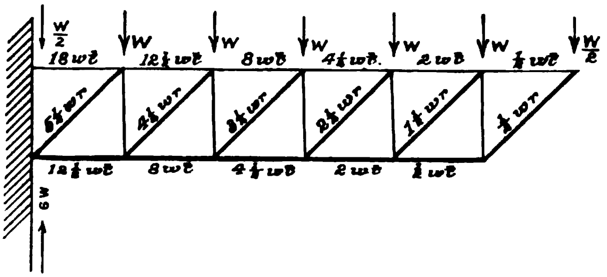


FIG. 286.

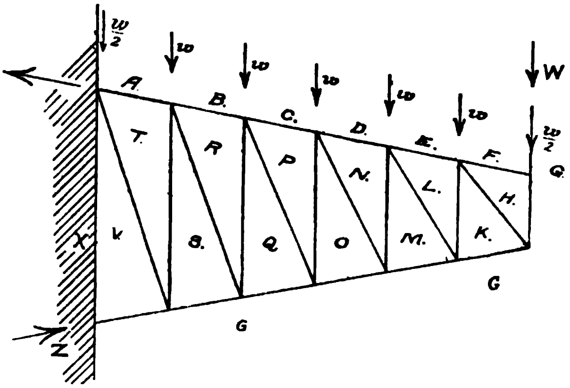


FIG. 287.

4. Bars sloping upwards towards the abutment are ties, and bars sloping upwards away from the abutment are struts.

Fig. 286 shows these stresses written on the members.

The retaining forces, or thrust against the abutment of the lower flange, and the pull away from the abutment of the upper flange, are equal and opposite to the stresses in the flanges in the end panel nearest the abutment, plus the horizontal component of the stresses in the end bracing.

The most useful application of the above rules is in solving the cantilever portions of fixed or continuous girders.

In the case of cantilevers, the stresses given in the diagonals are correct, and are the maximum shear stresses that can be caused in the bracing by any position of the loads.

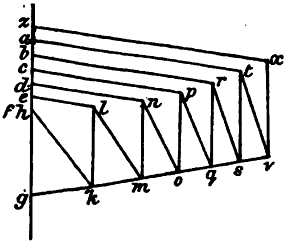


FIG. 288.

The equation giving the shear at any point on a cantilever distant x from the support, when the head of the load has reached that point, is:—

$$S = R_A = w\,(l - x),$$

which is the equation to a straight line; and the curve of shear ordinates at all points is a straight line with a minimum ordinate $= O$, and a maximum ordinate

$$= wl \text{ when } x = O,$$

i.e., the maximum shear stress curve is the same as the shear diagram for a uniform distributed load.

In practice cantilevers with parallel flanges are rarely used, but the depth is increased near the abutments in order to give increased M_R and for such a truss as Fig. 287 the rules given above would not apply.

Cantilevers which are not parallel are most easily solved by stress diagrams (Fig. 288), and the solution is not given in this book.

FIXED AND CONTINUOUS BRACED GIRDERS.

It has been shown at some length in Chapter III. how to find the *Max.* M_B in fixed and continuous beams, and also how to locate the points of contraflexure by M_B diagrams.

We can also write in the stresses by the rules given above, having once fixed the points of contraflexure.

These, however, must be known, and continuous girders should be hinged or so connected at these points as to be able to transmit vertical shear only and not M_B.

Take for instance the fixed girder *AB* with 10 panels, shown in Fig. 289, with a load *w* at each panel point.

First find the points of contraflexure *C* and *D*.

These are known to be nearly $\frac{1}{5}$ of the span from *A* and *B*.

Then write down the stresses on the girder between *C* and *D* as if supported at these points, and on the cantilevers *AC*, *BD*, as being loaded with half the weight on *CD* at their extremities as well as the load *w* at each panel point.

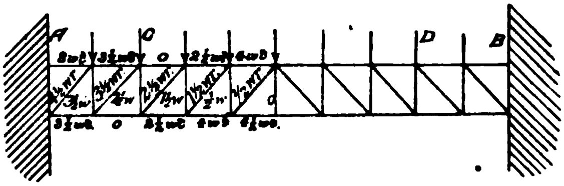


FIG. 289.

The solution is shown on Fig. 289.

Fig. 290 shows another variation of the same type of girder.

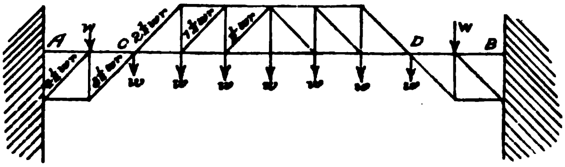


FIG. 290.

If preferred, the M_B and shear diagrams can be drawn as explained for continuous beams in Chapter VIII. from the actual wheel loads, and then stepped off to correspond with panel points.

Such girders are rare in practice under moving loads, except for very large bridges, or for swing bridges.

They are, however, very useful in the design of large span roofs where nothing but a uniform load has to be dealt with.

DEFLECTION.

Deflection of all braced girders is calculated as for plate girders.

$$\Delta = \frac{n'fl^2}{Ey}.$$

STRUTS IN GIRDERS—FIXATION.

As a general rule all struts in braced girders are made the same width as the flange of the girder.

A strut is generally composed of several angles or channels or plates, and is joined up together into one whole for sake of stiffness, by joining the various component angles and other sections together by plates, and flats, bars, etc.

It will be found that such struts are almost invariably amply strong to resist buckling at right angles to the plane of the girder.

But the dimensions of a strut or a member of the bracing or a portion of the flange, in the other direction, *i.e.*, in the plane of the girder, is a variable depending solely on the necessities of the case; and compression members will always have to be tested to ensure that this dimension is sufficient for stiffness.

In calculating such struts it must be remembered that they are to be rigidly attached to the members by the joints at their ends and may be regarded therefore as having their "ends fixed" against buckling in the plane of the girder.

At right angles to the plane of the girder this is not so, since the flange or other member to which they are secured might twist, and thereby allow, even if not actually assist, buckling.

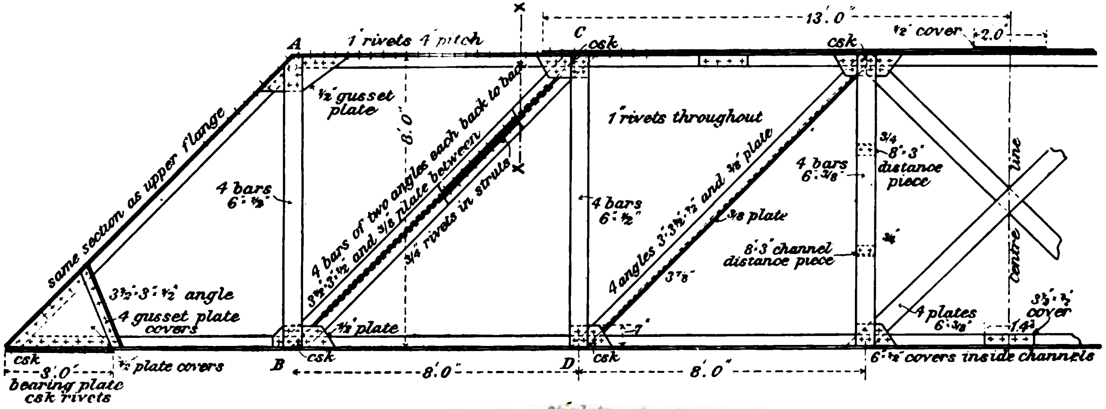
EXAMPLE.

To design a small girder of N type for a 50-foot span, single line.

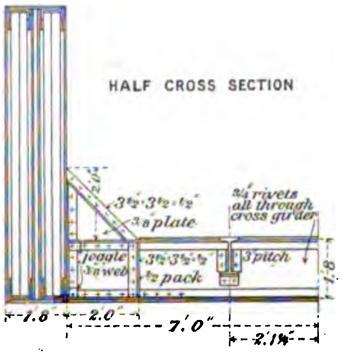
14 feet clear will be left between the girders.

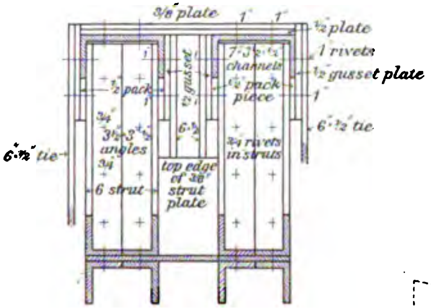
3 feet bearing will be given at either end, hence the total span = 56 feet.

PLATE XI.



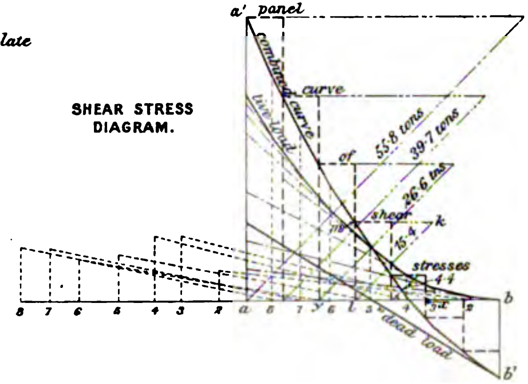
HALF CROSS SECTION





SECTION OF TOP FLANGE AT X. X.

SHEAR STRESS DIAGRAM.



CROSS GIRDERS.

Dead Load.—The dead load on one cross girder is its own weight, say 75 lbs. per foot run

$$= 1{,}050 \text{ lbs. (for 14-foot span).}$$

Also half the dead load on all four rail bearers which rest on it

$$= \frac{3{,}768}{2} \times 4$$

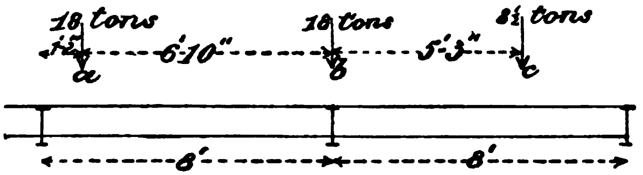
$$= 7{,}536 \text{ lbs.}$$

Therefore the total dead load on one cross girder

$$= 8{,}586 \text{ lbs.}$$

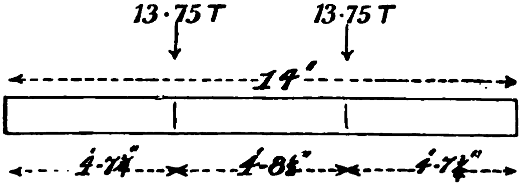
$$= 4 \text{ tons nearly.}$$

Live Load.—The live load is the heaviest pair of wheels (18 tons) plus shear due to the live loads on four rail bearers when the engine is in the position shown in Fig. 292.



CROSS GIRDERS IN SECTION

FIG. 292.



CROSS GIRDERS IN ELEVATION

FIG. 293.

Shear at *b* due to 18 tons at *a*.

$$= \frac{18 \times 1 \text{ foot } 2 \text{ inches}}{8} = 2{\cdot}63 \text{ tons.}$$

Shear at *b* due to 8½ tons at *c*

$$= \frac{17 \times 2 \text{ feet } 9 \text{ inches}}{2 \times 8} = 2{\cdot}9 \text{ tons.}$$

Total = 5·5 tons nearly.

All these loads may be considered as acting at the points of application of the rail bearers on the cross girders (Fig. 293), and the load at each of these points is half the sum of the live and dead loads; viz.:

$$= \frac{18}{2} + \frac{5{\cdot}5}{2} + \frac{4}{2}$$

$$= 13{\cdot}75 \text{ tons.}$$

The *Max.* M_B occurs at the points underneath the rails, and is

= Reaction × the distance of the rail from the end of girder.

= 13·75 × 4 feet 6½ inches.

= 749 inch-tons, the span of the cross girders being taken as 14 feet, *i.e.*, its length overall.

Now *f* is 4½ tons, and therefore

$$\begin{aligned} 4\tfrac{1}{2} \times a \times d &= M_R \\ &= M_B \\ &= 749 \text{ inch-tons.} \\ \therefore\ ad &= 166{\cdot}5. \end{aligned}$$

Assume *d* to be 20 inches.

Then *a* must be 8·33 square inches.

The cross girder may therefore be built up of a ⅜-inch web plate, a ⅝-inch × 7½-inch flange plate, and two 3½-inch × 3½-inch × ½-inch flange angles.

The sectional area of the flange plate, deducting two ⅞-inch rivet holes for ¾-inch rivets,

$$= \left(7\tfrac{1}{2} - \frac{14}{8}\right) \times \frac{5}{8}$$

$$= 3{\cdot}59 \text{ square inches.}$$

The area of the two angles, deducting two rivets in each = 4·75 square inches.

Total area = 8·34 square inches.

Therefore this section for the cross girder will suffice.

Testing for deflection.

$$\Delta = \frac{n' l^2 f}{E y}.$$

$$\therefore \Delta = \tfrac{1}{12} \times \frac{7 \times 14^2 \times 12^2}{13{,}400 \times 10} \text{ inches}$$

$$= \cdot 122 \text{ inches,}$$

which is safe.

CONNECTIONS OF RAIL BEARERS TO CROSS GIRDERS.

Maximum shear on rail bearer end (approx.) $= \dfrac{18}{2}$ tons.

$= 10$ tons (say).

See Plate XI. for joint.

Suppose six rivets to be employed, ¾ inch diameter.

Then the strength of the joint against shearing

$$= (\tfrac{3}{8})^2 \times \pi \times 4 \cdot 8 \times 6$$

$$= 12 \cdot 73 \text{ tons,}$$

and is ample.

For bearing, in the ⅜-inch web, the strength of the joint is

$$= \tfrac{3}{8} \times \tfrac{3}{4} \times 6 \times 6 \cdot 4$$

$$= 10 \cdot 8 \text{ tons,}$$

which is ample.

Hence six rivets, arranged as shown, will be used.

One stiffener will be added on both sides, 2 feet from the ends of the cross girder, of 3½-inch × 3½-inch × ½-inch angle steel.

Outside this stiffener the web plate is prolonged upwards to join the end stiffeners of the cross girder which are carried up the vertical member of the main girder at the point of support of the cross girder.

MAIN GIRDERS.

The main girders are to be considered as loaded with a uniform load; *i.e.*, the uniform equivalent load from Table XVI.

Table XVI. gives the equivalent uniform load for a 55-foot span as 2·5 tons per foot run, and this may be taken.

This is live load, and must be corrected by the addition of a percentage.

Table XIV. gives an addition of 13 per cent. to correct this live load.

The load therefore becomes $2·5 \times \frac{113}{100} = 2·825$ tons. Half this comes on each girder, or $1·412 \times 8$ tons or 11·296 tons on each panel point.

In this case the flange stresses will be obtained by using the rules given on p. 310, which enable us to write down the stresses on each portion of the flange direct.

A shear stress diagram will have to be drawn to obtain the stresses on the bracing.

Now the dead load per panel point on each girder = half the dead load on cross girders (2 tons)

+ small portion of ballast (·45 tons)

+ weight of main girders, estimated at 4·6 tons each girder (·66 tons),

making a total of 3·11 tons per panel.

Hence total load on one panel both live and dead = 14·4 tons,

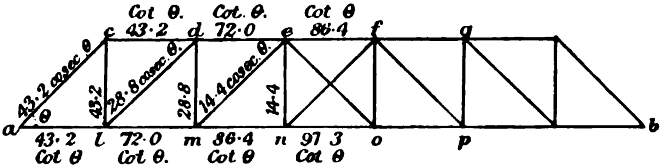


FIG. 294.

which is the value for w for writing the stresses on the members of the girder.

In Fig. 294 the stresses can now be written in on the various members, including the bracing.

(These latter stresses will be useful for comparison.)

The numbers now written have to be multiplied by cosec θ and cot θ.

$\theta = 45°$ in this case.

Therefore cot $\theta = 1$

and cosec $\theta = \sqrt{2}$.

All these stresses are given here in tabular form.

Member.	Stresses due to uniform load by rules.		Stresses from shear diagram.
ac	43·2 cosec 45°	61·1	55·8
ld	28·8 „	40·7	39·7
me	14·4 „	20·3	26·7
nf	0 „	0	15·4
cl	—	43·2	39·0
dm	—	28·8	28·2
en	—	14·4	18·9
al	43·2 cot 45°	43·2	
cd	do.	43·2	
lm	72·0 „	72·0	
de	do.	72·0	
mn	86·4 „	86·4	
ef	do.	86·4	
no	97·3 „	97·3	

Now the stresses found above are not the maximum stresses that can occur in the bracing, and a maximum shear stress diagram must be drawn for the load advancing on to the span. A short explanation is here introduced.

REMARKS ON SHEAR STRESS DIAGRAM.

The values of the stresses found above are those due to a uniform load covering the span; the uniform load is such that it produces moments of bending in the truss equal to the maximum that the heaviest engines could produce in their passage over the span. These stresses are therefore the maximum that can occur on the span due to M_B as far as the top and bottom flanges of the girder are concerned.

But the bracing does not resist M_B, but shear; and it will be found that higher shearing stresses are produced on the girder when the load is actually advancing on to the span; and the values of these stresses on the diagonals due to shear must be found before designing the girder.

It will also be noticed that under a purely uniform load there will be no stress in the bracing of the centre panel, so that no bracing is apparently required, whereas this is far from being the case when the load is unsymmetrical, *i.e.*, is not completely covering the span.

Reversal of stress will also occur, and the rules given above do not in any way locate this.

SHEAR DIAGRAM.

The shearing stress diagram must therefore be drawn; refer to Plate XI.

On the base line *ab* the shear diagram for the dead load is first constructed.

The dead load = 3·11 tons per panel on one girder, or 21·77 for the whole span.

Therefore ordinates are erected representing $\frac{21{\cdot}77}{2}$ tons, positive at *a* and negative at *b*, and the points so found are joined.

The curve of maximum shear for the live load may be found by using the uniform equivalent load of 2·5 tons per foot run.

Hence the total live load W on one girder

$$= \frac{2{\cdot}5 \times 56}{2} + 13 \text{ per cent.}$$

$$= 79{\cdot}1 \text{ tons.}$$

The parabolic curve of maximum shear may be drawn on *ab* as base line, by erecting an ordinate at *a* equal to $\frac{W}{2}$, and drawing a parabola through this point so found having its vertex at *b*.

If it is preferred not to use the uniform equivalent load from table, the shear curve may be drawn by the method of actual wheel loads.

This method is slightly the more accurate of the two, and is used in this case.

The diagram so obtained is compounded with or added to that for dead load, obtaining the final curve *a′b′*.

The position of panel points must now be marked off on this diagram along the base line which has been drawn 56 feet long, and they must be projected upwards to meet the curve of maximum shear *a'b'*.

The dotted rectangular curve shown in the diagram is the real shear stress curve for the girder.

Only the upper or positive portion of the shear diagram is drawn, since the lower or negative portion will be equal and opposite to it.

Now the vertical ordinates of the diagram represent the vertical shear, and they must be multiplied by cosec θ, in this case cosec 45°, in order to obtain the values of the whole shear stresses in the diagonal members of the bracing.

This may be done graphically by drawing the inclined lines shown in the shear diagram parallel to the bracing of the girder, and measuring them.

Their values are tabulated in column 4 of the stress table above.

Similarly the vertical members of the bracing will be subjected to the simple vertical shearing stress, the greater value at each panel point being taken. These results are also tabulated.

It will be noticed in the table that the stresses in the bracing and the verticals near the end of the span, *i.e.*, in panels No. 1 and 2, are slightly less than those found by the rules; this is due to the fact that actual wheel loads have been used for drawing the shear diagram, whereas the uniform equivalent load was used for the application of the rules.

The shearing stress diagram drawn from actual loads gives more accurate values for a particular engine than when drawn by the use of uniform equivalent loads.

The maximum values found in the stress table will be those to be used in design.

The shear diagram also shows the range and extent of reversal of stress.

The diagram shows that positive shear stresses may occur in the girder from *a* at the left of the span, up to the point *x*, where the shear curve cuts the base line *ab*.

The negative diagram would similarly show that negative shear stresses may occur from *b* at the right of the span up to a point y, similar to x, where the negative curve cuts the base line ab.

Between x and y, therefore, both negative and positive shears or reversal of stress may occur, which will affect the three centre panels.

The member *nf* (Fig. 294) has been drawn and is found to be liable to a compressive stress of 15·4 tons, which may be reversed when the load advances from the opposite direction to 15·4 tons tension.

In the panel at either side of the centre, the diagonal strut will have to carry either 26·6 tons compression (*vide* panel 3 from left of diagram), or 4·4 tons tension (*vide* panel 5 of diagram).

In the centre panel the strut *nf* (stress 15·4 tons) is replaced by a tie *eo* (carrying the same stress 15·4 tons), and the reverse stress is met by a tie bar *fn*. These two tie bars, crossbracing the centre panel, are simpler than struts and lighter.

So that the centre panel, which ought apparently to be braced with two struts is braced with two ties, and the verticals *ne* and *of* are relieved of the tensile stress to the value of the vertical component of the thrust in the compression members *nf* and *eo*.

On panels 3 and 5 no counterbracing is provided, and the struts *me* and *pf* are allowed to come into tension, since the amount of the tension is not great, but a sufficiently large sectional area must be given to these struts in order to avoid stressing them highly either in compression or tension.

With the stresses all found and tabulated, the design may now be proceeded with.

DEFLECTION.

To test the girder for deflection, the formula is:—

$$\Delta = \frac{n'fl^2}{Ey}$$

$$= \frac{5}{48} \times 7 \times \frac{56^2 \times 12^2}{13{,}400 \times 4 \times 12}$$

$$= \cdot 512 \text{ inches,}$$

which is amply safe.

FLANGES.

Assume a width of 18 inches for the booms of the girder, with four rows of rivets along it.

It is intended to use 1-inch rivets throughout at a pitch of 4 inches, so that 1⅛-inch holes must be deducted.

Consider the lower or tension boom.

Suppose the flange to consist of two 7-inch × 3½-inch × ½-inch channels, and a ½-inch plate.

The effective area of the plate, deducting four rivet holes, is :—

$$= (18 \text{ inches} - 4\tfrac{1}{2} \text{ inches}) \times \tfrac{1}{2} \text{ inch.}$$
$$= 6{\cdot}75 \text{ square inches.}$$

Also the effective area of the two channels, deducting four rivet holes in each channel, is :—

$$= 8{\cdot}5 \text{ square inches.}$$

Therefore the total effective area of the flange is :—

$$= 15{\cdot}25 \text{ square inches.}$$

Hence total safe stress in the flange is :—

$$= fa.$$
$$= 7 \times 15{\cdot}25.$$
$$= 106{\cdot}75 \text{ tons, tension.}$$

taking $f = 7$ tons per square inch.

This section will therefore suffice for any portion of the lower flange.

This section should now be tested to see whether it is suitable and stiff enough to act as a strut in the top, or compression flange, without buckling.

Consider the central portion of the upper flange.

It is 18 inches × 4 inches overall.

It is most likely to buckle in the direction of the smaller dimension.

The total sectional area of the upper flange is = 15·25 square inches.

The stress in the central portion is 86·4 tons; *vide* stress table.

This gives a stress per square inch of 5·66 tons.

Referring to Table V., it will be found that this intensity of stress is out of the table, being far too large for any but very short columns.

This means that the portion of the flange *ef* must be strengthened against buckling either by providing intermediate supports, or by reducing the intensity of stress upon it.

On examination it will be found that the same applies to the adjoining panels on either side of the centre, viz., members *de* and *fg*.

A ⅜-inch plate will therefore be added to the top of the flange in the three centre panels, having a net sectional area after deducting for rivets of 5·07 square inches.

This will increase the total net section of the upper flange to 20·32 square inches, giving a stress per square inch of 4·23 tons.

Referring to Table V. this is found to be equivalent to a value of $\frac{l}{k} = 90 = \frac{ln}{h}$.

The ends of the struts are taken as being "fixed" since the flange is a long continuous flange supported at each panel point.

From Table IV. $n = 3{\cdot}8$,
and we know that $h = 4\frac{3}{8}$ inches, *i.e.*, the thickness of both the flange plates and the smaller dimension of the channel steels.

Hence:—

$$\begin{aligned} l &= 90 \times 4\tfrac{3}{8} \times \frac{1}{3{\cdot}8} \\ &= 103{\cdot}6 \text{ inches} \\ &= 8 \text{ feet } 7\tfrac{1}{2} \text{ inches.} \end{aligned}$$

This will just be enough, the panel being 8 feet long.

The extra ⅜-inch plate of the upper flange must be carried about three pitches of rivets or 12 inches beyond the panel points *c* and *g*.

At the ends of the girder where the upper or compression flange joins the end diagonals, and the end diagonals join the lower flange, gusset plates will be required to strengthen and cover the joints.

BRACING.

Vertical tie bars.

cl. *cl* (tension), must have a net area of

$$a = \frac{43{\cdot}2}{7} \text{ square inches}$$
$$= 6{\cdot}2 \text{ square inches.}$$

Use four plates ½ inch × 6 inches, deducting for two rivets 1 inch diameter from each.

These will have a net area

$$= 4 \times \tfrac{1}{2} \times 3\tfrac{3}{4}$$
$$= 7{\cdot}5 \text{ square inches.}$$

For the joints at the ends of this member, viz., *c* and *l*, let *n* be the number of rivets required.

The full constants may be taken for bearing and shearing, viz., 8 tons and 6 tons per square inch.

Then for shear :—

$$n \times (\tfrac{1}{2})^2 \times \pi \times 6 = 43{\cdot}2.$$
$$\therefore n = 9 \text{ rivets.}$$

Suppose twelve rivets to be used; there will then be three rivets in each plate of this member.

A gusset plate will have to be used at both ends so as to fit these rivets in conveniently.

dm. *dm* is a tie bar, stress 28·8 tons.

Four ½-inch plates will be used as before, and the number of rivets will be eight, viz., two in each plate.

In this instance, although a ⅜-inch plate would amply suffice for the stress, it would not afford sufficient bearing area for the rivets.

The plates are therefore made the same thickness as the other member *cl*.

en. In *en* the stress is 18·9 tons.

Four plates 6 inches × ⅜-inch will suffice.

This completes the tie bars.

COMPRESSION BARS.

ac. This strut will be the same section as the main boom, or flange, and need not therefore be tested for stiffness again, *ld.* This strut is to be made up of four bars, each consisting of two angles, 3½ inches × 3 inches × ½ inch shown in section in Fig. 295.

These are not stiff enough by themselves, and it is proposed to stiffen them by joining them together by a ⅜-inch plate, so that all four shall form one member, and thus gain stiffness; *vide* Fig. 295.

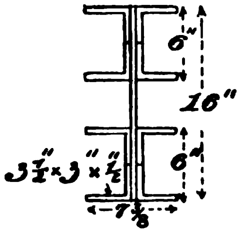


FIG. 295.

When thus joined together they form one strut, 16 inches × 7⅜ inches, and it is in the direction of the smaller of these two dimensions that the strut is likely to fail first, *i.e.*, in the plane of the girder, in which direction it may be considered as "ends fixed."

Each pair of angles back to back forming one bar will take a quarter of the total stress, or

$$\frac{40{\cdot}7}{4}, \text{ or } 10{\cdot}17 \text{ tons.}$$

Now the net sectional area is 3¾ square inches for each bar; therefore the intensity of pressure is :—

2·71 tons per square inch.

Referring to table, it will be found that for a load of 2·71 tons per square inch, on steel struts with ends fixed,

$$\frac{l}{k} = 150,$$

and also $\qquad \frac{l}{k} = n\frac{l}{h}.$

Now $\qquad n = 4{\cdot}2$ from Table IV.,

and $\qquad h = 7\tfrac{3}{8}$ inches,

therefore $\qquad l = 150 \times \frac{10 \times 59}{42 \times 8}$

$$= 263 \text{ inches, or } 22 \text{ feet,}$$

which is amply safe since the length of the strut is only $8\sqrt{2}$ feet.

me. This is also a strut, and is to be made up of four angles, 3½ inches × 3 inches × ½ inch with a ⅜-inch plate joining them together in order to obtain stiffness.

The stress to be borne is 26·6 tons, and the sectional area of the strut is 10·5 square inches, including a portion of the $\frac{3}{8}$-inch plate.

This gives a stress intensity of 2·53 tons per square inch.

Referring to Table V., the value of $\frac{l}{k}$ is found to be:—

$$= 160$$

$$= n\frac{l}{h} \text{ where } n = 4·9$$

$$h = 3\tfrac{1}{8} \text{ inches.}$$

Therefore $l = 130$ inches, nearly, which is greater than the length of the diagonal member.

Therefore the section selected is safe.

eo, *nf*. These two members are ties, carrying 15·4 tons each.

They will be made of four plates 6 inches by $\frac{3}{8}$ inch each.

JOINTS.

There will be a join in the $\frac{1}{2}$-inch steel plate of the top flange at the centre of the span.

This will be covered by a cover plate $\frac{1}{2}$ inch × 18 inches, *i.e.*, the same cross-section as the plate itself.

This plate, allowing for four rivets across it, will have to carry a stress:—

$$= 13\tfrac{1}{2} \text{ inches} \times \tfrac{1}{2} \text{ inch} \times 7 \text{ tons.}$$

$$= 47·25 \text{ tons.}$$

Let n be the number of rivets 1 inch diameter required on each side of the join.

Then for shearing:—

$$n \times (\tfrac{1}{2})^2 \times \pi \times 6 = 47·25$$

$$\therefore n = 10·02.$$

For bearing:—

$$n \times 1 \times \tfrac{1}{2} \times 8 = 47·25$$

$$\therefore n = 11·81.$$

Twelve rivets will therefore be used, *i.e.*, three pitches of rivets, and the cover plate will be 24 inches long.

The channel steels also require covers both in the top and bottom flanges.

These will be placed in the middle of the third panel from either end, in the top flange, and in the middle of the centre panel in the lower flange.

The sectional area of the two channels in the flange is 8·5 square inches, and the maximum stress in the channels is:—

$$= 8{\cdot}5 \times 7$$
$$= 59{\cdot}5 \text{ tons.}$$

The joint will be made with flat fish plates, two 3½-inch by ½-inch plates placed along the stems of each channel, and one 6-inch by ½-inch plate along the underneath of the table of each channel.

Let n be the total number of rivets required.

Then for bearing:—

$$n \times 8 \times 1 \times \tfrac{1}{2} = 569{\cdot}5.$$
$$\therefore n = 14{\cdot}9 \text{ rivets.}$$

Each of the 6-inch cover plates will therefore have four rivets, and each of the narrow 3½-inch covers will have two rivets, making sixteen rivets in all.

At the point A, Plate XI., the plate of the top flange will be bent down to form part of the end diagonal member, and the channels also will be bent.

BEARING PLATE.

A bearing plate of 1-inch steel is fitted to the underneath of the ends of the girder.

This is riveted with the ordinary rivets of the lower flange, but these must have countersunk heads, giving a flush surface underneath the bearing plate.

RIVETS AND JOINTS.

It is desirable to make the four rows of rivets along the top and bottom flanges break joint, *i.e.*, the two inner or middle rows will be set half a pitch, or 2 inches, in front of the two outer rows.

This will render the riveting of the flange plates to the channels a little easier, and will save effective section.

The rivets in the joints are very difficult to arrange in this girder as the members are so narrow.

They are pitched at 3 inches apart in nearly all the joints, and gusset plates are provided also.

Owing to the difficulty of executing the riveting, it is possible that some bad rivets may be put in, and therefore several spare rivets have been allowed at each joint.

The ⅜-inch plate which has been introduced into the struts for sake of stiffness has not been considered at all in the matter of stress per square inch, although it would have been quite permissible to do so.

This plate is moreover cut away, or "stopped" about 6 inches away from the joints of the struts with the upper and lower flanges, so as not to interfere with the vertical tie members.

FLOOR.

A ½-inch plate floor is to be fixed over all the rail bearers and cross girders, which will require support at its outer edges parallel to the main girders.

This floor will provide ample bracing against lateral sway due to wind and other causes.

GENERAL.

In the foregoing design, exceedingly small plates and sections have been used throughout.

It might have been better to use larger angles and channels and fewer of them, but the design has been prepared with a view to utilising small sections only, for sake of example, and this has led to a girder which is difficult to plate up and rivet together.

REVERSAL OF STRESS.

In theory under a uniform distributed load a truss with an odd number of panels requires no bracing in the centre panel, and of course no truss requires any counterbracing.

But this condition of absolutely perfect uniform loading is never obtained.

The supposition of uniform equivalent loading on the span only gives maximum values of stresses which may occur in certain members; it by no means follows that these will all occur at one time. Some may be less than the maximum values; in fact, we know that the load is anything but a uniformly distributed one, and therefore we must clearly remember that the use of a uniform equivalent load is a short road to obtaining certain maximum values of stresses produced by a number of concentrated axle loads.

The method fails entirely to show any values except the maximum.

The shear stress diagram must therefore be drawn in every case.

This gives the points between which reversal of stress takes place, and practically, also, gives the minimum values of stress in the diagonals and verticals.

These minimum values may be zero or may have a negative value, and this value is shown in the negative shear stress diagram, converting tension members into compression, and *vice versâ*.

In the type of girder considered in the foregoing example (Fig. 271), the diagonal struts are liable to be put into tension.

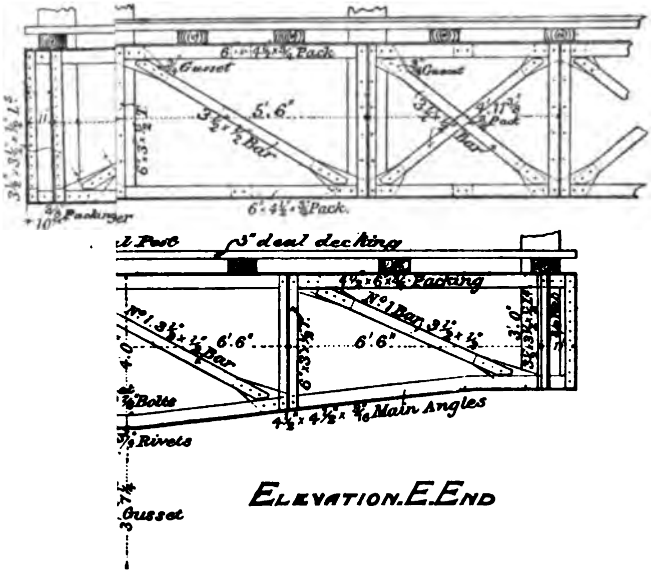
If this tension is not very great compared with the compressive thrust, which the strut is designed to take, no special precaution need be taken, beyond ensuring that the member cannot be very highly stressed either in tension or compression, *i.e.*, by designing it of ample proportions.

The safe resistance per square inch of a strut should be reduced by 25 *per cent. when it is liable to be put into tension by reversal of stress, in order to allow for fatigue.*

If, however, as in Fig. 270, the diagonals are ties, then they will be put into compression when reversal of stress takes place. To resist this they may be so designed as to be capable of acting as struts, without buckling, and the restriction as regards stress stated above must be adhered to.

In panels where ties are liable to be thrown into compression, however, it is customary to avoid this by supplementing the existing diagonal of the panel with another diagonal joining the two opposite corners of the panel, *i.e.*, counterbracing, which will also be put in tension when reversal takes place and will prevent any compression coming on to the other tie.

When the stress in any diagonal member is reversed the stress



ELEVATION. E. END

in any adjoining vertical member is affected thereby, and when a diagonal strut is thrown into tension, the vertical at its end may be thrown into compression and become a strut.

In the case of a girder with an even number of panels, the centre two will always be subject to the reversal of stress, and will require counterbracing or other treatment. In the case of girders with an odd number of panels, the centre panel will always require counterbracing and generally also the two panels on either side of it. These latter may sometimes be treated differently, without counterbracing, but will always require careful consideration.

The consideration of any shear stress diagram will show that the greater the dead load in proportion to the live or accidental load, the less range there will be of reversal of stress, or the greater the span the less will the range of reversal be in proportion to it.

In the actual drawing out of a design, it will generally be of great assistance to draw a line diagram of the truss properly spaced, such that the lines represent the lines of mean fibre of all the members.

The members can then be drawn round their lines of mean fibre, and the joints will arrange themselves so that all the stresses, either compressive or tensile, are symmetrically imparted from one member to another.

GENERAL.

Parallel braced girders are very easy to design up to moderate spans.

The location of shearing stresses in diagonal members leads to economy, since the exact stresses and their direction are known and can be arranged for accurately.

They are more sightly than plate girders, and are cheaper per foot run than plate girders for spans of 75 feet and up.

A braced signal girder is shown in Plate XII.; it will be noticed that the panels are not all of the same size. This is due to the position of the lines underneath which the signals control. The design of such a girder is simple, if the principles are understood.

RATIO OF DEPTH TO SPAN.

Until recent years it was the custom to design girders with a ratio of depth to span of 1 to 16, up to 1 to 10; 1 to 10 and 1 to 12 are common ratios now for plate girders, as they are economical in the matter of web plate; but for braced girders modern experience advocates a ratio of 1 to 7 or 1 to 5.

There is no doubt that a very deep girder with a ratio of depth to span of 1 to 5 is ugly; but it is cheap, and where appearances do not matter is very good.

Light sections can be used; no elaborate plating is required; struts are not complicated; the weight of the girder itself is reduced. All these advantages join to make a deep girder desirable.

CHAPTER XIX

BRACED GIRDERS (2)

DESIGN 75 FEET CLEAR SPAN.

EXAMPLE.

To design a braced girder for a single line track, and ordinary rolling stock. Clear span, 75 feet. The floor is to be of cross-girders and longitudinal troughing, and ballast with cross sleepers and flexible track (Plate XIII).

LENGTH.

The total length of the girder allowing for a bearing of 3 feet at either end will be 75 feet + 6 feet = 81 feet.

The span for calculations will be from centre to centre of bearings, *i.e.*, 78 feet.

PANELS.

There will be 9 panels of 9 feet each.

DEPTH.

The depth will be 10 feet. This is $\frac{1}{8}$ of the span and is about the usual ratio of depth to span for a braced girder.

LIVE LOAD.

The correction for the live load will be made by allowing a reduced value of "f" in the floors, and by adding a percentage to the total live load for the main girder calculations.

FLOORING TROUGHS.

The span of the troughs is 9 feet. The heaviest axle load on the floor is 19·4 tons, and this is the ruling load for the troughs.

This gives a load of 9·7 tons per rail.

Consider the trough which comes immediately under the rail.

$$Max.\ M_B = \frac{9{\cdot}7 \times 9 \times 12}{4} \text{ inch-tons,}$$

$$= 261{\cdot}9 \text{ inch-tons.}$$

Only half of this will be taken by this one trough, owing to the distribution of weight by the ballast and sleepers, hence the *Max.* M_B on one trough.

$$= 131 \text{ inch-tons.}$$

Dead Load.—Assume a likely section of trough, 2 feet wide, 9 inches deep, by 26·26 lbs per foot super,

Then the weight of one trough = 473 lbs.

Weight of ballast over one trough made 12 inches deep over the top of the troughs.

$$= 9 \text{ feet} \times 2 \times \frac{16}{12} \text{ inches} \times 110 \text{ lbs.}$$

$$= 2{,}640 \text{ lbs.}$$

Weight of one line of track with sleepers, etc.

$$= 9 \text{ feet} \times \frac{168}{2}$$

$$= 756 \text{ lbs.}$$

$$\text{Total dead load} = 3{,}869 \text{ lbs.}$$

$$= 1{\cdot}73 \text{ tons.}$$

$$Max.\ M_B = \frac{Wl}{8} = \frac{1{\cdot}73 \times 9 \times 12}{8}$$

$$= 23{\cdot}35 \text{ inch-tons.}$$

$$\text{Hence total } M_B = 131 + 23{\cdot}35$$

$$= 155 \text{ inch-tons.}$$

$$= f\frac{I}{y}$$

$$\text{Therefore } \frac{I}{y} = 155 \div 4\tfrac{1}{2}$$

$$= 34{\cdot}4 \text{ inch-tons.}$$

Therefore the trough selected will do.

CROSS GIRDERS.

Span.—The main girders will be high, and hence the maximum clearance laid down by the Board of Trade must be given, *i.e.*, 6 feet 7 inches from inner edge of main girder to the centre of the track.

The whole span of the cross girders is therefore 13 feet 2 inches.

The rails are symmetrically placed, being taken as 2 feet 5 inches each side of the centre line of the track.

The cross girder is subjected to dead and live loads, the latter being concentrated on the rails.

Dead Load.—The weight of the cross girder itself may be assumed to be 60 lbs. per foot run.

$$\text{Therefore its total weight} = 13 \text{ feet } 2 \text{ inches} \times 60$$
$$= 790 \text{ lbs.}$$

$$\text{Total weight of troughs, 6 widths.}$$
$$= 2{,}836 \text{ lbs.}$$

$$\text{Weight of ballast} = 12 \times 9 \times \frac{4}{3} \times 110 \text{ lbs.}$$
$$= 15{,}840 \text{ lbs.}$$

$$\text{Total} = \underline{19{,}466 \text{ lbs.}}$$

$$= 8{\cdot}7 \text{ tons.}$$

$$\text{Then } Max.\ M_B = \frac{8{\cdot}7 \times 13 \text{ feet } 2 \text{ inches}}{8}$$
$$= 171{\cdot}8 \text{ inch-tons.}$$

Live Load.—The maximum live load is brought on the cross girder when the engine shown in Fig. 226 has one axle actually

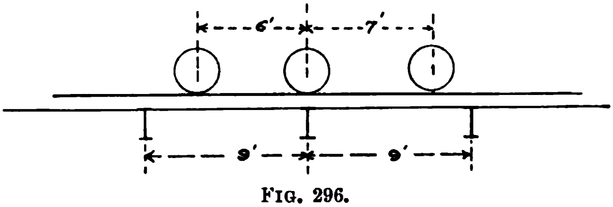


FIG. 296.

on the cross girder and one on each trough on each side of the cross girder (Fig. 296).

Then the load on the cross girder is:

$$= 17{\cdot}5 + 17{\cdot}5\left(\frac{3}{9} + \frac{2}{9}\right)$$
$$= 17{\cdot}5 \times \frac{14}{9} \text{ tons,}$$

This must be halved to get the load on each rail, and the whole load may be taken as being concentrated at the two rail points.

Hence the M_B at each rail is :

= Reaction at one end of the cross girder multiplied by the distance from the end to one rail point.

$$= 17{\cdot}5 \times \frac{7}{9} \times 4 \text{ feet } 2 \text{ inches.}$$

$$= 681 \text{ inch-tons.}$$

Therefore the total M_B due to all loads

$$\begin{aligned} &= 681 + 171{\cdot}8. \\ &= 853 \text{ inch-tons.} \\ &= f\frac{I}{y} \\ &= fad. \end{aligned}$$

Now $$f = \frac{9}{2}.$$

Therefore $ad = 190$.

And if d be assumed to be 20 inches, then $a = 9{\cdot}5$ square inches.

This is the area of flange required to give enough M_R.

The flange may be made up as follows :

Two 4½-inch × 4½-inch × ½-inch angles, and a 9½-inch × ½-inch plate.

Deducting two 1-inch rivet holes from each of the two angles and from the plate, the effective area is

$$= 10{\cdot}25 \text{ square inches.}$$

which is sufficient.

The web is assumed to be ½-inch plate.

The shear at the ends is important.

The shear at the end of the cross girder is half of the dead load, and the live load on one rail.

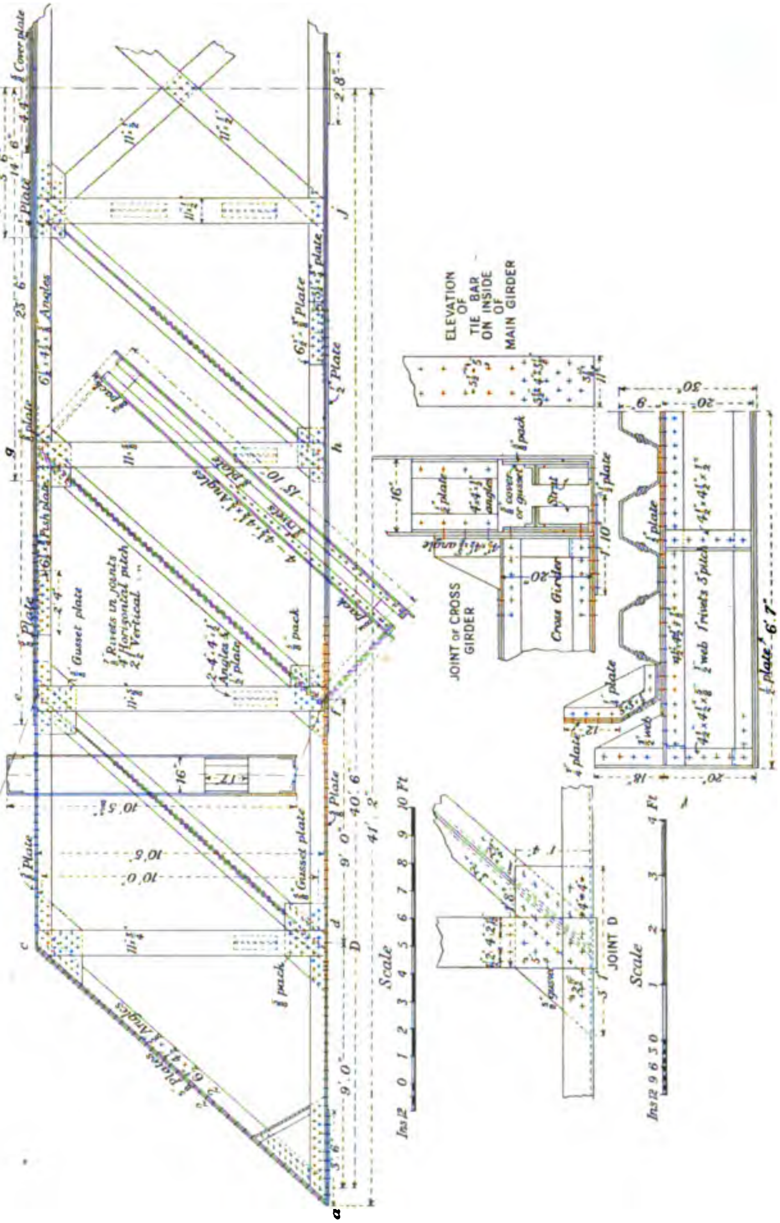
$$\begin{aligned} &= \frac{8{\cdot}7}{2} + 17{\cdot}5 \times \frac{7}{9} \\ &= 17{\cdot}95 \text{ tons} \\ &= 18 \text{ tons approx.} \end{aligned}$$

This is vertical shear.

Consider a vertical section at the end of the girder.

PLATE XIII.

75′ 0″ Clear Span



The depth is 20 inches. With rivets at 4 inches pitch there will be five of them in one vertical section.

$\frac{7}{8}$-inch rivets can only take 2·8 tons per rivet in bearing on a $\frac{1}{2}$-inch plate, using reduced constants. These will therefore not be large enough. It will be necessary to use 1-inch rivets at 3-inch pitch. There will be six of them in one vertical section of the girder. Then in bearing at 6·4 tons per square inch, six of these will take 19·8 tons.

These will therefore be used.

Since the horizontal shear is equal to the vertical shear, the rivets in the flanges joining the angles to the flange plates must also be 1 inch at 3-inch pitch. The shear continues the same from the end of the cross girder to the nearest rail. Consequently there is little gained by altering the rivet pitch at the middle part of the girder, and it will be kept the same all through.

Stiffeners.—Stiffeners will be fixed on both sides of the cross girder under the rail points. These will be made of one angle $4\frac{1}{2}$ inches × $4\frac{1}{2}$ inches × $\frac{1}{2}$ inch riveted to the web.

The ends of the cross girders will be made of two angle uprights $4\frac{1}{2}$ inches × $4\frac{1}{2}$ inches × $\frac{5}{8}$ inch, which will be continued up for a distance of 15 inches in order to provide attachment to the vertical member of the main girder.

MAIN GIRDERS.

Live Load.—Table XVI. gives a load of 2·40 tons (approx.) per foot run as the uniform equivalent distributed load for this span, and Table XIV. gives a percentage addition of 12 per cent. to convert this to dead load.

Hence the total load for calculation is

$$2{\cdot}40 \times \frac{112}{100} \times 78 \text{ tons for two girders,}$$

and *Max* M_B on the girder treating this as a uniform distributed load

$$\begin{aligned} &= \frac{Wl}{8} \\ &= 1{\cdot}20 \times \frac{112}{100} \times 78 \times \frac{78 \times 12}{8}, \\ &= 12{,}265 \text{ inch-tons,} \end{aligned}$$

Dead Load.—The whole dead load on each cross girder is 8·7 tons. Therefore the dead load on one main girder at the point of junction with each cross girder is = 4·35 tons.

This total load may be taken as being distributed over the whole span uniformly: and there are 8 cross girders.

Hence the total dead load on the whole span is:

$$= 4{\cdot}35 \times 8$$
$$= 34{\cdot}80 \text{ tons.}$$

The weight of the girder may be taken as 19 tons.

Therefore the total dead load = 53·80 tons

(say) = 54·0 tons.

$$\text{The } Max.\ M_B = 54 \times \frac{78 \times 12}{8}$$
$$= 6{,}318 \text{ inch-tons.}$$

Therefore the total M_B

$$= 12{,}265 + 6{,}318$$
$$= 18{,}583 \text{ inch-tons}$$
$$= fad.$$

If d be 10 feet or 120 inches, then

$$fa = 155 \text{ tons.}$$

And if $f = 7$ tons, then

$$a = 22 \text{ square inches.}$$

The M_B diagram is now drawn with this (18,583) as vertical ordinate, and is stepped for stresses in the top and bottom flanges. The diagram is shown in Plate XIV.

The ordinates measured are actual ordinates of M_B, and have to be divided by the depth in order to find the actual stresses in the members.

The horizontal scale is 16 feet = 1 inch and the vertical scale of moments is

5,000 inch-tons = 1 inch.

The shear stress diagram is also drawn.

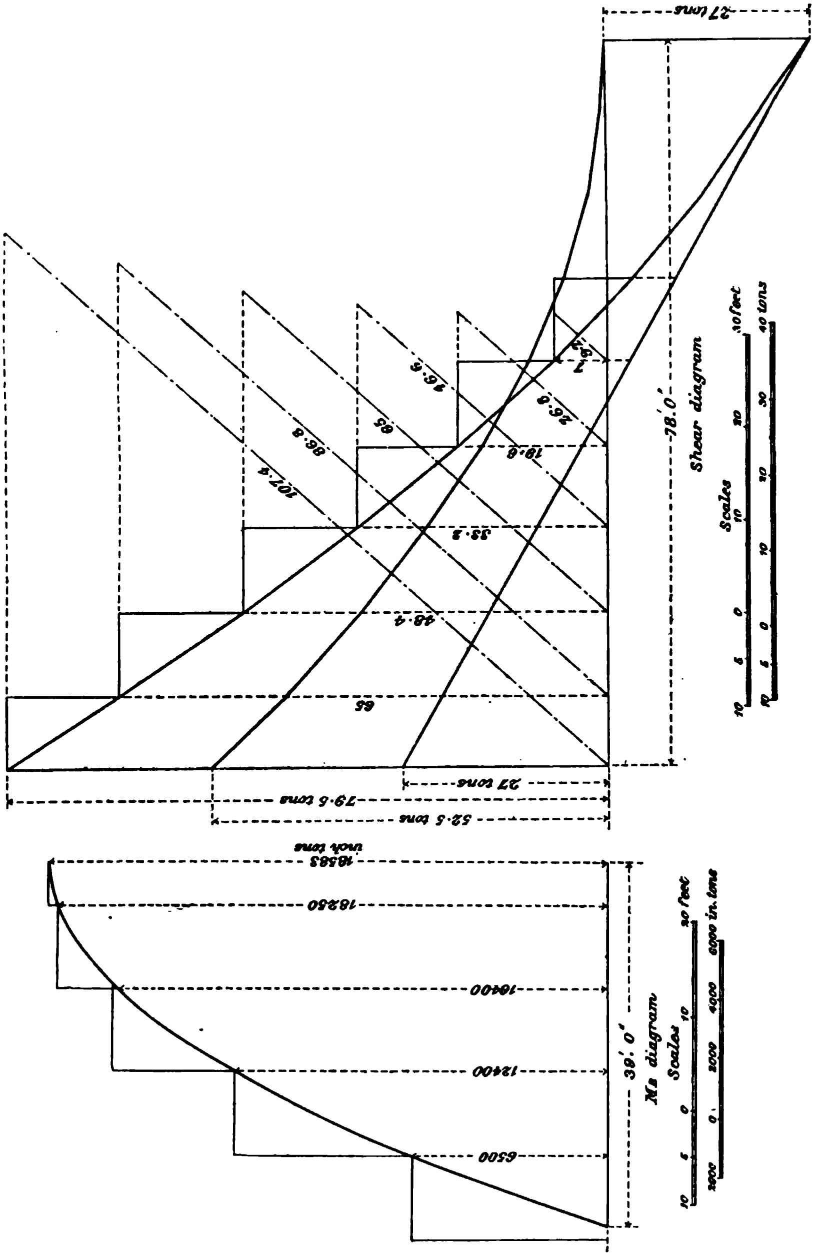
The total load on each girder is:

Corrected live load = 105 tons
Dead load = 54

The end shears are therefore 52·5 and 27 tons, the diagrams for these two being parabolic and rectilinear respectively.

This diagram is drawn and shown in Plate XIV., the diagram

PLATE XIV.


27 tons
78.0'
Shear diagram
Scales
10 5 0 10 20 30 feet
10 5 0 10 20 30 40 tons
107.4
86.8
65
46.6
26.6
7
18.6
33.2
48.4
65
27 tons
79.6 tons
52.5 tons
18583 inch tons
18280
16400
12400
6500
39' 0"
Ms diagram
Scales
10 5 0 10 20 feet
2000 0 2000 4000 6000 in. tons

being stepped off to give the vertical shear in each panel: and the values of stresses in the bracing are found graphically.

All the stresses found are now tabulated as under.

TABLE OF STRESSES.

ac	107·4	Strut.
de	86·8	,,
fg	65·0	,,
hi	46·6	,,
jk	26·8	,, or tie.
lm	9·2	,, ,,
cd	79·5	Tie.
ef	65·0	,,
gh	48·4	,,
ij	33·2	,, } similar members.
kl	19·6	,,
mn	7·0	,, similar to *gh*.
ad	54·2	,,
ce	,,	Strut.
df	103·3	Tie.
eg	,,	Strut.
fh	136·7	Tie.
gi	,,	Strut.
hj	152·0	Tie.
ik	,,	Strut.
jl	155·0	Tie.

The panel points are lettered in the elevation of the girder in Plate XIII. for reference, also in Fig. 297.

It will be seen from the diagrams that reversal of stress takes place in the three centre panels.

In panels 4 and 6 the struts have to carry a compressive force of 46·6 tons, or a tensile stress of 9·2 tons.

No counterbracing will be used here.

In the centre panel also, the struts are liable to be thrown into tension, and here counterbracing will be resorted to by using two ties each able to take 26·8 tons tension.

These ties in the centre panel can never come into compression, since the whole stress will be taken by the tie acting in the direction of the other diagonal of the centre panel.

It is now necessary to find out the effect of the reversal of stress in the diagonals on the tie bar *lk* (Fig. 297).

Suppose the load to have advanced far enough across the span to cause the maximum shear to occur in panel 5, and consider the vertical forces acting at panel point *l*. Since there is counterbracing in the central panel, there are no struts in this panel and *il* is a tie.

The vertical forces acting are: stress in *il* sin θ upwards (where θ is the angle of the diagonals with the horizontal), and the panel load 18·36 tons downwards.

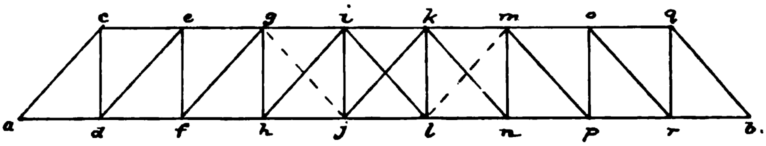


FIG. 297.

$$\text{Now} \qquad il \sin\theta = jk \sin\theta$$
$$= 26{\cdot}8 \sin\theta$$
$$= 26{\cdot}8 \times \frac{10{\cdot}00}{13{\cdot}45}$$
$$= 20 \text{ tons.}$$

And panel load = 18·36 tons.

Therefore the stress in *lk*

$$= 20 - 18{\cdot}36$$
$$= 1{\cdot}64 \text{ tons downwards.}$$

Therefore *lk* is a strut and has to carry 1·64 tons.

Again, suppose that the load is in such a position as to cause the maximum shear in panel No. 6, *klnm* (Fig. 297).

The stress found and tabulated for the member *lm* is the reversed stress in the member *kn*.

Consider the vertical forces acting at *k*.

These are *kn* sin θ downwards, and

The force in *lk* upwards. The member *jk* is redundant.

$$\text{Now} \qquad kn \sin\theta = 9{\cdot}2 \times \frac{10{\cdot}0}{13{\cdot}4}$$
$$= 7 \text{ tons.}$$

Therefore the force in *lk* is 7 tons upwards.

This means that the member *kl* may be put into compression up to 7 tons.

Some means must therefore be found in designing it to arrange for this compression.

The flange design can now be proceeded with.

LOWER FLANGE.

The general design will consist of plates 16 inches wide, and on top of them there will be two angles $4\frac{1}{2}$ inches × $6\frac{1}{2}$ inches × $\frac{5}{8}$ inch.

To these will be attached the members of the bracing.

The net sectional area of these two angles, deducting two rivet holes 1 inch diameter

$$= 8{\cdot}65 \text{ square inches.}$$

Assume that the first plate is $\frac{5}{8}$ inch thick, then its net sectional area deducting four rivet holes is

$$= 7{\cdot}5 \text{ square inch.}$$

Total fa taking f at 7 tons per square inch due to these two areas is

$$= 113{\cdot}05 \text{ tons.}$$

This will suffice for the first two panels of the lower boom.

Then assume another plate $\frac{1}{2}$ inch thick of the same width.

Net sectional area of this plate is

$$= 6{\cdot}0 \text{ square inches}$$

and the fa due to this is

$$= 42 \text{ tons.}$$

$$\text{Total} \qquad = 155{\cdot}05 \text{ tons.}$$

This section will suffice all through the lower flange.

UPPER FLANGE.

This is in compression and it will be found that the sections allowed above are not enough for stiffness.

The flange is not likely to bend in the plane at right angles to the girder, *i.e.*, in the direction of its large dimension, 16 inches; and in the other direction, *i.e.*, in the plane of the girder it may be assumed to be half fixed at the ends since it is continuous, and is joined by other members as well, which help to keep it stiff.

It is necessary to ascertain the maximum value of f_c that may be allowed on this strut; the formula is $\frac{l}{k} = \frac{nl}{h}$.

Here $n = 4{\cdot}0$ from table.
$l = 108$ inches.
$h = 7{\cdot}5$ inches approximate.

From this f_c is found to be,
for ends fixed $= 4{\cdot}87$ tons per square inch;
for ends free $= 4{\cdot}12$ tons.

Mean value $f_c = 4{\cdot}50$ tons per square inch.

Now assume the same members as in the lower flange. These are two angles and one plate ⅝ inch thick, and they have a combined area of 16·15 square inches.

This will give a total strength, or "fa"
$= 16{\cdot}15 \times 4{\cdot}5$ tons.
$= 72{\cdot}67$ tons.

This will therefore suffice for the first panel, *ce*.

Now add another ⅝-inch plate.

Then the additional area is 7·5 square inches, and the additional strength or fa is
$= 33{\cdot}75$ tons
making a total of 106·42 tons.

This will be sufficient for the second panel, *eg*.

Then assume another plate ⅝ inch thick.

The extra strength from this is 33·75 again, making a total of 140·17 tons.

This will do for the third panel, *gi*.

To this add ¼-inch plate, with an additional strength of 13·5 tons.

This makes a total of 153·67 tons and is sufficient for the centre panel.

This completes the selection of plates for the upper boom.

DIAGONAL STRUTS.

ac. This end strut always partakes of the nature of the main or upper flange, and is generally made like it.

In this case it will be made the same section as the member *eg*.

The remainder of the struts are different in their type of construction, not having a large plate at their back.

The general type of construction to be adopted is shown in Fig. 298.

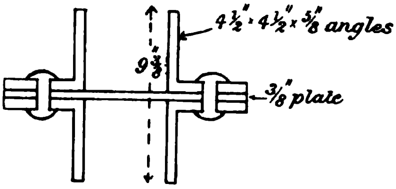


FIG. 298.

These struts are 13·45 feet long, *i.e.*, 162 inches, nearly, between points of intersection of lines of mean fibre.

They are most likely to fail by buckling in the direction of their least dimension, *i.e.*, in the plane of the girder.

It is now necessary to find the maximum value of f_c that may be allowed in these struts.

Consider one pair of angles, 4½ inches × 4½ inches × ⅝ inch.

Formula is $\frac{l}{k} = \frac{n}{h}$

Here $n = 4{\cdot}2$ from Table.
$l = 162$ inches.
$h = 9\frac{3}{8}$ inches.

Therefore $\frac{l}{k} = 72{\cdot}6$

From table it is found that f_c

for ends fixed $= 4{\cdot}60$,
for ends free $= 3{\cdot}59$,
Mean value $f = 4{\cdot}10$.

ed. Now the net sectional area of one pair of angles as above is 9·37 square inches deducting two rivet holes ⅞ inch diameter.

Add to this the area of the portion of ⅜ inch plate included between the two angles (Fig. 298).

Its area is = 1·36 square inches.

Total area = 10·73 square inches.

Therefore the total safe stress on one pair of angles as above allowing f_c to be 4·10 tons, is:—

$= 4{\cdot}10 \times 10{\cdot}73$ tons
$= 44$ tons.

This on the two pairs of angles in one strut amounts to 88 tons.

This will therefore suffice for *ed*.

fg. The same section may be used again.

hi. In this strut the stress is 46·6 tons compression, or 9·2 tons tension.

The reversal is to be met by allowing the strut to take tension and in this case the permissible value of f_c must be reduced by 25 per cent., to allow for fatigue; *vide* p. 338.

The safe stress has been found to be 4·10 tons per square inch.

This when reduced is 3·07 tons.

One single pair of 4½-inch × 4½-inch × ⅝-inch angles will be found to be insufficient. It is not worth while altering the size of angle in use and trying another. The double strut had best be used as in the other struts. There is only about 30 per cent. waste in this arrangement.

CENTRE PANEL.

This panel, as has been said, is to be counterbraced with two tie bars, each consisting of a pair of plates.

The stress is 26·8 tons, and the combined area of each member must be $\frac{26·8}{7}$, or 4 square inches,

or 2 inches for each bar.

They will be made of plates 11 inches × ½ inch, which deducting for three rivet holes gives an area of 4 square inches per plate.

It is not convenient in the design to make them any smaller.

VERTICAL TIE BARS.

cd. Stress is 79·5 tons.

f_t may be taken as 7 tons.

Net area is therefore 11·4 square inches.

These ties will all be made of two plates riveted one to each angle of the flanges. The area of each plate is therefore 5·7 square inches.

The width of each vertical member is 11 inches so as to fit in with the riveting. There will be three rivet holes in this width.

Plates ¾ inch thick will therefore be used for this member.

The section is = ¾ inch × (11 inches — 3 inches)

= 6 square inches.

ef. Stress is 65 tons, or 32·5 per tie bar.

Use two plates 11 inches × ⅝ inch, having net section 5 square inches.

gh. Stress is 48·4 tons or 24·2 per tie bar.

Use two plates 11 inches × ½ inch having net section 4 square inches.

ij and *lk*. The stress is 33·2 tension or 7 tons compression, *i.e.*, 16·6 per bar tension, or 3·5 compression per bar.

The area required for tension is

$$= \frac{16{\cdot}6}{7}$$

$$= 2{\cdot}4 \text{ square inches.}$$

Therefore two plates will be used of the same section as before, viz., 11 inches × ½ inch. Deducting three rivets these will have an area of 4 square inches each.

Now each of these bars must also be able to take 3·5 tons compression, or 0·9 tons per square inch.

These must therefore be tested as struts to ascertain at what intervals they will require lateral support to prevent failure from buckling.

Their ends will be considered free.

Referring to table it will be found that for a value of ·9 tons per square inch, $\frac{l}{k} = 180$.

And $\frac{l}{k} = \frac{nl}{h}$

Here $h = \frac{1}{2}$ inch.

$n = 3{\cdot}5$.

$l = ?$

From this l is obtained = 26 inches.

Hence these plates must be supported every 26 inches.

They are 16 inches apart in the clear, and they will be supported by diaphragms placed between them consisting of ⅛-inch plate, fixed to the tie bar by means of two 4-inch × 4-inch × ½-inch angles on each vertical edge.

These diaphragms are the same as those provided at the lower portion of every tie member in the girder, as will be explained later on, for the support of the cross girders.

This completes the general design.

RIVETS AT JOINTS.

cd. This is a tie bar, consisting of two plates, each 11 inches by ¾ inch, and each carrying a stress of 39·7 tons. The rivets to be used are ⅞ inch diameter. The flange angle to which they are to be secured is ⅝ inch thick.

Hence, if n be the number of rivets required; then in bearing $n \times \frac{7}{8} \times \frac{5}{8} \times 8 \text{ tons} = 39{\cdot}7 \text{ tons}$.

$n = 10$ rivets.

For single shearing 11 will be required.

ef. This is a tie consisting of ⅝-inch plates.

The stress in each is 32·5 tons.

Ten rivets, ⅞ inch diameter, are required.

gh. This is a tie composed of two plates, 11 inches × ½ inch.

The stress in each is 24·2 tons.

Seven rivets will be required.

ij. This is a tie consisting of two 11-inch by ½-inch plates. The stress on each is 16·6 tons.

Five rivets will suffice.

de. This is a strut consisting of two pairs of angles, 4½ inches × 4½ inches × ⅝ inch joined together by a ⅜-inch plate.

The stress is 43·3 tons on each pair of angles.

Twelve rivets will be required.

fg. This is also a strut similar to the last.

The stress is 32·5 tons on each pair of angles.

Ten rivets will be required.

hi. This is a strut with a stress of 23·3 tons on each pair of angles.

Seven rivets will be required.

jk, *il.* These are ties with 26·8 tons on each, or 13·4 tons on each plate of the member.

Four rivets will be required per bar.

JOINTS IN MEMBERS.

Flange Plates.—44 feet is about the maximum length of plate obtainable of the size required for the flanges.

In the upper flange, there will be joints in the two longest ⅝-inch plates. These will be made at the centre.

The cover plate will be ⅝ inch thick, and will be arranged so as to be central. The joints of the two plates will both be under the same cover plate, this latter being made long enough to cover both the joints.

Each joint will be a little to one side of the centre. The number of rivets required to transfer the stress from one ⅝-inch plate to the cover is 15, and 16 will be used to fit the rivet rows in the flange. There are four rows of rivets in the flange, and, therefore, the cover plate must extend four pitches of rivets to the outside of the joint of each ⅝-inch plate. Between the two joints of the plates, five pitches of rivets will be enough. This makes a total length of cover plate of 4 feet 4 inches.

The other plates of the upper flange can be got long enough in one piece.

The same calculations for the lower flange must be made. The ⅝-inch plate in the lower flange should have a join at the centre, the cover plate being ⅝ inch thick and 2 feet 8 inches long.

Angles.—These will also have to be joined. It is not convenient to join them at the centre where the joints in the flanges occur. A convenient place for the join appears to be panel 3 in the upper flange and panel 4 in the lower flange. The two portions of angle on either side of the joint will not be equal, but they will be similar in both girders. A suitable cover plate is as follows:—

6½-inch × ¾-inch plate at the back of the 6½-inch face of the angle, and another plate 3½ inches by ¾ inch on the front of the shorter face of the angle.

The maximum safe stress in a 6½-inch by 4½-inch × ⅝-inch angle is 30·28 tons, and the number of ⅞-inch rivets required to take this stress is:—9 rivets; 6 in the larger plate and 3 in the smaller. This concludes the joints.

CONNECTION OF CROSS GIRDERS.

The tie plates of the main girders are carried down the outside of all the members outside the edges of the flange plates. This is done so as to make them available for the attachment of the cross girders to them.

The ends of the cross girders are made of two angles 4½ inches by 4½ inches by ⅝ inch fixed to either side of the web of the cross girder and carried up through the angles of the top flange of the cross girder, for a distance of 15 inches, so as to give extra riveting space for the joint to the tie plates of the main girder.

The web of the cross girder is also carried up with these angles, the top flange plate of the cross girder being stopped 1 foot short of the end, so as to allow the web plate to pass through.

The number of rivets required for this attachment is only 7, *i.e.*, 4 in each vertical angle, of ⅞ inch diameter. These are of course computed at the reduced value of "*f*" to allow for live load on the floor.

Now the rivets of the joint of the tie plates and the flange and the struts of the main girder are already sufficiently used to make it undesirable to load them also with the floor load. The floor weight is therefore taken by the rivets in the upper part of the end angles of the cross girder. Underneath the bottom of the flange of the cross girder a short piece of plate is riveted to the underneath of the flange of the main girder as a support and steadier.

The attachment of the cross girder to the main girder is all the lateral bracing that the latter will be given, and is all that there will be to hold the main girder upright.

WIND BRACING.

This is provided by the floor. The floor troughing is continuous, and being riveted together, is strong against lateral deflection.

For open braced girders of such small span this is quite enough.

JOINTS.

Each joint in the main girder, with the exception of two, requires a gusset plate. This will be $\frac{5}{8}$ inch thick in all cases.

It will be placed between the struts and the vertical face of the $6\frac{1}{2}$-inch × $4\frac{1}{2}$-inch × $\frac{5}{8}$-inch angle of the flanges.

The tie bars are outside this vertical member of the flange angles, and there will have to be a packing piece between the tie bars and the gusset plate, $\frac{5}{8}$ inch thick, where the ties are to be riveted to the gusset plates.

Reference to the drawings in Plate XIII. will explain the arrangement of the rivets in each joint.

DEFLECTION.

This must be calculated by the formula

$$\Delta = \frac{n'fl^2}{Ey}.$$

Here

$$n' = \frac{5}{48}$$

$$E = 13{,}400 \text{ tons}$$

$$f = 7 \text{ tons}$$

$$y = 5 \text{ feet} = 60 \text{ inches.}$$

$$l = 78 \text{ feet} = 936 \text{ inches}$$

$$\therefore \Delta = \frac{5}{48} \times 7 \times \frac{936^2}{13{,}400 \times 60}$$

$$= \cdot 7945 \text{ inches,}$$

which is not excessive.

CAMBER.

At the allowance of $\frac{1}{2}$ inch per 20 feet of span, 2 inches camber should be given.

WEIGHT.

The weight of the girder works out to 16·73 tons.

This is well within the weight estimated for the purposes of calculations.

The discrepancy from the weight given in the table is due to the fact that the table is for plate girders.

CHAPTER XX

SHOP PRACTICE AND GENERAL DETAILS

THE labour in a steel building yard is generally priced under the following headings :—

(1) Drawing ; (2) Template making ; (3) Marking ; (4) Rolling ; (5) Shearing ; (6) Planing ; (7) Punching ; (8) Drilling ; (9) Sawing ; (10) Smith's work ; (11) Plating up ; (12) Riveting (13) Painting.

1. *Drawing.*—The designer's plans are always redrawn by the makers before the work is given out into the shops.

2. *Templates.*—Templates are made for practically every portion of the girder.

The longest plate in the flange is selected and on it are marked all rivet holes, cover plates, beginnings and endings of all additional plates, etc., etc. ; so that this one template conveys all the information for that boom.

Templates are also made for all other portions of the structure, gusset plates, angle brackets, stiffeners, etc.

3. *Marking.*—These templates all go to the marker, who marks the plates and angles and passes them out to the shop.

4. *Rolling.*—All plates are rolled after delivery from the mills, so as to ensure their being properly flat.

5. *Shearing.*—The plates and angles are then cut to size by the marks on them ; they are generally ordered from the mills with ½-inch margin all round the required size which has to be cut out.

6. *Planing.*

7. *Punching.*—The centres of rivet holes are marked by the marker with a punch, the marks being about $\frac{1}{16}$ inch deep.

These are then punched out.

Practically all the material for any work passes through the punching machine.

8. *Drilling.*—This is done after the plating has been begun, so

as to get all the holes that have to be riveted together properly centred, so that they may be drilled in the solid.

9. *Sawing.*

10. *Smith's work.*—This consists of various bends, joggles and odd jobs.

It is expensive, and should be avoided in the design as much as possible.

11. *Plating.*—On leaving the punching machine, the material is taken by the platers, who are responsible for collecting all the pieces of the job and fitting them together.

The work would then be put together and bolted up, and taken to the drill and all the holes drilled out in the solid to the full size.

12. *Riveting.*—It is then riveted up.

The platers are responsible for removing all roughness at the edges of the drilled holes before riveting up.

They also do the scraping and painting, with boiled linseed oil, of all the portions of the metal that will be inaccessible after plating up.

13. *Painting*—Scraping and painting then follow, and the girder or structure is complete and ready for erection.

It is at this period between the scraping and the painting that the engineer's inspection is most important.

RIVETS.

Rivet steel is much softer than plate steel. This is necessary in order to enable the rivet to thoroughly fill the hole and to ensure good work.

Even when holes are drilled in the solid through plates it is rarely that they are absolutely concentric when finally put together for riveting up.

The rivet hole is made $\frac{1}{16}$ inch larger than the rivet.

The reason for this is twofold.

When hot the rivet increases in diameter and will not go into a hole of the exact size; and also if the holes be a little untrue the net diameter of the completed rivet comes about to what was intended in the design.

It is therefore absolutely necessary that when the rivet is put into the hole it should be so compressed that it shall completely fill the hole. Hence the steel must be soft and ductile.

Rivet steel is usually specified to stand a tensile stress of 26 to 30 tons per square inch.

It must also be subjected to a bending test.

A rivet should stand being bent into a U shape, so that the head and tail meet, without showing any cracks at all on the convex side.

RIVET HOLES.

Steel plates are marked with a punch where the centres of holes are to come. These punch marks are made $\frac{1}{16}$ inch deep.

Where several plates are to be riveted together each is thus marked, and then taken to the punching machine.

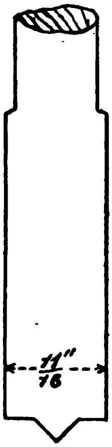


FIG. 299.

Holes are then punched $\frac{1}{4}$ inch less than the finished size, *i.e.*, if $\frac{7}{8}$-inch rivets are shown on drawings, $\frac{15}{16}$-inch holes will be the final size. These will therefore be punched out to $\frac{11}{16}$ inch.

The punch is something similar to that shown in Fig. 299.

Should the workman not have got the plate in the exact correct position under the punch, the centre bit will strike the edge of the punch hole, in the plate and will centre itself.

There is always a certain amount of play in the tool.

After punching, the girders or parts of girders are put together with bolts and nuts, and are then taken to the drilling machine, where the holes through all the plates are drilled out together in the solid to the correct size.

This process of punching first and drilling out afterwards saves cost of construction, and ensures the utmost accuracy in concentricity of holes.

Many engineers still dislike this process and specify nothing but drilled work, fearing the distress of metal caused by punching.

But the distressed portion is all drilled out by the above process and good holes are obtained.

Any divergence from the usual practice of the steel works that get the contract for construction may entail additional cost.

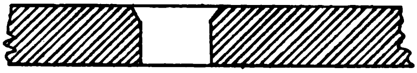


FIG. 300.

The above remarks apply to important structures such as bridges.

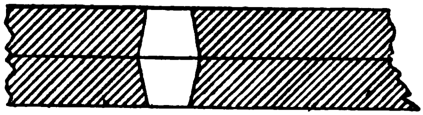


FIG. 301.

For light work of no great importance or where no great strains are expected, plain punched holes are quite good enough and are much cheaper. For example: small tank stands, grille lattice-work for the handrail of a long girder, etc.

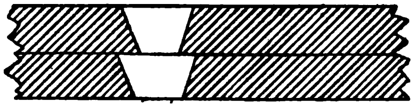


FIG. 302.

Drilled holes have always got a sharp edge, and some engineers specify that this should be rimered down, as shown in Fig. 300. This is, however, not necessary.

Punched holes are never cylindrical, but are similar to the frustrum of a cone (Fig. 302).

If two punched holes are to be riveted together, Fig. 301 is a better arrangement than 302 provided that the rivet fills the hole.

RIVETING.

Riveting may be done by three different processes:—

1. Hand riveting; 2. Machine riveting; 3. Pneumatic riveting.

1. *Hand Riveting.*—This is the least efficient of the three. It takes longer to do, and a rivet gets cold before it can be got to fill up the hole.

The proof of this is that a hand-formed rivet may have its head cut off, and the tail and the shank can then be knocked out with ease from the rivet hole.

In structures such as steel masts, built up of curved plates, where rivets are countersunk on the outside, and have their heads inside, hand riveting is employed, on account of inaccessibility for any other tools. Also in girder erection, where parts, sent separate from the shops, have to be put together on the site, hand rivets will sometimes have to be used.

A hand-made rivet can be told at once by the hammer marks, and generally by the flatness of its head.

Hand-made riveting should be avoided wherever possible.

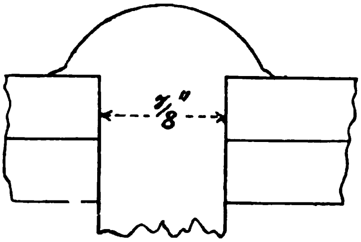


Fig. 303.

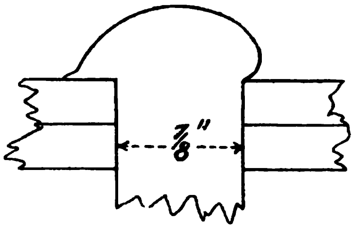


Fig. 304.

2. *Machine Riveting.*—In this process the rivet is pressed by hydraulic pressure into its hole, so that it is compelled to fill the hole.

The head is formed in the same process.

A machine rivet is seldom found to be anything but the best and completely fills its hole irrespective of the number of edges that there may be in that hole.

When the head of a machine-made rivet is cut off, the rivet can rarely be punched out, but must be drilled out like a piece of the solid metal.

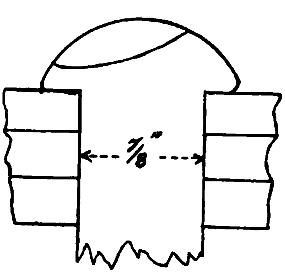


Fig. 305.

This is by far the best riveting. The head is always fair, and usually has a rim round it, as in Fig. 303.

3. *Pneumatic Riveting.*—This is done with pneumatic tools, and is invaluable for use in erection, or work anywhere out of the shops, where heavy hydraulic tools are not available, or convenient, or in inaccessible places.

The rivet is formed by a succession of blows, instead of by sheer pressure; but the results are practically as good as machine riveting. The head is usually marked with a spiral line showing the edge of the snap at the last stroke (Fig. 305).

General Remarks on Rivets.—In riveting up any steel work the parts must first be bolted tightly together; bolts every sixth hole.

As the riveting proceeds the parts immediately in front of the riveters should be drawn tightly together by extra bolts. This ensures contact of the plates, and gets the maximum value out of the contractive or drawing together action of the rivets on cooling.

The length required for a rivet is important and must be accurately gauged for good work.

In calculating out the necessary length, there is of course the length of shank due to the thickness of plates to be riveted up, and then a projection of metal for forming the head.

The portion of the rivet shank that is contained in the hole in the plates is called the "grip," and the projecting part of the shank necessary for forming the head is called the "clink."

The amount of clink required depends on the grip, the number of plates in the grip, the diameter of rivet, and the nature of riveting that is to be employed, since a good deal of the clink is pressed up into the hole in the plates, which is bigger than the rivet.

For machine riveting the length of shank is equal to the length of grip :—

+ 1 inch for the head.
+ $\frac{1}{4}$ inch for a two-plate joint.
+ $\frac{1}{8}$ inch for every additional plate.

This must be accurately ascertained so that rivets of the right sizes can be got and cutting avoided.

Too much clink produces a head rather like Fig. 303, only rather more spreading.

The diameter of the head should be equal to the diameter of the rivet + $\frac{1}{2}$ inch.

For hand riveting ($\frac{7}{8}$ inch rivets) $1\frac{1}{8}$ inch to $1\frac{1}{4}$ inch is enough clink for two-ply joints.

For three-ply joints and over, ¼ inch is added, irrespective of the number of plates.

This is based on experience, and proves that on three-ply joints and over, the rivet metal only fills up the outer portions of the hole, when formed by hand.

The length of a rivet in the grip should not exceed five times its diameter.

Rivets of this length are subjected to a tremendous tensile stress when they cool. Long rivets have been cut out of old work, and are found to have actually passed the elastic limit and elongated, becoming smaller in diameter at their middle.

PITCH.

The pitch of rivets is not always an easy question. It is regulated in nearly every case to a certain extent by some stringent practical requirement.

4 inches is the usual pitch.

As a rule in flanges, 6 inches or 8 inches would suffice for strength, but if rivets be too far apart, the plates buckle between rivets and allow dampness to get in between and corrode the metal. Moreover, it must not be forgotten that in a compression member, rivets add greatly to the stiffness of the member against buckling, by keeping the component plates or bars of the member together so that they act as one member.

As a rule ⅞-inch rivets will be used in flooring and ⅞-inch to 1-inch rivets in main girders.

Larger rivets than 1 inch are very rarely used.

In riveted joints for braced girders it is better to use large rivets, than to increase the thickness of plates to obtain bearing area. Gusset plates are also used to get extra rivets at a joint.

In plate girders it will be found that the bearing area of rivets is nearly always deficient near the abutments. The pitch is nearly always a fixed quantity, and therefore the diameter will have to be increased.

To alter the pitch of rivets at a joint is bad design.

SMITH'S WORK.

Smith's work includes nearly all manual labour; it is both slow and tedious, and runs into a lot of waste of time and expense.

It should be kept as little as possible by the designer, by adhering to plain and straight plates and angles, etc., throughout.

All bends and other odd jobs go to the smiths, who are kept employed at special work.

Small bends in angles can be made cold; large bends are made hot.

With the hydraulic presses now in use a tee or angle steel can be bent inwards through 90° or outwards through 45°, without any cutting of the stem at the bend.

Such bends are very simple.

If curves or bends are to be formed on the end of a long bar, the labour is increased greatly.

SUNK WORK.

Both in large, and more particularly in small plates, sunk work is undesirable. In "sunk work" are included edges of members which are not straight, but have a concave form or re-entrant angle in them; *vide* right hand side of Fig. 306.

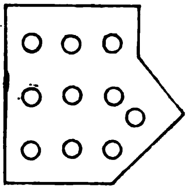


FIG. 306.

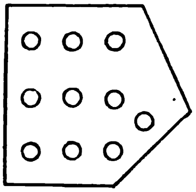


FIG. 307.

The saving of weight is more than covered by the extra expense of labour in cutting the piece out of the plate to waste.

The shearing machine cannot get at these concave edges and they have to be cut by punching a row of holes all round the proposed cut, and then the edge is ground smooth on an emery wheel. The process is slow and expensive.

The appearance of a structure may, however, justify such expense, or if there be many plates of a kind, the saving of weight may demand it.

Compare Figs. 306 and 307; the latter is good.

In the case of large plates with a re-entrant edge two plates can generally be joined together to form the required shape, instead of cutting portions of one whole plate to waste.

CURVED WORK.

Curved edges are also more expensive than straight, even when convex, as they require grinding by emery tools and a lot of additional labour and handling.

INSPECTION.

The inspection of the material is an important matter, but is a subject by itself and apart from the purpose of this short chapter.

But the inspection of the labour of construction and plating is a most important matter and deserves mention.

This inspection usually takes place in the building yard after the structure has been put together complete, except for scraping and painting.

After the full growth of mill scale the whole structure is cleaned bright by scraping, or by the sand blast, and is then ready for inspection. The inspector should be present through all this part of the work until the first coat of paint is set and dry. Too much care cannot be given to this part of the work, forming as it does, the most tedious part from the contractor's point of view.

It, moreover, often happens that the work has got a little behind hand, and somebody is in a hurry and wants to rush through this part of the contract.

The main points to see are:

Cleanliness from mill scale, dirt, rust, etc., before any coat of paint or other skin covering is put on.

Proper fit and thickness of plates.

Riveting.

Inspection of rivet work.

This, like all other inspections, must be carried out before a single dab of paint is put on the structure.

Rivets should be sounded with a light hammer.

To distinguish the sound and feel of a bad rivet, viz., one that does not fill its hole, requires a little practice.

The sound is, however, very similar to that given by any defective piece of metal, and compares with the sound of a good rivet much as the sound of a bad coin does with that of a good one.

RIVET HEADS.

Theoretically, a rivet head should terminate exactly where the snap meets the plate.

But in practice a head like that in Fig. 303 is good.

Too much metal in the head is far better than too little.

An eccentric head is shown in Fig. 304.

It is characterised by a rounded undercut edge on the one side and a flat on the other.

If it is visibly out of line, or appears very wrong on comparison with the template or plans, it should be cut out.

Ruthless cutting out is, however, inadvisable, for the rivet may sometimes have to be replaced by a hand-made one, and it is doubtful whether this latter will be as good as the rejected one.

The metal of the head should touch the plate all round to prevent the entrance of water under the rivet head. This fault is particularly liable to occur when pneumatic riveting tools are used.

It should be impossible to insert the point of a penknife more than $\frac{1}{8}$ inch under the head of the rivet. Of course if the knife can go in $\frac{1}{4}$ inch, it means that the shank of the rivet is exposed. This is bad.

While contractors expect and rely upon a certain amount of inspection, they dislike too much of it.

This is not for any nefarious reason, but simply because too frequent inspection delays the work.

PAINT OR SKIN COVERINGS.

The protection of steel from rust is a question of vital importance. Until lately it has not been studied very much, and the consequent corrosion of metal structures erected only 50 years ago is very large.

By an ordinary natural law, steel and iron when exposed to water, or air laden with water, will oxidise, forming red rust.

This will in time reduce the metal section of the structure to such a size as to render it no longer safe.

The most obvious method of excluding moisture is to provide an absolutely waterproof coating over the metal.

Provided this coating be efficiently maintained all moisture will be excluded.

But this is not in itself enough; the coating must be applied to the steel when it is absolutely clean, so that there is nothing between it and the steel.

Fe_2O_3, which is ordinary red rust, always contains a large percentage of water, and if rust spots on metal are painted over without scraping, the water that is imprisoned in the Fe_2O_3 under the paint will form more Fe_2O_3, liberating a lot of hydrogen.

This H, having a large volume, will break through the paint covering and admit a little more moist air to the affected part. By raising the paint covering it will also increase the surface over which corrosion is going on.

Steel is found to rust rather more than wrought iron, and therefore additional care is required in modern structures.

The metal section of girders, etc., should not be cut too fine but a slight margin should be left for corrosion and loss by scraping.

MILL SCALE.

When steel leaves the mills where it is made it wears a scale known as "mill scale." This mill scale is not very apparent at first, but appears to grow on the surface of the steel, and finally becomes quite loose, flaking off and leaving underneath itself a rusty surface.

It is of no avail to try to hurry this scaling process. To put a coat of paint over it is useless, for the mill scale will break it up again, and the state of things explained above will occur.

The mill scale on the steel is gradually growing all the time while it is being kept stored in the yard or is being worked ready for use, and takes about four to six months to grow and begin flaking off.

If time admits, it is best to allow the steel to wait until all the scale is loose.

It can then be scraped clean from all mill scale and rust, and paint properly applied then will keep the steel from further corrosion.

As a rule, however, time is not available, and the girders or other work, when constructed, are despatched to the site of erection (perhaps abroad) with a great deal of mill scale still ungrown upon them.

In such a case it is wise to apply a coat of boiled linseed oil to the structure while at the yard. Where the steel appears fairly clean and blue, this may be applied without scraping. Where the steel is red rusty, scraping is necessary. This coating merely preserves the metal from excessive oxidisation; it in no way prevents the mill scale from forming and finally breaking up the whole coating of linseed oil. This coating must not be looked upon as anything but a temporary measure during the formation of mill scale.

(It may be noted that a linseed oil coat renders the subsequent scraping of the steel more difficult than it otherwise would be.)

If, however, there is no violent hurry for the work, it is best to allow all mill scale to form while the structure is in the yard. The whole structure can then be cleaned thoroughly, and painted immediately after.

Before painting, the scraping must be most thorough.

The steel should be absolutely bright, without a trace of rust, and the inspection should be most searching.

The painting should be done within a very short time (say one hour, if possible) of the completion of the cleaning process, or rust will begin, due to the moisture of the air. The painting must therefore closely follow the scraping, and the inspector should be present during these processes.

It will be seen that the whole question hangs on the time taken for mill scale to grow. Until it has grown it is no use trying to apply a permanent coat.

It should be specified that the hidden parts of all steel should be scraped and coated with boiled oil before being riveted together, though this is done as a matter of course in all steel yards.

When the riveting is done, the metal contact is so complete and intimate between the metal surfaces that there is no longer any fear of air or moisture getting into the joint. The rivets, however, should not be spaced more than 4 inches to 5 inches apart in order to ensure this contact.

CLEANING.

Cleaning steel is not easy, and scraping is a process that is frequently shirked. It is slow and tedious, but must be thorough.

After mill scale has been roughly scraped off, steel plates can be cleaned by an acid bath. This must be done before the plates are put together. It is an expensive process, but good. This is the process employed by the Admiralty in English dockyards.

SAND BLAST.

Another process for cleaning is the sand blast.

Air at 50 to 60 lbs. pressure is forced through a small nozzle, and sand is blown through with it.

The sand should be sharp and gritty.

The sand blast produces the cleanest surface on the steel, and renders all parts accessible, clean and bright.

For cleaning steel in the building yard, after mill scale has formed, it is most excellent.

The painters should follow the sand blast without the lapse of more than one hour.

The sand blast is also good for use after erection. It is more expensive than scraping, but is more thorough. It is well worth spending a little more per ton on the scraping and painting, if thereby perfection is as nearly as possible attained, and the life of the structure is prolonged.

In painting after erection, some precautions must be taken to avoid the sand blowing on to the newly painted work, etc.

So far it is the difficulties due to mill scale that have been discussed; to summarise them:—

No coating applied before the mill scale has flaked off is of any value as a permanent covering.

No coating applied on steel improperly cleaned is any real use.

The next most important question is, "What paint to use." The word paint has so far been used in a broad sense, meaning any covering that excludes water.

Some coverings wear better than others, some are more impervious, and each has its own peculiarities.

PORTLAND CEMENT.

Cement or concrete forms the best covering for steel that is known. Steel in ferro-concrete construction appears to be practically indestructible provided the concrete is sound and waterproof.

Concrete is very useful on bridge floors. They should be cleaned and covered with concrete all over.

A layer of some waterproof material is then usually applied underneath the ballast.

PAINT.

The whole subject of paint selection is a very difficult one.

The best paint varies with each site.

Experiments at the site of each proposed structure may be useful.

Red lead mixed with boiled linseed oil is the most usual paint for the first coat.

It is impossible to exaggerate the necessity for using the very best quality paint, whatever its nature may be.

The market teems with bad paints and inferior oils, and the utmost care is necessary.

SPECIFICATION.

For a complete specification for structural steel for bridgework, the reader is referred to the publications of the British Standards Committee.

TABLE I.

SAFE WORKING STRESSES ON IRON AND STEEL.

Factor of Safety, 4.

Material.	Transverse Stress f.	Tension f_t.	Compression f_c.	Bearing f_b.	Shearing f_s.	Modulus of Elasticity E.
	Safe Stress in tons per square inch.					lbs. per sq. inch.
Steel	7	$7\frac{1}{2}$	7	8	$5\frac{1}{2}$ to 6	30,000,000
Wrought Iron	6	6	5	6	4 to 5	29,000,000
Cast Iron	$3\frac{1}{2}$	$1\frac{1}{2}$	8	10	$2\frac{1}{2}$	17,000,000

TABLE II.

$$\text{Formula } \Delta = \frac{nWl^3}{EI} = \frac{n'fl^2}{Ey}.$$

n and n' are constants depending upon the method of fixation or support of the beam and method of loading. Values of n and n' will be found on the table below.

W is the total load in same unit as E.

l is the span in inches.

E is the modulus of elasticity of the material to be found on Table I.; E must be in the same unit as W.

I is the moment of inertia of the section.

f is the modulus of rupture of the material, Table I.

y is the maximum distance of the extreme fibre from the neutral axis in inches.

VALUES OF n AND n'.

Arrangement of Beam and Load.	Beam of uniform cross-section throughout its length.	
	Value of n.	Value of n'
Supported at both ends and loaded uniformly	$\frac{5}{384}$	$\frac{5}{48}$
Supported at both ends and loaded at centre	$\frac{1}{48}$	$\frac{1}{12}$
Fixed at both ends and loaded uniformly	$\frac{1}{384}$	$\frac{1}{16}$
Fixed at both ends and loaded at centre	$\frac{1}{192}$	$\frac{1}{24}$
Fixed at one end and loaded at the other (cantilever)	$\frac{1}{3}$	$\frac{1}{3}$
Fixed at one end and loaded uniformly (cantilever)	$\frac{1}{8}$	$\frac{1}{4}$

TABLE III.

ULTIMATE RESISTANCE, ETC., ETC., OF MATERIALS.

Showing the weight per cubic foot and the ultimate resistance of the materials used under stress in building construction, to Tension, Compression, and Shearing, also the Modulus of Rupture and Modulus of Elasticity for each.

Material.	Ultimate Resistance in tons per square inch to							Elastic limit, tons per square inch, ordinary.	*f* Modulus of Rupture in lbs. per square inch.			*E* Modulus of Elasticity in lbs. per square inch.			Weight per cubic foot in lbs. (dry).
	Tension.			Compression.			Shearing.								
	From	To	Ordinary.	From	To	Ordinary.	Ordinary.		From	To	Ordinary.	From	To	Ordinary.	
Cast Iron	5	15	9	20	64	48	8	none stated	—	—	40,000	12,000,000	23,000,000	17,000,000	434 to 456 av. 444
Wrought Iron Bars . .	15	31	25	16	22	20	20	10 to 16	—	—	42,000 for plate girders.	24,000,000	29,000,000	29,000,000	474 to 487 av. 480
Wrought Iron Plates, grain lengthways . .	20	25	21	—	—	—	—	—	—	—	—	—	—	—	—
Wrought Iron Plates, grain crossways . .	19	23	20	—	—	—	—	—	—	—	—	—	—	—	—
Wrought Iron Angle Irons	17	28	24	—	—	—	—	—	—	—	—	—	—	—	—
Structural Steel . .	30	40	32	30	40	30	24	15 to 20	—	—	—	—	—	30,000,000	487 to 493 av. 490
	Ultimate resistance in lbs. per sq. inch.														
	Tension.			Compression.											
	From	To	Ordinary.	From	To	Ordinary.									
Oak, British . . .	5,800	19,300	10,000	3,946	10,000	8,000	—	—	5,260	13,700	10,000	444,680	1,750,000	1,200,000	39 to 69
Fir (red), Baltic . .	2 240	84,000	7,000	2,500	10,000	6,000	—	—	3,683	9,540	6,600	752,420	2,274,000	1,440,000	25 to 48
Pitch Pine . . .	4,200	7,650	5,000	6,090	6,790	6,400	—	—	5,700	15,000	11,000	755,240	2,274,000	1,500,000	31 to 58
American Yellow Pine .	1,800	2,800	2,000	1,570	5,600	4,000	—	—	5,650	9,230	6,300	621,050	1,890,000	1,500,000	26 to 37
Elm, English . . .	4,620	14,000	5,500	5,600	10,300	6,000	—	—	4,320	9,700	6,000	250,830	699,800	600,000	38 to 48
Beech	4,850	22,000	11,000	7,730	9,360	8,000	—	—	9,000	12,000	10,000	1,350,000	1,354,000	1,350,000	39 to 53
Teak	2,590	21,000	10,000	4,240	12,000	9,000	—	—	8,130	19,000	13,000	530,990	2,414,000	1,300,000	41 to 57
Greenheart . . .	7,980	9,380	8,800	12,960	15,540	13,000	—	—	16,360	27,500	20,000	436,880	2,656,400	1,100,000	67 to 73

NOTE.—The figures in the above table are taken from various authorities. Those given in the columns headed "Ordinary" are not averages, but show the figures that may be used for ordinary work. The figures for timber in the columns headed "To" are from experiments upon very small and perfect specimens as given by Barlow, Bevan, Tredgold, etc. Those in the columns headed "From" are from more recent experiments by Laslett, Kirkaldy, etc., on large scantlings.

When nothing special is known about the material to be used, the figures in the columns headed "Ordinary" may be taken. If the timber is known to be very good and well seasoned, and is to be used in small scantlings, free from defects, then higher figures may be used, after consulting the columns headed "To," which, however, are very extreme. If the material is likely to be inferior, unseasoned, and to be used in scantlings full of defects, smaller figures approximating to those in the columns headed "From" may be used. In the column for weight of timber, the higher figures are for unseasoned, the lower for well-seasoned material.

TABLE IV.

Values of *n* for use with Table V for Struts.

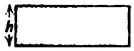
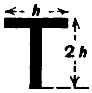
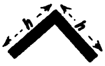
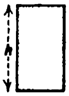
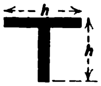
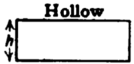
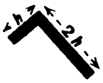
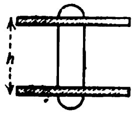
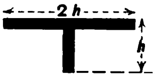
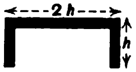
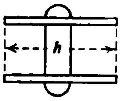
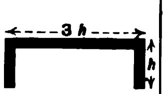
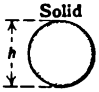
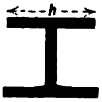
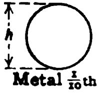
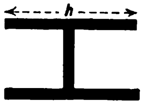
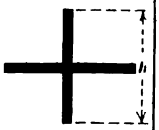
Section	*n*	Section	*n*	Section	*n*
	3·5		6·0		4·9
	2·0		4·0		
Hollow  Metal $\frac{1}{10}$th. b	2·5				3·9
	3·0		4·2		3·8
	2·0				4·3
Solid 	4·0	 Area of Web = $\frac{1}{2}$,, of Flanges	3·0		
Hollow  Metal $\frac{1}{10}$th. h	3·1	 Area of Web = $\frac{1}{4}$,, of Flanges	4·2		4·0

TABLE V.

SHOWING STRENGTH OF COLUMNS OF CAST IRON, WROUGHT IRON, AND STEEL.

Here l = length of strut between ends in inches.

k = least radius of gyration, *i.e.*, the square root of "Moment of Inertia ÷ area of cross section."

f_c = safe stress in tons per square inch.

$\frac{l}{k}$	f_c.					
	Cast Iron.		Wrought Iron.		Mild Steel.	
	Ends Rounded.	Ends Fixed.	Ends Rounded	Ends Fixed.	Ends Rounded.	Ends Fixed.
10	8·68	8·85	4·00	4·00	5·33	5·34
15	8·41	8·76	3·98	4·00	5·26	5·31
20	8·07	8·65	3·92	3·99	5·20	5·29
25	7·58	8·46	3·88	3·98	5·13	5·24
30	6·98	8·21	3·80	3·95	5·02	5·20
35	6·32	7·91	3·72	3·92	4·90	5·15
40	5·68	7·56	3·64	3·89	4·76	5·09
45	5·02	7·19	3·54	3·86	4·58	5·03
50	4·43	6·82	3·44	3·82	4·40	4·98
55	3·84	6·46	3·31	3·78	4·22	4·92
60	3·35	6·10	3·17	3·73	4·02	4·83
65	2·92	5·75	3·04	3·68	3·80	4·75
70	2·57	5·39	2·90	3·65	3·59	4·67
75	2·23	5·02	2·76	3·55	3·37	4·56
80	1·96	4·68	2·60	3·48	3·15	4·45
85	1·74	4·33	2·46	3·40	2·96	4·35
90	1·56	4·00	2·33	3·32	2·77	4·25
95	1·42	3·66	2·18	3·22	2·56	4·13
100	1·29	3·35	2·03	3·17	2·40	4·00
105	1·17	3·07	1·92	3·08	2·24	3·88
110	1·07	2·80	1·79	3·00	2·08	3·74
115	0·99	2·57	1·67	2·91	1·95	3·61
120	0·93	2·37	1·57	2·82	1·83	3·46
125	0·86	2·19	1·47	2·74	1·71	3·32
130	0·80	2·03	1·39	2·66	1·61	3·21
135	0·75	1·90	1·32	2·58	1·50	3·09
140	0·70	1·78	1·24	2·48	1·42	2·96
145	0·66	1·66	1·17	2·40	1·36	2·85
150	0·61	1·56	1·10	2·32	1·28	2·72
160	0·56	1·40	0·98	2·14	1·13	2·51
170	0·49	1·25	0·88	2·00	1·01	2·32
180	0·43	1·14	0·80	1·84	0·91	2·13
190	0·39	1·03	0·72	1·70	0·83	1·97
200	0·36	0·93	0·66	1·57	0·75	1·83
210	0·32	0·84	0·58	1·46	0·68	1·68
220	0·30	0·77	0·55	1·35	0·62	1·55
230	0·28	0·70	0·50	1·26	0·58	1·44
240	0·25	0·64	0·46	1·18	0·53	1·34
250	0·23	0·59	0·42	1·11	0·49	1·25
260	0·22	0·56	0·40	1·04	0·46	1·16
270	0·20	0·52	0·37	0·97	0·42	1·08
280	0·19	0·49	0·35	0·91	0·39	1·01
290	0·18	0·46	0·32	0·86	0·37	0·96
300	0·17	0·43	3·30	0·80	0·35	0·92

For practical purposes the value of $\frac{l}{k}$ can be found with sufficient accuracy from $\frac{l}{k} = n\,\frac{l}{h}$ where n and h are taken from Table IV. For accurate work k can be found from Table VI., and $\frac{l}{k}$ calculated direct. This table will then give f_c.

TABLE VI.

Showing Value of k (the Radius of Gyration) for Various Shapes of Cross Section.

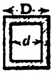
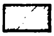
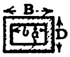
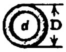
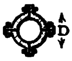
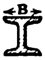
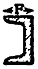
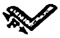
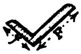
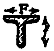
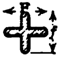
	Shape of Cross Section	Least Radius of Gyration	Remarks
	Solid square	$\frac{\text{Side}}{\sqrt{12}}$	$\sqrt{12} = 3{\cdot}46$
	Hollow square (thickness uniform)	$\sqrt{\frac{D^2+d^2}{12}}$	
	Solid rectangle	$\frac{\text{Least side}}{\sqrt{12}}$	
	Hollow rectangle (thickness uniform)	$\sqrt{\frac{BD^3-bd^3}{12\,(BD-bd)}}$	
	Solid circle	$\frac{\text{Diameter}}{4}$	
	Hollow circle (thickness uniform)	$\sqrt{\frac{D^2+d^2}{16}}$	
	Phœnix column	D × 0·3636	
	I beam	$\frac{B}{4{\cdot}58}$	
	Channel iron	$\frac{F}{3{\cdot}54}$	
	Angle with equal legs	$\frac{F}{5}$	
	Angle with unequal legs	$\frac{Ff}{2{\cdot}6\,(F+f)}$	
	T with F=f	$\frac{F}{4{\cdot}74}$	
	Cross with F=f		

TABLE VII.

Values of "a" for Gordon's Formula for Struts.

Factor of Safety = 4.

	Shape of Cross Section	Values of a			f_c
		ends free	ends fixed	one end free one fixed	
Timber	Rectangular □	$\frac{1}{60}$	$\frac{1}{250}$	$\frac{1}{140}$	1450 lbs.
	Circular ○	$\frac{1}{60}$	$\frac{1}{250}$	$\frac{1}{140}$	,, ,,
Cast Iron	Circular Solid ○	$\frac{1}{100}$	$\frac{1}{400}$	$\frac{1}{160}$	8 tons
	,, Hollow ◯	$\frac{1}{200}$	$\frac{1}{800}$	$\frac{1}{320}$	,, ,,
	Rectangular Solid ▨	$\frac{3}{400}$	$\frac{3}{1600}$	$\frac{1}{210}$	,, ,,
	,, Hollow ▣	$\frac{3}{440}$	$\frac{3}{1760}$	$\frac{1}{232}$	,, ,,
	Cross Shape ✚	$\frac{3}{200}$	$\frac{3}{800}$	$\frac{3}{320}$	,, ,,
Wrought Iron	Rectangular □ Circular Solid ○ ,, Hollow ◯	$\frac{4}{2500}$	$\frac{1}{2500}$	$\frac{1}{1000}$	4 ,,
	Other Shapes L T ✚ ▣ ‡ I ⊔	$\frac{4}{900}$	$\frac{1}{900}$	$\frac{1}{360}$	,, ,,
Steel; Mild Structural	Rectangula. □	$\frac{1}{600}$	$\frac{1}{2000}$	$\frac{1}{1000}$	7½ ,,
	Circular ○	$\frac{1}{350}$	$\frac{1}{1400}$	$\frac{1}{775}$	,, ,,
	,, Hollow ◯	$\frac{1}{635}$	$\frac{1}{2500}$	$\frac{1}{1400}$	,, ,,
	L, T ⅂ L ⊔ ▣ I, Ⅱ, etc.	$\frac{1}{250}$	$\frac{1}{1000}$	$\frac{1}{550}$	,, ,,
Hard Steel	Rectangular □	$\frac{1}{350}$	$\frac{1}{1400}$	$\frac{1}{775}$	12½ ,,
	Circular ○	$\frac{1}{225}$	$\frac{1}{900}$	$\frac{1}{500}$	,, ,,
	,, Hollow ◯	$\frac{1}{375}$	$\frac{1}{1500}$	$\frac{1}{1850}$	,, ,,

$$F_c = \frac{f_c \times A}{1 + a\left(\frac{l}{d}\right)^2}$$

Where F_c = Total safe pressure in lbs. or tons.
f_c = Safe resistance to crushing in lbs. or tons.
A = Cross sectional area in square inches.

Where l = Length of bar in same unit as d.
d = Least transverse dimension of the bar.
a = Constant from above table.

TABLE IX.

STRENGTH OF RIVETS.

Rivet.	Shear. (6 tons per sq. inch.)		Resistance to bearing of *one* rivet in plates of various thicknesses—at 8 tons per square inch.							
	Single.	Double [1]	1/16	1/4"	3/8"	1/2"	5/8"	3/4"	7/8"	1"
5/8"	1·84	3·22	0·31	1·25	1·87	2·50	3·12	3·75	4·37	5·00
3/4"	2·65	4·64	0·37	1·50	2·25	3·00	3·75	4·50	5·25	6·00
7/8"	3·6	6·3	0·44	1·75	2·62	3·50	4·37	5·25	6·12	7·00
1"	4·7	8·2	0·50	2·00	3·00	4·00	5·00	6·00	7·00	8·00

Rivet.	Shear at 6·5 tons per sq. inch.		Bearing. (At 10 tons per square inch.)					
Diameter.	Single.	Double.	1/4"	3/8"	1/2"	5/8"	3/4"	7/8"
5/8"	1·99	3·47	1·562	2·34	3·13	3·91	—	—
3/4"	2·87	5·03	1·875	2·81	3·75	4·69	5·63	—
7/8"	3·91	6·84	2·187	3·28	4·38	5·47	6·56	7·66
1"	5·10	8·93	—	3·75	5·00	6·25	7·5	8·75

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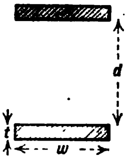
[1] In this case double shear is taken at 1¾ times single shear.



Distance between plates in the clear in inches.	1⅞″	2″	2⅛″	2¼″	2⅜″	2½″
6	9·24	65·33				
6½	6·85	73·58				
7	4·94	82·33				
7½	3·49	91·58				
8	2·52	101·33				
8½	2·01	111·58				
9	1·97	122·33	133·10	144·28		
9½	2·40	133·58	145·18	157·22		
10	3·30	145·33	157·80	170·72		
10½	4·66	157·58	170·95	184·78		
11	6·50	170·33	184·63	199·40		
11½	8·81	183·58	198·84	214·59		
12	1·58	197·33	213·58	230·34	247·62	265·42
12½	4·82	211·58	228·86	246·65	264·98	283·85
13	8·53	226·33	244·66	263·53	282·95	302·92
13½	2·71	241·58	261·00	280·97	301·50	322·60
14	7·36	257·33	277·86	298·97	320·65	342·92
14½	2·48	273·58	295·26	317·53	340·39	363·85
15	8·06	290·33	313·19	336·65	360·72	385·42
15½	4·12	307·58	331·65	356·34	381·66	407·60
16	0·64	325·33	350·64	376·60	403·18	430·42
16½	7·63	343·58	370·17	397·40	425·30	453·85
17	5·10	362·33	390·22	418·78	448·01	477·92
17½	3·02	381·58	410·81	440·72	471·31	502·60
18	1·42	401·33	431·93	463·22	495·21	527·92
18½	0·29	421·58	453·57	486·28	519·70	553·85
19	9·63	442·33	475·75	509·90	544·79	580·42
19½	9·43	463·58	498·47	534·09	570·47	607·60
20	9·71	485·33	521·71	558·84	596·74	635·42
20½	0·45	507·58	545·48	584·15	623·61	663·85
21	1·66	530·33	569·79	610·03	651·07	692·92
21½	3·34	553·58	594·62	636·47	679·12	722·60
22	5·49	577·33	619·99	663·47	707·77	752·92
22½	8·10	601·58	645·89	691·03	737·02	783·85
23	1·19	626·33	672·32	719·15	766·85	815·42
23½	4·74	651·58	699·28	747·84	797·28	847·60
24	8·77	677·33	726·77	777·10	828·31	880·42

TABLE X.

Safe Loads in Tons for Single T Struts (Steel).

Free ends.

Length. Ft.	Ins.	Inches. 2¼ × 2¼ × ¼	Inches. 2½ × 2½ × ¼	Inches. 3 × 3 × ⅜	Inches. 3½ × 3½ × ⅜	Inches. 4 × 4 × ½	Inches. 4 × 5 × ½
4	0	3·25	4·00				
	3	3·06	3·80				
	6	2·86	3·60	7·53			
	9	2·70	3·40	7·20			
5	0	2·53	3·22	6·88	9·14		
	3	2·39	3·04	6·58	8·85		
	6	2·25	2·88	6·27	8·50		
	9	2·12	2·73	5·97	8·18		
6	0	2·00	2·59	5·72	7·86	13·39	
	3		2·46	5·44	7·59	12·94	
	6		2·32	5·19	7·29	12·56	
	9			4·98	6·99	12·10	
7	0			4·78	6·72	11·70	12·85
	3			4·56	6·42	11·32	12·40
	6			4·37	6·20	10·91	11·94
	9				5·98	10·54	11·48
8	0				5·73	10·17	11·14
	3				5·50	9·79	10·67
	6				5·31	9·41	10·25
	9				5·13	9·15	9·95
9	0				4·91	8·85	9·60
	3					8·55	9·31
	6					8·25	8·97
	9					8·00	8·63
10	0					7·75	8·37

Note.—For thicker sections of same external dimensions as the above, increase the load proportionately to the area. For rivet holes deduct proportionately to the area.

TABLE XI.

Beams of Uniform Section.

Method of loading.	Value of *Max.* M_B.		Position of *Max.* M_B.		Deflection.		Position of points of contraflexure.
	Fixed.	Supported.	Fixed.	Supported.	Fixed.	Supported.	
Load W in centre .	$\frac{Wl}{8}$	$\frac{Wl}{4}$	Centre and at fixing	Centre	$\frac{1}{192}\frac{Wl^3}{EI}$	$\frac{1}{48}\frac{Wl^3}{EI}$	$\frac{1}{4}l$ from end.
Load W uniform .	$\frac{Wl}{12}$	$\frac{Wl}{8}$	At fixing	Centre	$\frac{1}{384}\frac{Wl^3}{EI}$	$\frac{5}{384}\frac{Wl^3}{EI}$	·211l from end.
Load W centre and W uniform . .	$\frac{5}{24}Wl$	$\frac{3}{8}Wl$	”	Centre	$\frac{3}{384}\frac{Wl^3}{EI}$	$\frac{13}{384}\frac{Wl^3}{EI}$	·23l from end.

TABLE XII.

CONTINUOUS BEAMS.

W = Distributed Load on One Span. l = Length of One Span.

No. of Spans.	Reactions at Supports (Abutments or Piers).						Moments at Supports.						Shearing Stresses at Support on Either Side.					
Supports.	1st.	2nd.	3rd.	4th.	5th.	6th.	1st.	2nd.	3rd.	4th.	5th.	6th.	1st.	2nd.	3rd.	4th.	5th.	6th.
2	$\frac{3}{8}W$	$\frac{10}{8}W$	$\frac{3}{8}W$	—	—	—	0	$\frac{Wl}{8}$	0	—	—	—	$0 \mid \frac{3}{8}W$	$\frac{5}{8} \mid \frac{5}{8}W$	$\frac{3}{8}W \mid 0$	—	—	—
3	$\frac{4}{10}W$	$\frac{11}{10}W$	$\frac{11}{10}W$	$\frac{4}{10}W$	—	—	0	$\frac{Wl}{10}$	$\frac{Wl}{10}$	0	—	—	$0 \mid \frac{4}{10}W$	$\frac{6}{10} \mid \frac{5}{10}W$	$\frac{5}{10} \mid \frac{6}{10}W$	$\frac{4}{10}W \mid 0$	—	—
4	$\frac{11}{28}W$	$\frac{32}{28}W$	$\frac{26}{28}W$	$\frac{32}{28}W$	$\frac{11}{28}W$	—	0	$\frac{3Wl}{28}$	$\frac{Wl}{14}$	$\frac{3Wl}{28}$	0	—	$0 \mid \frac{11}{28}W$	$\frac{17}{28} \mid \frac{15}{28}W$	$\frac{13}{28} \mid \frac{13}{28}W$	$\frac{15}{28} \mid \frac{17}{28}W$	$\frac{11}{28}W \mid 0$	—
5	$\frac{15}{38}W$	$\frac{43}{38}W$	$\frac{37}{38}W$	$\frac{37}{38}W$	$\frac{43}{38}W$	$\frac{15}{38}W$	0	$\frac{4Wl}{38}$	$\frac{3Wl}{38}$	$\frac{3Wl}{38}$	$\frac{4Wl}{38}$	0	$0 \mid \frac{15}{38}W$	$\frac{23}{38} \mid \frac{20}{38}W$	$\frac{18}{38} \mid \frac{19}{38}W$	$\frac{19}{38} \mid \frac{18}{38}W$	$\frac{20}{38} \mid \frac{23}{38}W$	$\frac{15}{38}W \mid 0$

TABLE XIII

Weight of plate girders. span "end to end of Girders"

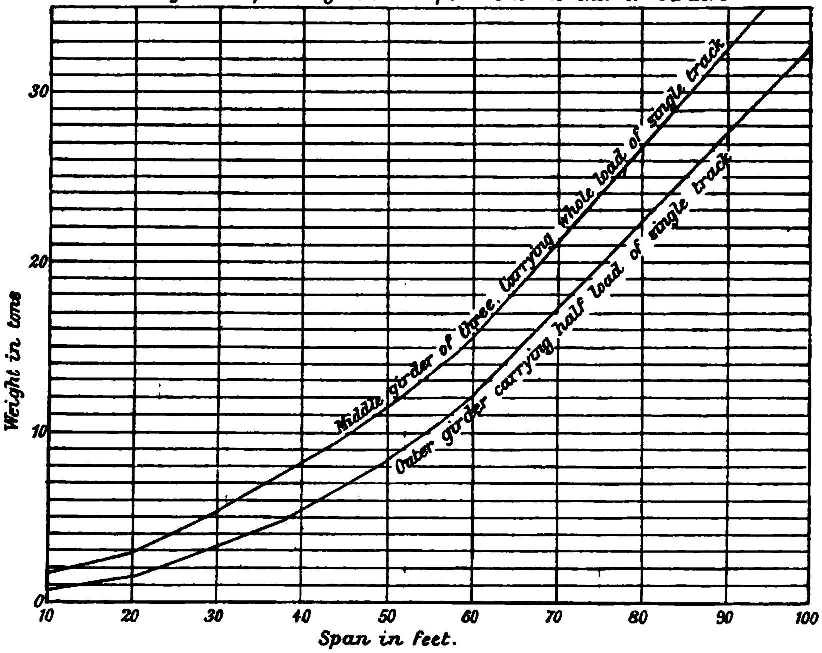
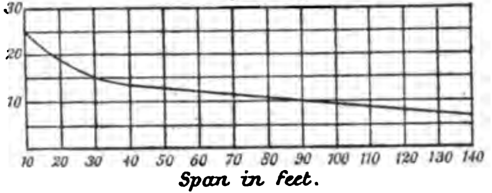


TABLE XIV

Percentage addition for live loads on Railway Bridges.



For cross girders allow 100%
For main girders allow above percentages.

TABLE XV
TABLE OF PERCENTAGES

For correction of live load on Road Bridges for good macadam road or wood block.

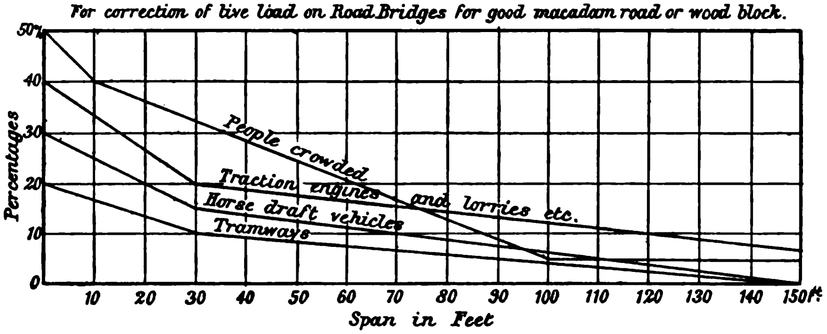


TABLE XVI.

UNIFORM EQUIVALENT LIVE LOADS FOR HEAVIEST LOCOMOTIVES ON SINGLE LINE TRACK.

Span in feet.[1]	Tons per foot of Span.
10	3·99
15	3·72
20	3·44
25	3·35
30	3·28
35	3·06
40	2·88
45	2·67
50	2·60
55	2·50
60	2·47
65	2·44
70	2·41
75	2·38
80	2·35
85	2·32
90	2·30
100	2·29
120	2·28
140	2·27
160	2·26
180	2·25
200	2·25

---

[1] The span above is the effective span between bearing centres. The table is based on the rolling stock of the G.W.R., England, 1911.

TABLE XVII.

STANDARD EQUIVALENT UNIFORM LOAD IN TONS PER FOOT RUN OF SINGLE TRACK (for use in calculating Bending Moments).

Span.	5 feet 6 inches Gauge.	Metre Gauge.
5	9·0	5·0
10	4·5	3·313
15	4·184	2·750
20	3·775	2·463
25	3·455	2·285
30	3·225	2·166
35	3·05	2·071
40	2·919	2·00
45	2·818	1·945
50	2·738	1·900
55	2·673	1·859
60	2·616	1·823
65	2·570	1·789
70	2·529	1·759
75	2·493	2·735
80	2·463	1·711
90	2·408	1·660
100	2·360	1·613
110	2·313	1·564
120	2·270	1·523
130	2·235	1·485
140	2·200	1·454
150	2·166	1·425
200	2·021	1·319

NOTE.—The span given in the first column is the effective span, *i.e.*, between the centres of bearings.

TABLE XVIII.

Table of Weights of Miscellaneous Substances,

LBS. PER FOOT CUBED.

Asphalte	156 lbs.
Bitumen solid	87 ,,
Brickwork pressed	150 ,,
,, ordinary common	125 ,,
,, soft	100 ,,
Bricks in mortar, best pressed	140 ,,
,, facing bricks	130 ,,
,, common stocks	100 to 125 ,,
Coal, ordinary loose	56 ,,
Concrete (Breeze)	100 ,,
,, ordinary	112 to 125 ,,
,, reinforced (includes steel)	144 ,,
Cast iron	450 ,,
Earth, loose dry	72 to 80 ,,
,, slightly rammed compact	90 to 100 ,,
Wet flowing mud	110 ,,
Gravel	70 to 80 ,,
Lead	712 lbs.
Rocks, broken and piled, the bulk increasing from 1 to 1¾. ¾ being voids:	
Granite gneiss	98 lbs. per ft.
Quartz	96 ,,
Sandstone	86 ,,
Shale	93 ,,
Limestone	98 ,,
Shingle ballast	88 lbs.
Sand, Thames	102 ,,
,, pit sharp	100 ,,
,, dry loose river	50 to 70 ,,
,, rammed	100 to 120 ,,
,, very fine	117 ,,
Steel rolled sections	490 ,,
Turf	20 to 30 ,,
Gas tubing:—	
¾ inch	1·17 foot run
1 inch	1·68 ,,
1¼ inches	2·46 ,,
1½ inches	3·14 ,,
Masonry:—	
Granite or limestone well dressed	165 lbs.
,, scabbled rubble mortar	154 ,,
,, average	160 ,,
,, dry rubble	138 ,,
Sandstone rubble	140 ,,
Mud, dry close	80 to 110 ,,
,, wet pressed	110 to 130 ,,
,, fluid	104 to 120 ,,

TABLE XIX.

Live Load on Floors.

Bridge floors, crowd of people	84 to 112 lbs. per foot sup.
Floors of private dwellings, offices	56 to 112 lbs. per foot.
Public halls, churches, theatres	100 to 160 lbs. per foot sup.
Floors of stores, warehouses, etc.	100 to 300 lbs. per foot sup.
,, workshops	200 to 400 lbs. per foot sup.

TABLE XX.

Table of Road Bridge Live Loads.

Vehicle.	Length O. A.	Width O. A.	Wheel Base.	Track.	Total Weight.	Weight Back Axle.	Width of Tyre. Front.	Width of Tyre. Back.	Diam. of Wheel. Front.	Diam. of Wheel. Back.	Remarks.
	Ft. Ins.	Ft. Ins.	Ft. Ins.	Ft. Ins.	Tons.	Tons.	Ft. Ins.	Ft. Ins.	Ft. Ins.	Ft. Ins.	
Steam Roller . . .	21 0	9 0	12 7	8 6	20	13½	5 4	2 0	4 6	6 9	J. Fowler & Co.
" " . . .	20 0	8 0	12 0	7 6	18	12	4 8	1 8	4 6	5 6	" " "
Ploughing Engine .	23 0	10 0	15 0	9 7	22	16	1 4	2 2	4 11	6 6	" " "
Road " . .	18 4	7 8	10 2	7 2	19	14¼	0 9	1 4	4 6	7 0	" " "
" " . .	20 0	8 6	11 6	8 0	16¼	12¼	0 9	1 6	4 0	7 0	Aveling & Porter.
Stone Waggon . .	—	6 8	9 0	—	16	8	—	—	—	—	Government of India.
Boiler " . .	12 0	8 0	8 0	7 10	32	16	1 8	1 8	3 0	3 0	L.C.C., London.
Wagon loaded . .	21 3	—	10 2	6 4	13·25	7·0	0 9	0 9	3 4	4 0	Allows for load of 9·25 tons in waggon.
Commercial Lorry . .	—	—	14 6	7 5	10·35	6·85	—	—	—	—	Commercial Cars, Ltd.
Trailer to Lorry, loaded .	—	—	10 6	—	4·5	2·35	—	—	—	—	" " "
Tramcars, L.C.C., bogie, double deck.	33 1	7 1	14 6	4 8½	18·5	5·14	—	—	2 8	2 8	L.C.C., 2 bogies, centres 10′ 6″ apart.
Tramcars, single deck, bogie.	35 0	7 0	17 0	5 0	15	6	—	—	—	—	Indian.
Tramcars, double deck .	35 0	7 0	16 0	5 0	17·75	6·5	—	—	—	—	"
4·7 gun on trav. carriage	—	6 7	10 2	5 2	5·1	4·3	—	—	—	—	
Elephants, loaded . .	11 0	9 0	6 6	—	3·6	2·2 1 leg	—	—	—	—	

TABLE XXI.

Dead Loads on Railway Bridge Floors.

Ballast	110 lbs. per foot cube.
Rails, fish plates, joints, etc., per foot run of single track	100 lbs. per foot run.
Rails, fish plates, joints and guard rails, per foot run of single track . .	160 lbs.
Track on sleepers 2 feet apart (no ballast), with fish plates, chairs, etc. (100-lb. rails), per foot of single track . .	168 lbs.
Ditto with longitudinal sleepers per foot run of track	168 lbs.

TABLE XXII.

Maximum Dimensions to which Steel Plates are Rolled.

Thickness.	Length in feet.	Width in inches.	Area in feet.
$\frac{1}{8}''$	14	48	48
$\frac{3}{16}''$	20	60	80
$\frac{1}{4}''$	26	72	100
$\frac{5}{16}''$	32	75	150
$\frac{3}{8}''$	36	81	235
$\frac{7}{16}''$	44	81	235
$\frac{1}{2}''$	44	81	235
$\frac{9}{16}''$	44	81	235
$\frac{5}{8}''$	44	81	235
$\frac{11}{16}''$	44	81	235
$\frac{3}{4}''$	44	81	235
$\frac{13}{16}''$	44	81	220
$\frac{7}{8}''$	44	81	200
$\frac{15}{16}''$	44	81	185
$1''$	44	81	175
$1\frac{1}{16}''$	44	81	165
$1\frac{1}{8}''$	44	81	155
$1\frac{1}{4}''$	44	81	138
$1\frac{3}{8}''$	44	81	125
$1\frac{1}{2}''$	44	81	115

$\frac{1}{8}''$ plates are rolled full to the gauge and about 6 lbs. to the square foot.

Note.—The area given in the last column is the limiting figure: for instance, if a $1\frac{1}{2}''$ plate is required 44′ long, it can only be 31″ wide; or if the plate must be 4′ wide, it can then only be 28′ long.

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